ELECTRIC POTENTIAL ENERGY
AND
ELECTRIC POTENTIAL

REVIEW OF POTENTIAL ENERGY

- Energy of single particle — Particle energy$=mc^2+K$
- Approximate kinetic energy — $K=1/2mv^2$
- Energy change — $\Delta K = K_f - K_i$

Checkpoint 1 A proton initially travels at a speed of 3000 m/s. After it passes through a region in which there is an electric field, the proton's speed is 5000 m/s. **(a)** What is the initial kinetic energy of the proton? **(b)** What is the final kinetic energy of the proton? **(c)** What is the change in kinetic energy of the proton?

CHANGE IN ENERGY

- Energy of single particle
- Approximate kinetic energy
- Energy change $\Delta K = K - K$

$$W = F_x \Delta x + F_y \Delta y + F_z \Delta z$$

$$\vec{A} \cdot \vec{B} = (A_x B_x + A_y B_y + A_z B_z) \quad \text{(a scalar quantity)}$$

$$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos\theta$$

A dust particle with charge 2×10^{-11}C moves from $\langle 0.1, -0.3, 0.4 \rangle$ m to $\langle 0.2, -0.3, -0.2 \rangle$ m. In this region there is an electric field of $\langle 2000, 0, 4000 \rangle$ N/C. How much work is done on the dust particle by the electric force?

ENERGY CONSIDERATIONS

When a force, F, acts on a particle, work is done on the particle in moving from point *a* to point *b*

$$W_{a\to b} = \int_a^b \vec{F} \cdot d\vec{l}$$

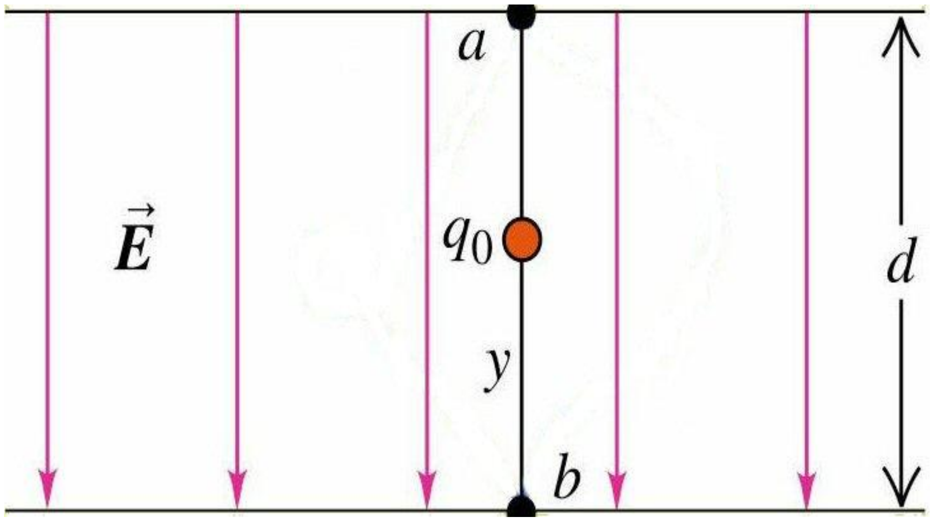
If the force is a conservative, then the work done can be expressed in terms of a change in potential energy

$$W_{a\to b} = -\left(U_b - U_a\right) = -\Delta U$$

Also if the force is conservative, the total energy of the particle remains *constant*

$$KE_a + PE_a = KE_b + PE_b$$

WORK DONE BY UNIFORM ELECTRIC FIELD



Force on charge is

$$\vec{F} = q_0\vec{E}$$

Work is done on the charge by field

$$W_{a\to b} = Fd = q_0Ed$$

The work done is *independent* of path taken from point a to point b because

The Electric Force is a *conservative force*

ELECTRIC POTENTIAL ENERGY

The work done by the force is the same as the change in the particle's potential energy

$$W_{a \to b} = -\left(U_b - U_a\right) = -\Delta U$$

$$U_b - U_a = -\int_a^b \vec{F} \cdot d\vec{s} = -qE_{uniform}\left(y_b - y_a\right)$$

