KAZAKH BRITISH TECHNICAL UNIVERSITY

Physics 1

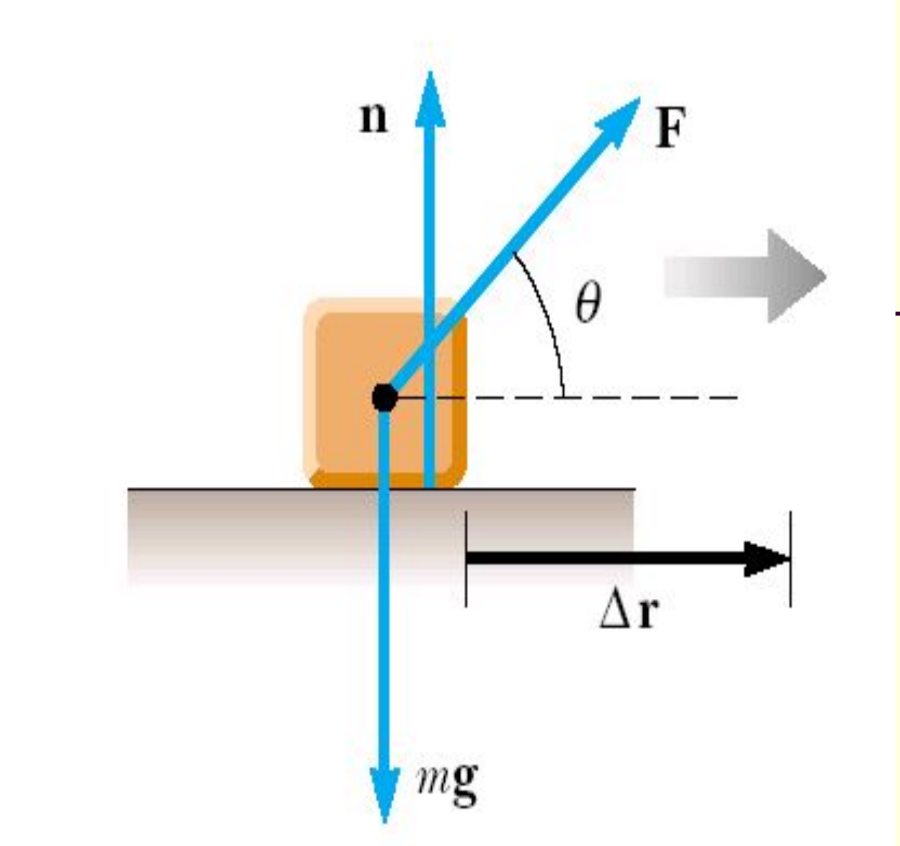
Lecture 3

- Work, energy and power
- Conservation of energy
- Linear momentum.
- Collisions.

Work

- A force acting on an object can do work on the object when the object moves.

