



OKAN ÜNİVERSİTESİ
İSTANBUL

BBA182 Applied Statistics

Week 7 (2)

Discrete Probability Distributions: Binomial and Poisson Distribution

DR SUSANNE HANSEN SARAL

EMAIL: SUSANNE.SARAL@OKAN.EDU.TR

[HTTPS://PIAZZA.COM/CLASS/IXRJ5MMOX1U2T8?CID=4#](https://PIAZZA.COM/CLASS/IXRJ5MMOX1U2T8?CID=4#)

WWW.KHANACADEMY.ORG

Mid-term exam

23/03/2017 11:45 – 13:00 hours

Bring:

- Calculator
- Pen
- Eraser

Probability distribution of a discrete random variable

In the last class we saw how to calculate the probability of a **specific discrete random variable**, such as $P(x = 3)$ and the **cumulative probability** such as $P(x \geq 3)$ in *n trials* with the following formula:

$$f(x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{(n-x)}$$

We need to know n , x and P (probability of success)

The binomial distribution is used to find the probability of a **specific or cumulative number of successes in n trials.**

NUMBER OF HEADS (r)	PROBABILITY =	$\frac{5!}{r!(5-r)!} (0.5)^r (0.5)^{5-r}$
0	$P(X = 0) = 0.0778$	$\frac{5!}{0!(5-0)!} (0.5)^0 (0.5)^{5-0}$
1	$P(X = 1) = 0.2592$	$\frac{5!}{1!(5-1)!} (0.5)^1 (0.5)^{5-1}$
2	$P(X = 2) = 0.3456$	$\frac{5!}{2!(5-2)!} (0.5)^2 (0.5)^{5-2}$
3	$P(X = 3) = 0.2304$	$\frac{5!}{3!(5-3)!} (0.5)^3 (0.5)^{5-3}$
4	$P(X = 4) = 0.0768$	$\frac{5!}{4!(5-4)!} (0.5)^4 (0.5)^{5-4}$
5	$P(X = 5) = 0.0102$	$\frac{5!}{5!(5-5)!} (0.5)^5 (0.5)^{5-5}$



Probability distribution of a discrete random variable

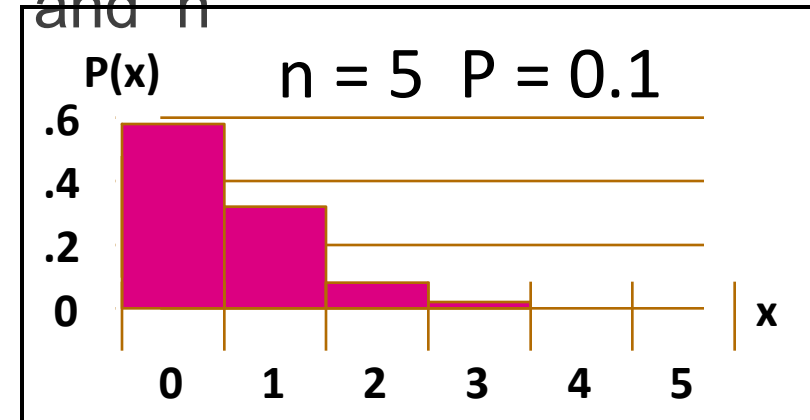
Ali, real estate agent has 5 potential customers to buy a house or apartment with a probability of 0.40. The probability and cumulative probability table of the sale of houses here below. We are able to answer what is $P(x=3)$, $P(x \leq 4)$, $P(x > 3)$ etc.

x	P(x)	F(x)
0	0.0778	0.0778
1	0.2592	0.337
2	0.3456	0.6826
3	0.2304	0.913
4	0.0768	0.9898
5	0.0102	1