The work done only depends upon the *change* in position

ELECTRIC POTENTIAL ENERGY

General Points

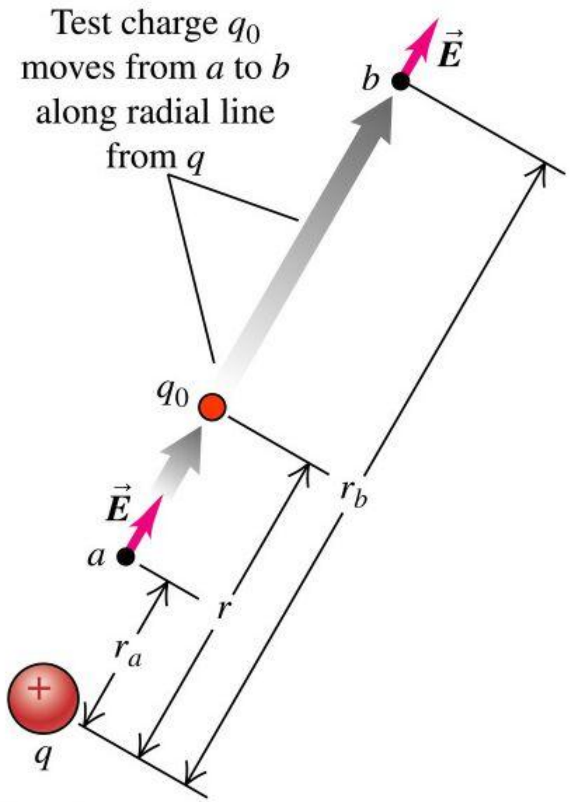
1) Potential Energy *increases* if the particle moves in the direction *opposite* to the force on it

Work will have to be done by an external agent for this to occur

and

2) Potential Energy *decreases* if the particle moves in the *same* direction as the force on it

POTENTIAL ENERGY OF TWO POINT CHARGES



Suppose we have two charges q and q_0 separated by a distance r

The force between the two charges is given by Coulomb's Law

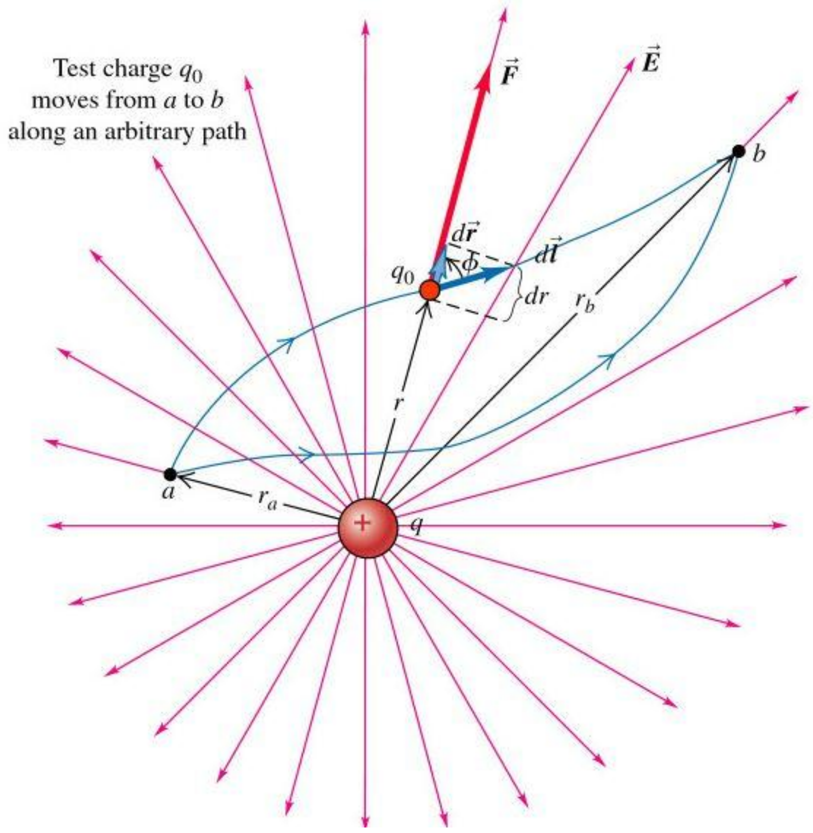
$$F = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}$$