$$W \equiv F \Delta r \cos\theta$$



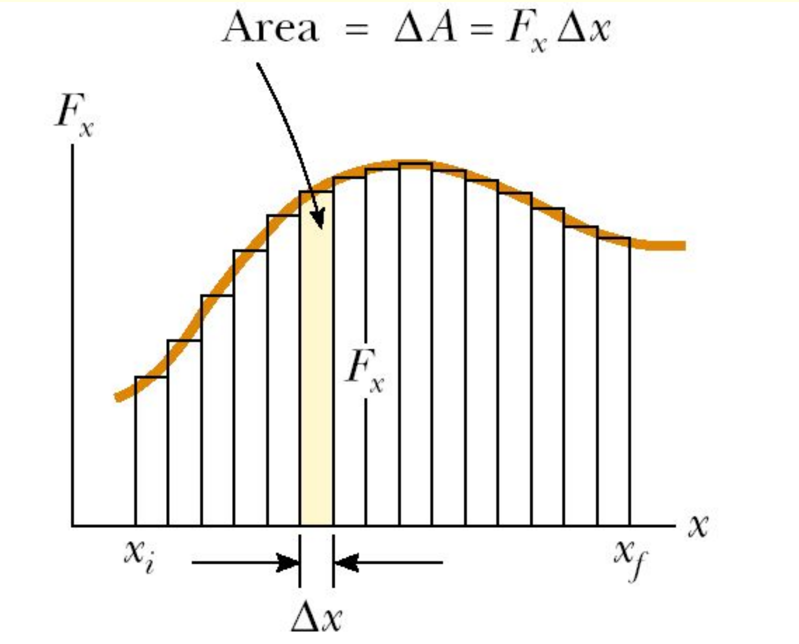
$$W \equiv F \Delta r \cos \theta$$

When an object is displaced on a frictionless, horizontal surface, the normal force n and the gravitational force *m*g do no work on the object. In the situation shown here, F is the only force doing work on the object.

Work Units

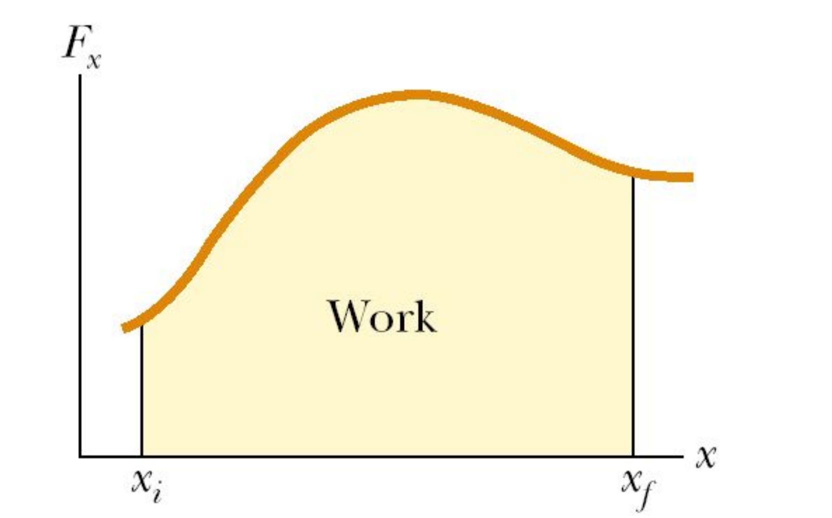
- Work is a scalar quantity, and its units are force multiplied by length. Therefore, the SI unit of work is the newton • meter (N • m). This combination of units is used so frequently that it has been given a name of its own: the joule (J).

Work done by a varying force



$$W \approx \sum_{x_i}^{x_f} F_x \, \Delta x$$

$$\lim_{\Delta x \to 0} \sum_{x_i}^{x_f} F_x \, \Delta x = \int_{x_i}^{x_f} F_x \, dx$$