We now displace charge q_0 along a radial line from point a to point b

The force is not constant during this displacement

$$W_{a\to b} = \int_{r_a}^{r_b} F_r dr = \int_{r_a}^{r_b} \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}dr = \frac{qq_0}{4\pi\varepsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$

POTENTIAL ENERGY OF TWO POINT CHARGES



The work done is not dependent upon the path taken in getting from point *a* to point *b*

The work done is related to the component of the force along the displacement

$$\vec{F} \cdot d\vec{r}$$

POTENTIAL ENERGY

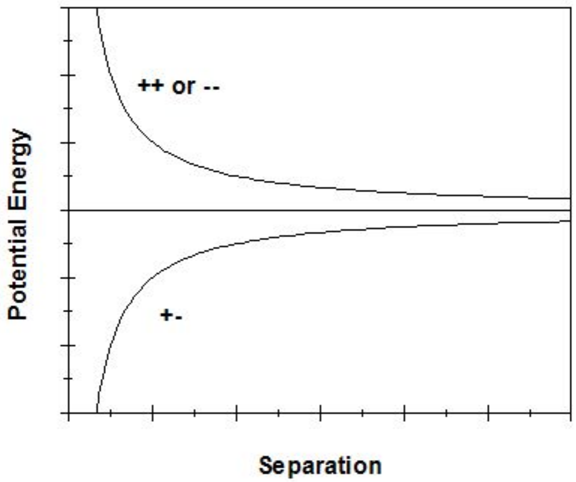
Looking at the work done we notice that there is the same *functional* at points a and b and that we are taking the difference

$$W_{a\to b} = \frac{qq_0}{4\pi\varepsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$

We define this functional to be the potential energy

$$U = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r}$$

The signs of the charges are included in the calculation



The potential energy is taken to be zero when the two charges are infinitely separated

A SYSTEM OF POINT CHARGES

SUPPOSE WE HAVE MORE THAN TWO CHARGES

HAVE TO BE CAREFUL OF THE QUESTION BEING ASKED

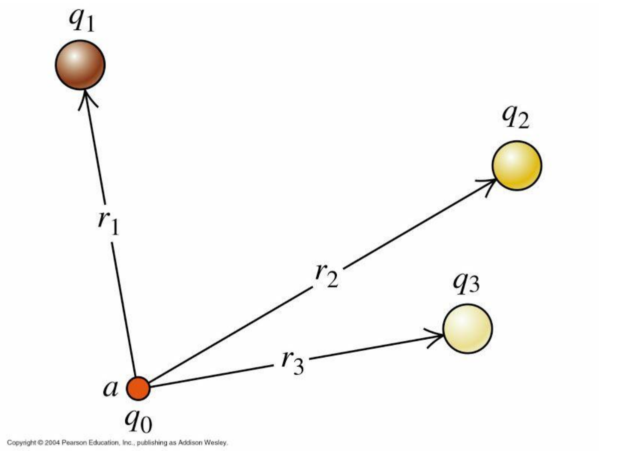
TWO POSSIBLE QUESTIONS:

1) TOTAL POTENTIAL ENERGY OF ONE OF THE CHARGES WITH RESPECT TO REMAINING CHARGES

OR

2) TOTAL POTENTIAL ENERGY OF THE SYSTEM

CASE 1: POTENTIAL ENERGY OF ONE CHARGE WITH RESPECT TO OTHERS



Given several charges, $q_1 \ldots q_n$, in place

Now a test charge, q_0, is brought into position

Work must be done against the electric fields of the original charges

This work goes into the potential energy of q_0

We calculate the potential energy of q_0 with respect to each of the other charges and then

Just sum the individual potential energies

$$PE_{q_0} = \sum_i \frac{1}{4\pi\varepsilon_0} \frac{q_0 q_i}{r_i}$$

Remember - Potential Energy is a Scalar

CASE 2: POTENTIAL ENERGY OF A SYSTEM OF CHARGES

Start by putting first charge in position

No work is necessary to do this

Next bring second charge into place

Now work is done by the electric field of the first charge. This work goes into the potential energy between these two charges.

Now the third charge is put into place

Work is done by the electric fields of the two previous charges. There are two potential energy terms for this step.

We continue in this manner until all the charges are in place

The total potential is then given by

$$PE_{system} = \sum_{i<j} \frac{1}{4\pi\varepsilon_0} \frac{q_i q_j}{r_{ij}}$$

EXAMPLE 1

The work done by the electric force as the test charge($q_0 = +2.0\times10^{-6}$ C)moves from A to B is $W_{AB} = +5.0\times10^{-5}$ J. **(a)** Find the value of the difference, Δ(EPE) = $\text{EPE}_B - \text{EPE}_A$, in the electric potential energies of the charge between these points. **(b)** Determine the potential difference, $\Delta V = V_B - V_A$, between the points.

CHECK YOUR ANSWER

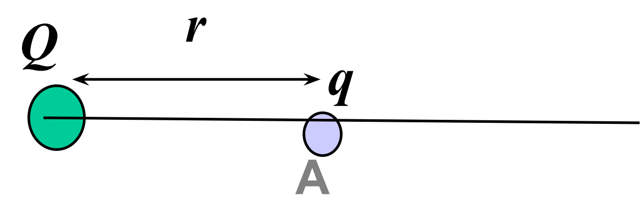
$$\underbrace{\text{EPE}_B - \text{EPE}_A}_{=\Delta(\text{EPE})} = -W_{AB} = \boxed{-5.0 \times 10^{-5}\ \text{J}}$$

$$\underbrace{V_B - V_A}_{=\ \Delta V} = \frac{\text{EPE}_B - \text{EPE}_A}{q_0} = \frac{-5.0 \times 10^{-5}\ \text{J}}{2.0 \times 10^{-6}\ \text{C}} = \boxed{-25\ \text{V}}$$

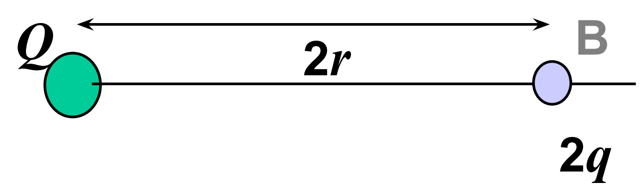
EXAMPLE 2

Two test charges are brought separately to the vicinity of a positive charge Q

Charge $+q$ is brought to pt A, a distance r from Q



Charge $+2q$ is brought to pt B, a distance $2r$ from Q



I) Compare the potential energy of q (U_A) to that of $2q$ (U_B)

(a) $U_A < U_B$ (b) $U_A = U_B$ (c) $U_A > U_B$

II) Suppose charge $2q$ has mass m and is released from rest from the above position (a distance $2r$ from Q). What is its velocity v_f as it approaches $r = \infty$?

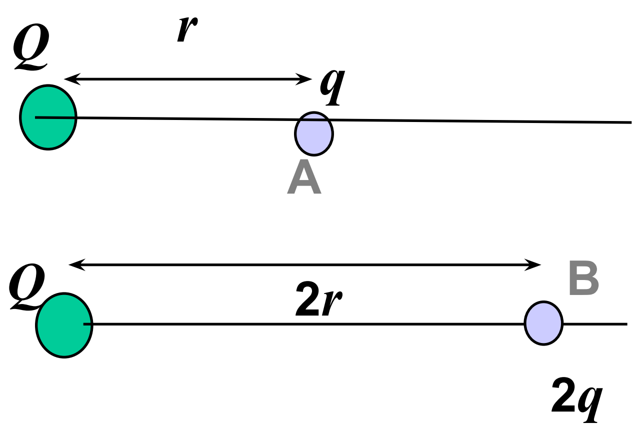
(a) $v_f = \sqrt{\frac{1}{4\pi\varepsilon_0}\frac{Qq}{mr}}$ (b) $v_f = \sqrt{\frac{1}{2\pi\varepsilon_0}\frac{Qq}{mr}}$ (c) $v_f = 0$

EXAMPLE 2

Two test charges are brought separately to the vicinity of a positive charge Q

Charge $+q$ is brought to pt A, a distance r from Q

Charge $+2q$ is brought to pt B, a distance $2r$ from Q



I) Compare the potential energy of q (U_A) to that of $2q$ (U_B)

(a) $U_A < U_B$

(b) $U_A = U_B$

(c) $U_A > U_B$

The potential energy of q is proportional to Qq/r

The potential energy of $2q$ is proportional to $Q(2q)/(2r) = Qq/r$

Therefore, the potential energies U_A and U_B are EQUAL!!!