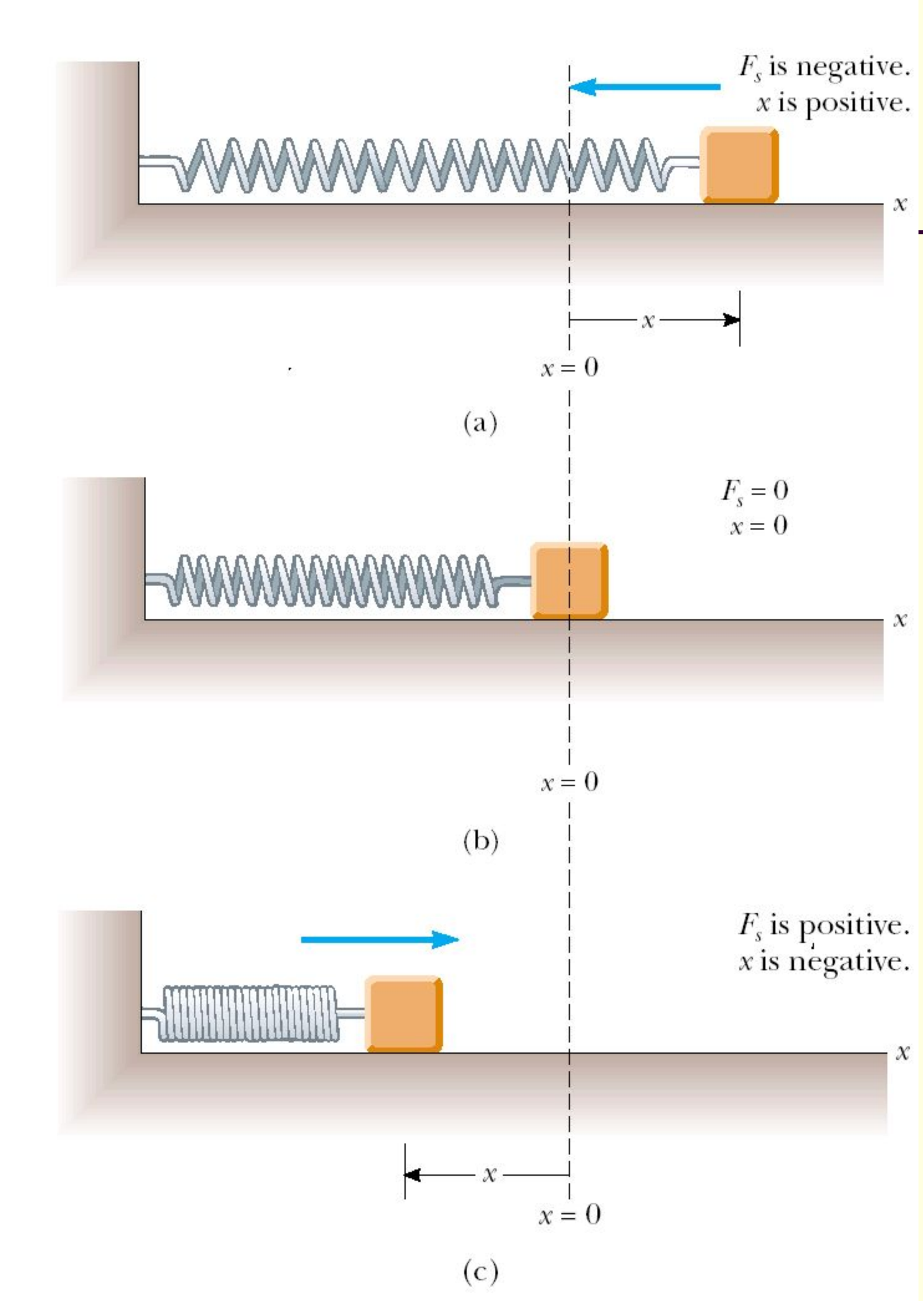


$$W = \int_{x_i}^{x_f} F_x \, dx$$

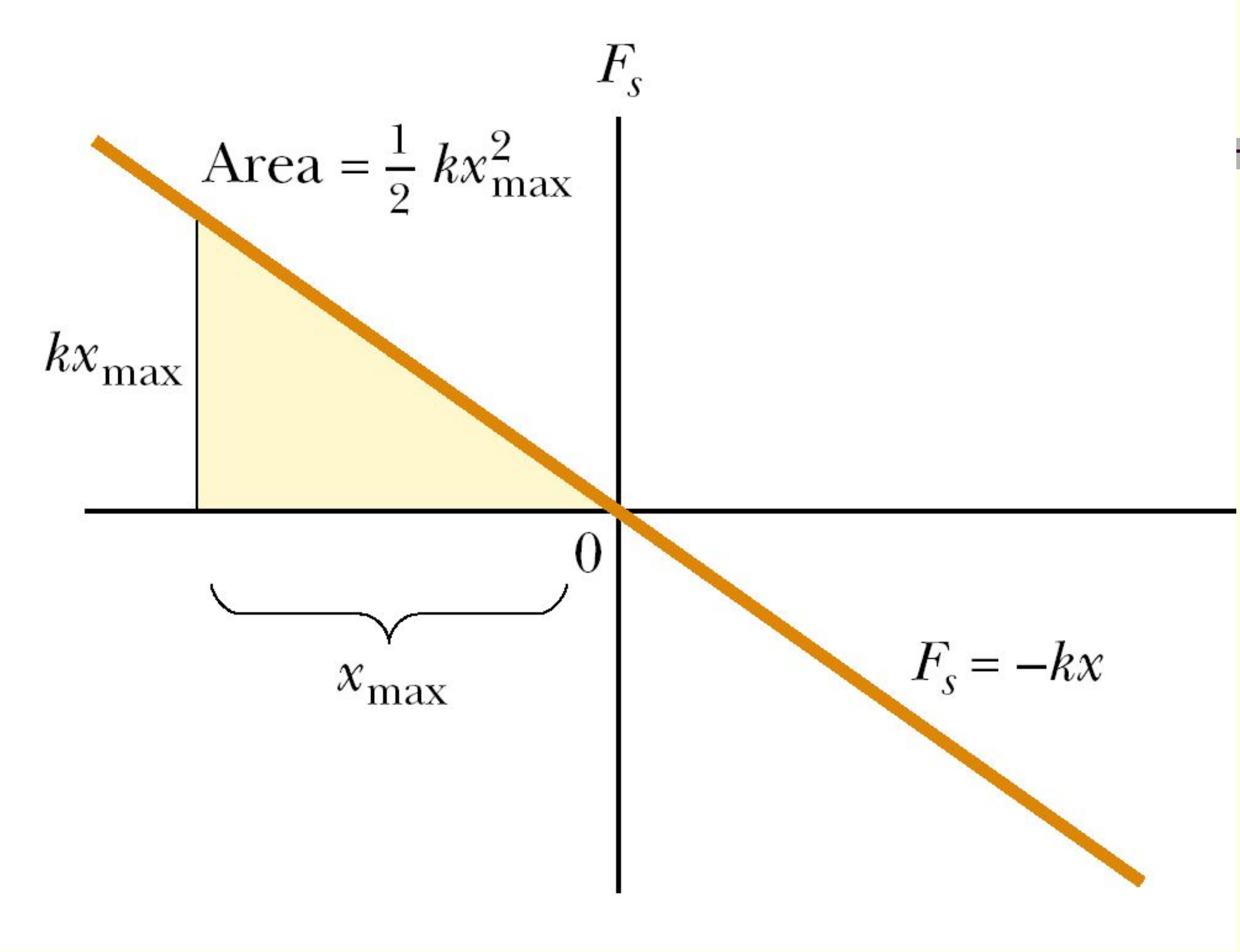
Work done by a spring

- If the spring is either stretched or compressed a small distance from its unstretched (equilibrium) configuration, it exerts on the block a force that can be expressed as

$$F_s = -kx$$



F_s is negative.
x is positive.
x
x
x = 0
(a)
F_s = 0
x = 0
x
x = 0
(b)
F_s is positive.
x is negative.
x
x
x = 0
(c)


F_s
Area = $\frac{1}{2}kx^2_{max}$
kx_{max}
0
x_{max}
$F_s = -kx$

Work of a spring

- So, the work done by a spring from one arbitrary position to another is:

$$W_s = \int_{x_i}^{x_f} (-kx)\,dx = \frac{1}{2}k{x_i}^2 - \frac{1}{2}k{x_f}^2$$

Kinetic energy

- Work is a mechanism for transferring energy into a system. One of the possible outcomes of doing work on a system is that the system changes its speed.
- Let's take a body and a force acting upon it:

$$\sum W = \int_{x_i}^{x_f} \sum F dx$$

- Using Newton's second law, we can substitute for the magnitude of the net force

$$\sum F = ma$$

- and then perform the following chain-rule manipulations on the integrand:

$$\sum W = \int_{x_i}^{x_f} ma\,dx = \int_{x_i}^{x_f} m\,\frac{dv}{dt}\,dx = \int_{x_i}^{x_f} m\,\frac{dv}{dx}\,\frac{dx}{dt}\,dx = \int_{v_i}^{v_f} mv\,dv$$

- And finally:

$$\sum W = \frac{1}{2}m{v_f}^2 - \frac{1}{2}m{v_i}^2$$

- This equation was generated for the specific situation of one-dimensional motion, but it is a general result. It tells us that the work done by the net force on a particle of mass *m* is equal to the difference between the initial and final values of a quantity

$$K \equiv \frac{1}{2}mv^2$$

Work-energy theorem:

$$\sum W = K_f - K_i = \Delta K$$

- In the case in which work is done on a system and the only change in the system is in its speed, the work done by the net force equals the change in kinetic energy of the system.
- This theorem is valid only for the case when there is no friction.

Conservative and Nonconcervative Forces

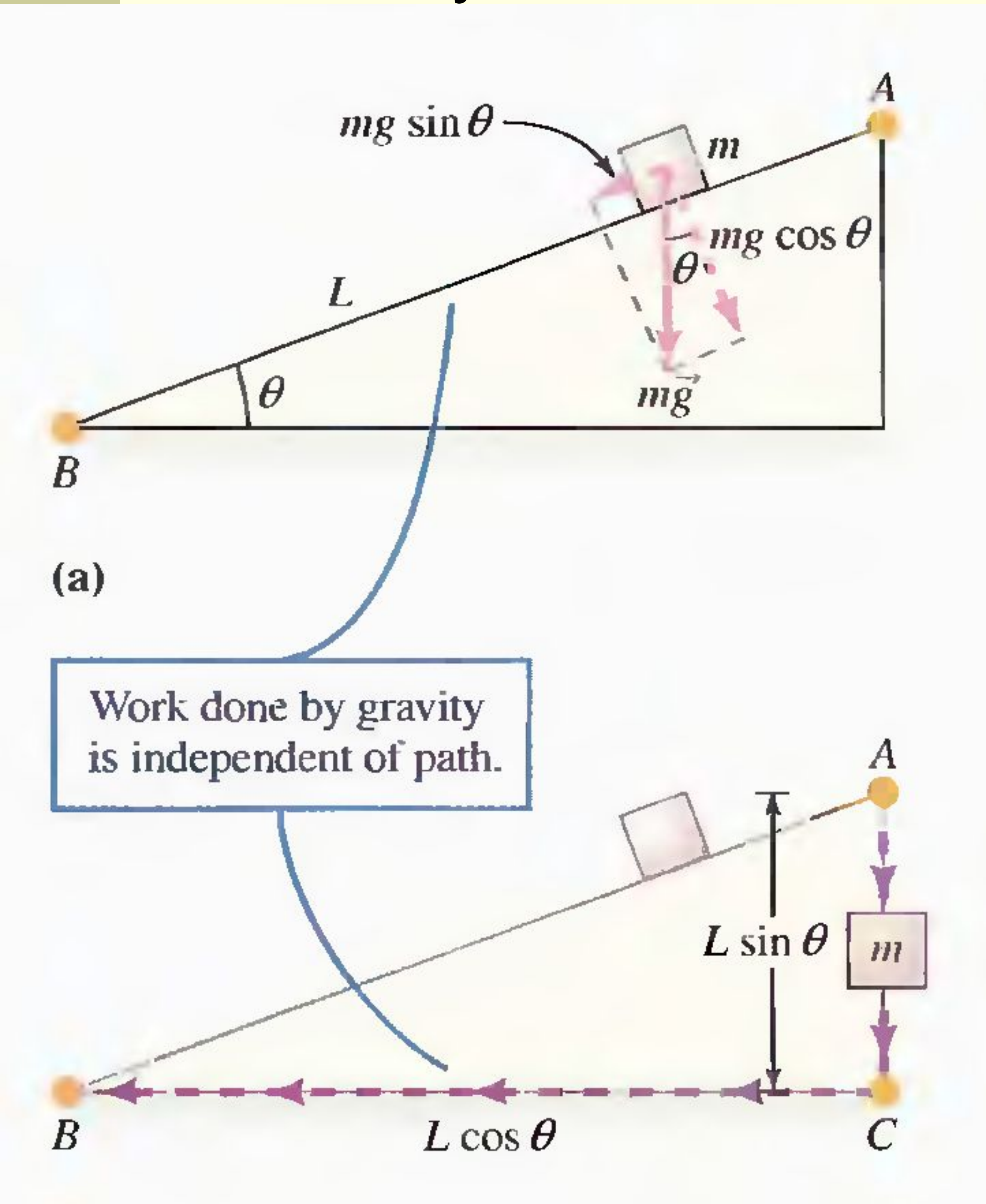
- Forces for which the work is **independent** of the path are called *conservative forces.*
- Forces for which the work **depends** on the path are called *nonconservative forces*

- **The work done by a conservative force in moving an object along any closed path is zero**.