II) Suppose charge $2q$ has mass m and is released from rest from the above position (a distance $2r$ from Q). What is its velocity v_f as it approaches $r = \infty$?

(a) $v_f = \sqrt{\frac{1}{4\pi\varepsilon_0}\frac{Qq}{mr}}$

(b) $v_f = \sqrt{\frac{1}{2\pi\varepsilon_0}\frac{Qq}{mr}}$

(c) $v_f = 0$

The principle at work here is ***CONSERVATION OF ENERGY***.

Initially:

The charge has no kinetic energy since it is at rest.

The charge does have potential energy (electric) = U_B.

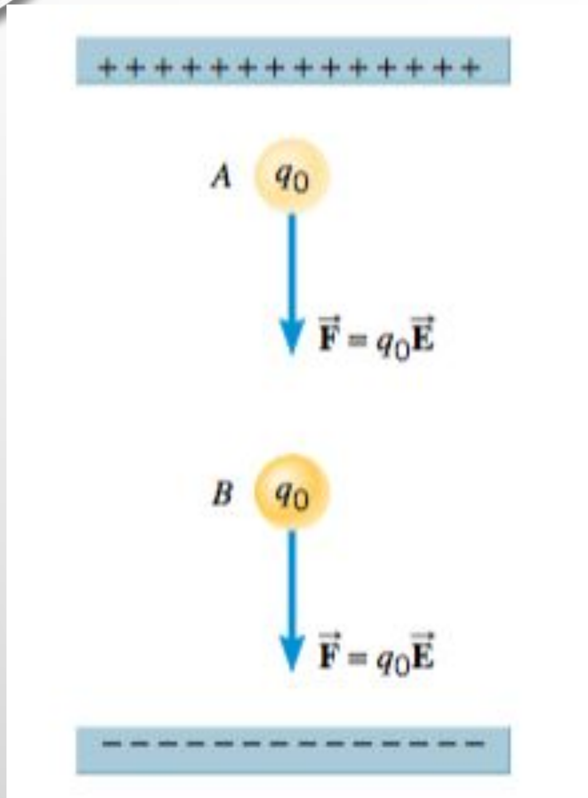
Finally:

The charge has no potential energy ($U \propto 1/R$)

The charge does have kinetic energy = KE

$$U_B = KE \longrightarrow \frac{1}{4\pi\varepsilon_0}\frac{Q(2q)}{2r} = \frac{1}{2}mv_f^2 \longrightarrow v_f^2 = \frac{1}{2\pi\varepsilon_0}\frac{Qq}{mr}$$

ELECTRIC POTENTIAL



The electric potential *V* at a given point is the electric potential energy EPE of a small test charge q_0 situated at that point divided by the charge itself.

$$\frac{W_{AB}}{q_0} = \frac{EPE_A}{q_0} - \frac{EPE_B}{q_0}$$

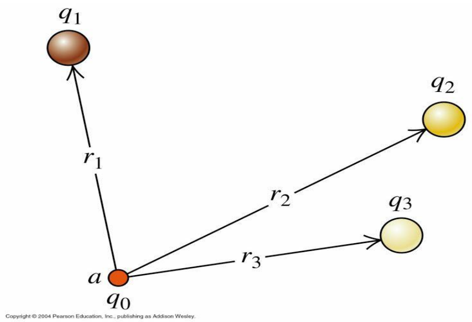
$$V = \frac{EPE}{q_0}$$

Because of the electric field **E**→, an electric force, **F** = q_0**E**→, is exerted on a positive test charge $+q_0$. Work is done by the force as the charge moves from *A* to *B*.