Examples

- Conservative Forces:
  - Spring
  - central forces
    - Gravity
    - Electrostatic forces
- Nonconcervative Forces:
  - Various kinds of Friction

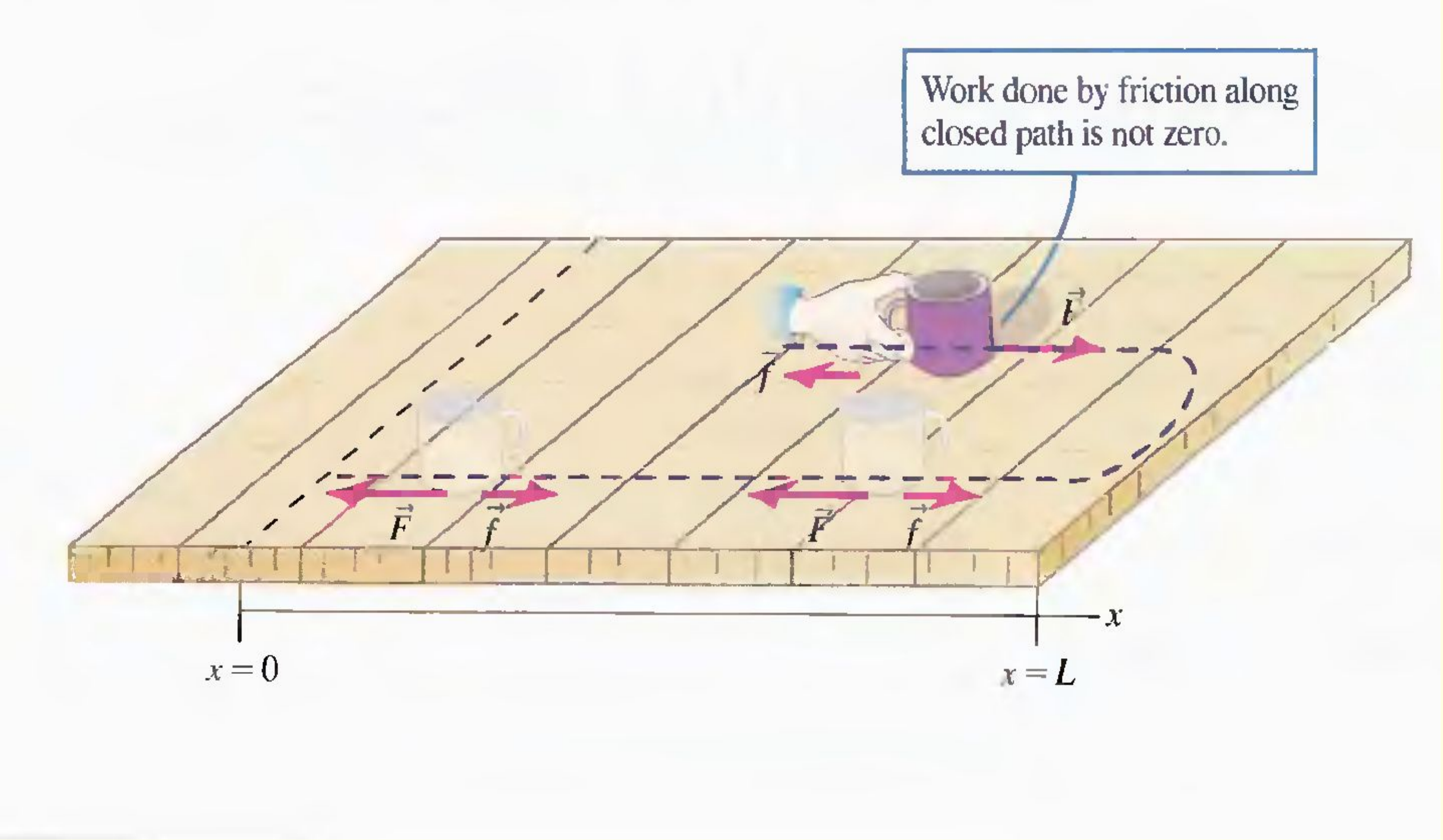
- Gravity is a conservative force:



An object moves from point A to point B on an inclined plane under the intluence of gravity. Gravity does positive (or negative) work on the object as it move down (or up) the plane.