ELECTRIC POTENTIAL

Recall Case 1 from before

The potential energy of the test charge, q_0, was given by



$$PE_{q_0} = \sum_i \frac{1}{4\pi\varepsilon_0} \frac{q_0 q_i}{r_i}$$

Notice that there is a part of this equation that would remain the same regardless of the test charge, q_0, placed at point *a*

The value of the test charge can be pulled out from the summation

$$PE_{q_0} = q_0 \sum_i \frac{1}{4\pi\varepsilon_0} \frac{q_i}{r_i}$$

ELECTRIC POTENTIAL

We define the term to the right of the summation as the electric potential at point *a*

$$Potential_a = \sum_i \frac{1}{4\pi\varepsilon_0}\frac{q_i}{r_i}$$

Like energy, potential is a *scalar*

We define the potential of a given point charge as being

$$Potential = V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

This equation has the convention that the potential is zero at infinite distance

ELECTRIC POTENTIAL

The potential at a given point

Represents the potential energy that a positive unit charge would have, if it were placed at that point

It has units of

$$\mathbf{Volts} = \frac{\mathbf{Energy}}{\mathbf{charge}} = \frac{\boldsymbol{joules}}{\boldsymbol{coulomb}}$$

ELECTRIC POTENTIAL

General Points for either positive or negative charges

The Potential *increases* if you move in the direction *opposite* to the electric field

and

The Potential *decreases* if you move in the *same* direction as the electric field

Example 4

Points A, B, and C lie in a uniform electric field.

What is the potential difference between points A and B?

$\Delta V_{AB} = V_B - V_A$

a) $\Delta V_{AB} > 0$ b) $\Delta V_{AB} = 0$ c) $\Delta V_{AB} < 0$

The electric field, *E*, points in the direction of decreasing potential

Since points A and B are in the same relative horizontal location in the electric field there is on potential difference between them

Example 5

Points A, B, and C lie in a uniform electric field.

Point C is at a higher potential than point A.

True **False**

As stated previously the electric field points in the direction of *decreasing* potential

Since point C is further to the right in the electric field and the electric field is pointing to the right, point C is at a lower potential

The statement is therefore false

Example 6

Points A, B, and C lie in a uniform electric field.

If a negative charge is moved from point A to point B, its electric potential energy

a) Increases. **b) decreases.** **c) doesn't change.**

The potential energy of a charge at a location in an electric field is given by the product of the charge and the potential at the location

As shown in Example 4, the potential at points A and B are the same

Therefore the electric potential energy also doesn't change

Example 7

Points A, B, and C lie in a uniform electric field.

Compare the potential differences between points A and C and points B and C.

a) $V_{AC} > V_{BC}$ b) $V_{AC} = V_{BC}$ c) $V_{AC} < V_{BC}$

In Example 4 we showed that the the potential at points A and B were the same

Therefore the potential difference between A and C and the potential difference between points B and C are the same

Also remember that potential and potential energy are scalars and directions do not come into play

WORK AND POTENTIAL

- **The work done by the electric force in moving a test charge from point *a* to point *b* is given by**

$$W_{a\to b} = \int_a^b \vec{F}\cdot d\vec{l} = \int_a^b q_0 \vec{E}\cdot d\vec{l}$$

Dividing through by the test charge q_0 we have

$$V_a - V_b = \int_a^b \vec{E}\cdot d\vec{l}$$

Rearranging so the order of the subscripts is the same on both sides

$$V_b - V_a = -\int_a^b \vec{E}\cdot d\vec{l}$$

POTENTIAL

From this last result $$V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{l}$$

We get $$dV = -\vec{E} \cdot d\vec{l} \text{ or } \frac{dV}{dx} = -E$$

We see that the electric field points in the direction of *decreasing* potential

We are often more interested in potential differences as this relates directly to the work done in moving a charge from one point to another

Example 8

If you want to move in a region of electric field without changing your electric potential energy. You would move

a) Parallel to the electric field

b) Perpendicular to the electric field

The work done by the electric field when a charge moves from one point to another is given by

$$W_{a\to b} = \int_a^b \vec{F} \cdot d\vec{l} = \int_a^b q_0 \vec{E} \cdot d\vec{l}$$

The way no work is done by the electric field is if the integration path is perpendicular to the electric field giving a zero for the dot product

Example 9

A positive charge is released from rest in a region of electric field. The charge moves:

a) towards a region of smaller electric potential

b) along a path of constant electric potential

c) towards a region of greater electric potential

A positive charge placed in an electric field will experience a force given by $F = qE$

But E is also given by $E = -\frac{dV}{dx}$

Therefore $F = qE = -q\frac{dV}{dx}$

Since q is positive, the force F points in the direction opposite to increasing potential or in the direction of decreasing potential

UNITS FOR ENERGY

There is an additional unit that is used for energy in addition to that of joules

A particle having the charge of *e* (1.6 x 10^{-19} C) that is moved through a potential difference of 1 Volt has an increase in energy that is given by

$$W = qV = 1.6 \times 10^{-19}\ joules$$

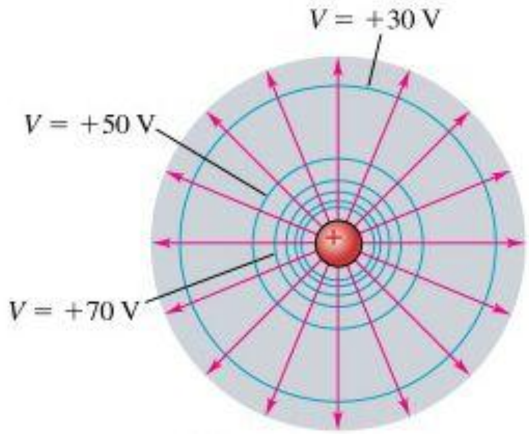
$$= 1\ eV$$

EQUIPOTENTIAL SURFACES

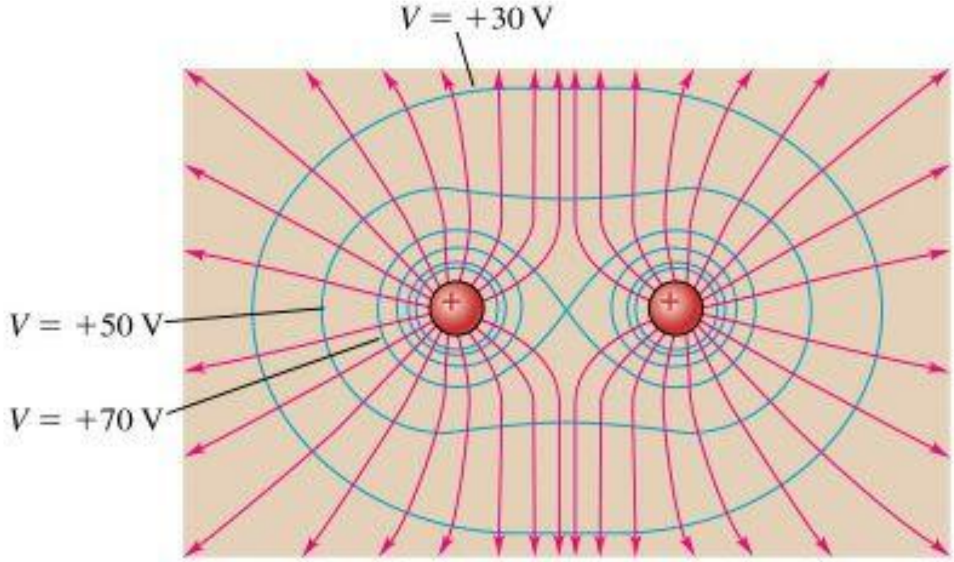
- IT IS POSSIBLE TO MOVE A TEST CHARGE FROM ONE POINT TO ANOTHER WITHOUT HAVING ANY NET WORK DONE ON THE CHARGE
- THIS OCCURS WHEN THE BEGINNING AND END POINTS HAVE THE SAME POTENTIAL
- IT IS POSSIBLE TO MAP OUT SUCH POINTS AND A GIVEN SET OF POINTS AT THE SAME POTENTIAL FORM AN *EQUIPOTENTIAL SURFACE*