The object now moves from point A to point B by a different path: a vertical motion from point A to point C followed by a horizontal movement from C to B. The work done by gravity is exactly the same as in part (a).

Friction is a nonconcervative force:



Power

- Power P is the rate at which work is done:

$$P \equiv \frac{dW}{dt}.$$

$$P = \vec{F} \cdot \frac{d\,\Delta\vec{r}}{dt} = \vec{F} \cdot \vec{v}.$$

Potential Energy

- Potential energy is the energy possessed by a system by virtue of position or condition.

- We call the particular function U for any given conservative force the **potential energy** for that force.

$$F(x) = -\frac{dU(x)}{dx}$$

- Remember the minus in the formula above.

$$W(x_0, x) = \int_{x_0}^{x} F(z)\, dz.$$

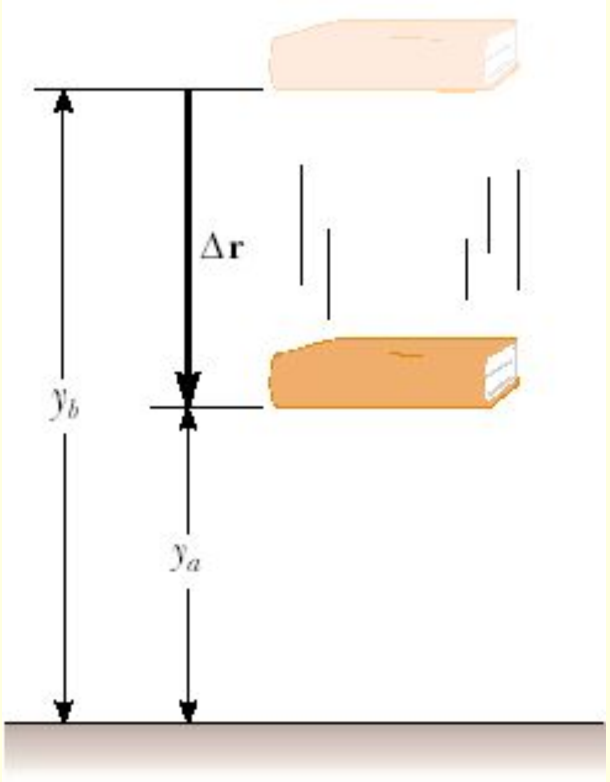
$$U(x) - U(x_0) = -W(x_0, x).$$

$$U(x) - U(x_0) = -\int_{x_0}^{x} F(z)\, dz.$$

$$F(x) = -\frac{dU(x)}{dx}$$

Potential Energy of Gravity

$$W_{\text{on book}} = (m\mathbf{g}) \cdot \Delta\mathbf{r} = (-mg\hat{\mathbf{j}}) \cdot [(y_a - y_b)\hat{\mathbf{j}}] = mgy_b - mgy_a$$



$$W_{\text{on book}} = \Delta K_{\text{book}}$$

$$\Delta K_{\text{book}} = mgy_b - mgy_a$$

$$\Delta K = -\Delta U_g$$

$$\Delta K + \Delta U_g = 0$$

$$E_{\text{mech}} = K + U_g$$

$$\frac{1}{2}mv_f^{\,2} + mgy_f = \frac{1}{2}mv_i^{\,2} + mgy_i$$

Conservation of mechanical energy

- $E = K + U(x) = \frac{1}{2} mv^2 + U(x)$ is called total mechanical energy
- If a system is
  - isolated (no energy transfer across its boundaries)
  - having no nonconservative forces within

then the mechanical energy of such a system is constant.

Linear momentum

- Let's consider two interacting particles:

$$\mathbf{F}_{21} + \mathbf{F}_{12} = 0$$

and their accelerations are:

$$m_1\mathbf{a}_1 + m_2\mathbf{a}_2 = 0$$

using definition of acceleration:

$$m_1 \frac{d\mathbf{v}_1}{dt} + m_2 \frac{d\mathbf{v}_2}{dt} = 0$$

masses are constant:

$$\frac{d(m_1\mathbf{v}_1)}{dt} + \frac{d(m_2\mathbf{v}_2)}{dt} = 0$$

$$\frac{d}{dt}(m_1\mathbf{v}_1 + m_2\mathbf{v}_2) = 0$$

- So, the total sum of quantities $m\mathbf{v}$ for an isolated system is conserved – independent of time.
- This quantity is called linear momentum.

$$\vec{p} \equiv m\vec{v}.$$

- General form for Newton's second law:

$$\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt}.$$

- It means that the time rate of change of the linear momentum of a particle is equal to the net for force acting on the particle.
- The kinetic energy of an object can also be expressed in terms of the momentum:

$$K = \frac{1}{2}mv^2 = \frac{p^2}{2m},$$

The law of linear momentum conservation

- The sum of the linear momenta of an isolated system of objects is constant, no matter what forces act between the objects making up the system.

Impulse-momentum theorem

$$\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt}.$$

$$\Delta \mathbf{p} = \mathbf{p}_f - \mathbf{p}_i = \int_{t_i}^{t_f} \mathbf{F}\, dt$$

- The impulse of the force F acting on a particle equals the change in the momentum of the particle.
- Quantity $\mathbf{I} \equiv \int_{t_i}^{t_f} \mathbf{F}\, dt$ is called the impulse of the force ***F.***

Collisions

Let's study the following types of collisions:

1. Perfectly elastic collisions:
    1. no mass transfer from one object to another
    2. Kinetic energy conserves (all the kinetic energy before collision goes to the kinetic energy after collision)
2. Perfectly inelastic collisions: two objects merge into one. Maximum kinetic loss.

Perfectly elastic collisions

We can write momentum and energy conservation equations:

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2', \qquad (1)$$

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2. \qquad (2)$$

(1)=> $$m_1(v_1 - v_1') = -m_2(v_2 - v_2').$$

(2)=> $$\frac{1}{2} m_1 (v_1 - v_1')(v_1 + v_1') = -\frac{1}{2} m_2 (v_2 - v_2')(v_2 + v_2').$$

(4)/(3): $$v_1 + v_1' = v_2 + v_2'. \qquad (5)$$

- Denoting $u_i = v_1 - v_2$ and $u_f = v_1' - v_2'$.

We can obtain from (5) $u_i = -u_f$.

Here U_i and U_f are initial and final *relative* velocities.

- So the last equation says that when the collision is elastic, the relative velocity of the colliding objects changes sign but does not change magnitude.

Perfectly inelastic collisions

$$M\vec{v} = m_1\vec{v}_1 + m_2\vec{v}_2.$$

$$M = m_1 + m_2.$$

$$v = \frac{m_1 v_1 + m_2 v_2}{M}.$$

Energy loss in perfectly inelastic collisions

$$\Delta E = \frac{1}{2}Mv^2 - \left(\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2\right)$$

$$= \frac{1}{2}\frac{M(m_1v_1 + m_2v_2)^2}{M^2} - \left(\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2\right)$$

$$= \frac{1}{2}\frac{m_1^2v_1^2 + 2m_1m_2v_1v_2 + m_2^2v_2^2 - M(m_1v_1^2 + m_2v_2^2)}{M}$$

$$= \frac{1}{2}\frac{m_1m_2(-v_1^2 - v_2^2 + 2v_1v_2)}{M} = -\frac{1}{2}\frac{m_1m_2}{M}(v_1 - v_2)^2.$$

Units in SI

- Work,Energy W,E $J=N*m=kg*m^2/s^2$
- Power P $J/s=kg*m^2/s^3$
- Linear momentum p $kg*m/s$