EQUIPOTENTIAL SURFACES

Examples of equipotential surfaces



Point Charge



Two Positive Charges

EQUIPOTENTIAL SURFACES

THE ELECTRIC FIELD DOES NO WORK AS A CHARGE IS MOVED ALONG AN EQUIPOTENTIAL SURFACE

SINCE NO WORK IS DONE, THERE IS NO FORCE, QE, ALONG THE DIRECTION OF MOTION

THE ELECTRIC FIELD IS *PERPENDICULAR* TO THE EQUIPOTENTIAL SURFACE

WHAT ABOUT CONDUCTORS

IN A STATIC SITUATION, THE SURFACE OF A CONDUCTOR IS AN EQUIPOTENTIAL SURFACE

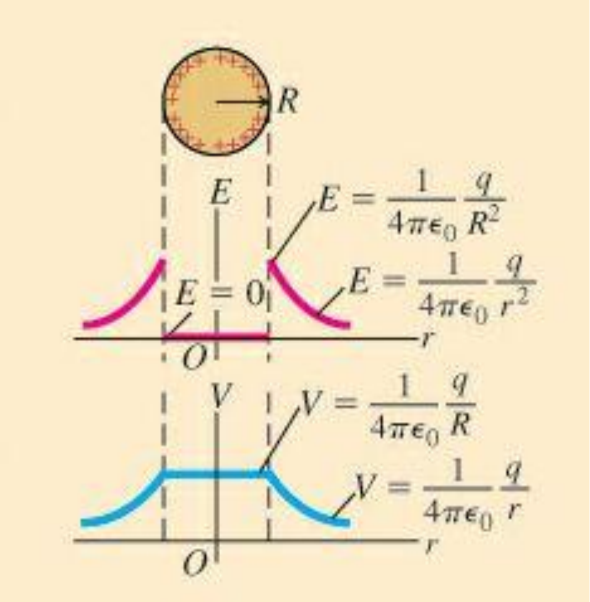
BUT WHAT IS THE POTENTIAL INSIDE THE CONDUCTOR IF THERE IS A SURFACE CHARGE?

WE KNOW THAT E = 0 INSIDE THE CONDUCTOR

THIS LEADS TO

$$\frac{dV}{dx} = 0 \text{ or } V = \text{constant}$$

WHAT ABOUT CONDUCTORS



The value of the potential inside the conductor is chosen to match that at the surface

POTENTIAL GRADIENT

The equation that relates the derivative of the potential to the electric field that we had before

$$\frac{dV}{dx} = -E$$

can be expanded into three dimensions

$$\vec{E} = -\vec{\nabla} V$$

$$\vec{E} = -\left(\hat{i}\frac{dV}{dx} + \hat{j}\frac{dV}{dy} + \hat{k}\frac{dV}{dz} \right)$$

POTENTIAL GRADIENT

- FOR THE GRADIENT OPERATOR, USE THE ONE THAT IS APPROPRIATE TO THE COORDINATE SYSTEM THAT IS BEING USED.

Example 10

The electric potential in a region of space is given by

$$V(x) = 3x^2 - x^3$$

The x-component of the electric field E_x at $x = 2$ is

(a) $E_x = 0$ (b) $E_x > 0$ (c) $E_x < 0$

We know $V(x)$ "everywhere"

To obtain E_x "everywhere", use

$$\vec{E} = -\vec{\nabla} V \longrightarrow E_x = -\frac{dV}{dx} \longrightarrow E_x = -6x + 3x^2$$

$$E_x(2) = -6(2) + 3(2)^2 = 0$$

POTENTIAL DIFFERENCE

$$\Delta U_{\text{electric}} = q\Delta V \quad \text{and therefore} \quad \Delta V = \frac{\Delta U_{\text{electric}}}{q}$$

POTENTIAL DIFFERENCE IN A UNIFORM FIELD

$$\Delta V = -(E_x \Delta x + E_y \Delta y + E_z \Delta z)$$

or, using dot product notation:

$$\Delta V = -\vec{E} \bullet \Delta\vec{l} = -\langle E_x, E_y, E_z \rangle \bullet \langle \Delta x, \Delta y, \Delta z \rangle$$