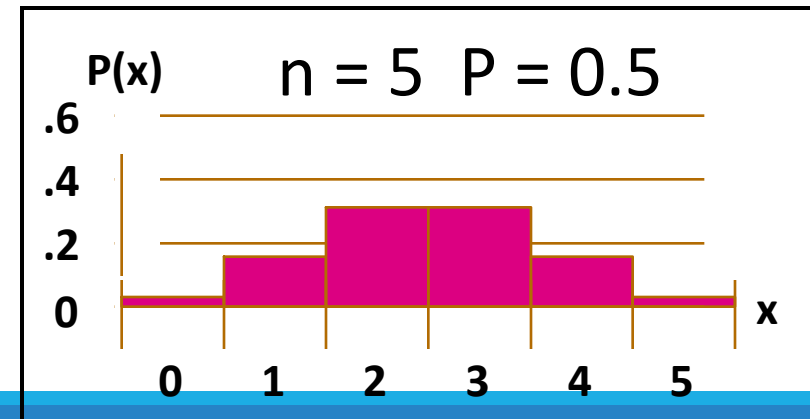
Shape of Binomial Distribution

The shape of the binomial distribution depends on the values of P and n

- Here, $n = 5$ and $P = 0.1$



- Here, $n = 5$ and $P = 0.5$





Binomial Distribution shapes

When $P = .5$ the shape of the distribution is ***perfectly symmetrical*** and resembles a bell-shaped (normal distribution)

When $P = .2$ the distribution is ***skewed right***. This skewness increases as P becomes smaller.

When $P = .8$, the distribution is ***skewed left***. As P comes closer to 1, the amount of skewness increases.



Using Binomial Tables instead of to calculating Binomial probabilities manually

N	x	...	p=.20	p=.25	p=.30	p=.35	p=.40	p=.45	p=.50
10	0	...	0.1074	0.0563	0.0282	0.0135	0.0060	0.0025	0.0010
	1	...	0.2684	0.1877	0.1211	0.0725	0.0403	0.0207	0.0098
	2	...	0.3020	0.2816	0.2335	0.1757	0.1209	0.0763	0.0439
	3	...	0.2013	0.2503	0.2668	0.2522	0.2150	0.1665	0.1172
	4	...	0.0881	0.1460	0.2001	0.2377	0.2508	0.2384	0.2051
	5	...	0.0264	0.0584	0.1029	0.1536	0.2007	0.2340	0.2461
	6	...	0.0055	0.0162	0.0368	0.0689	0.1115	0.1596	0.2051
	7	...	0.0008	0.0031	0.0090	0.0212	0.0425	0.0746	0.1172
	8	...	0.0001	0.0004	0.0014	0.0043	0.0106	0.0229	0.0439
	9	...	0.0000	0.0000	0.0001	0.0005	0.0016	0.0042	0.0098
	10	...	0.0000	0.0000	0.0000	0.0000	0.0001	0.0003	0.0010

Examples:

$$n = 10, x = 3, P = 0.35: \quad P(x = 3 | n = 10, p = 0.35) = .2522$$

$$n = 10, x = 8, P = 0.45: \quad P(x = 8 | n = 10, p = 0.45) = .0229$$



Solving Problems with Binomial Tables

MSA Electronics is experimenting with the manufacture of a new USB-stick and is looking into the number of defective USB-sticks

- Every hour a random sample of 5 USB-sticks is taken
- The probability of one USB-stick being defective is 0.15
- What is the probability of finding 3, 4, or 5 defective USB-sticks ?
 $P(x = 3)$, $P(x = 4)$, $P(x = 5)$

$$n = 5, p = 0.15, \text{ and } r = 3, 4, \text{ or } 5$$



Solving Problems with Binomial Tables

TABLE 2.9 (partial) – Table for Binomial Distribution, $n= 5$,

n	r	P		
		0.05	0.10	0.15
5	0	0.7738	0.5905	0.4437
	1	0.2036	0.3281	0.3915
	2	0.0214	0.0729	0.1382
	3	0.0011	0.0081	0.0244
	4	0.0000	0.0005	0.0022
	5	0.0000	0.0000	0.0001

MSA Electronics is experimenting with the manufacture of a new USB-stick

• Every hour a random sample of 5 USB-sticks is taken

• The probability of one USB-stick being defective is 0.15

• What is the probability of finding more than 3 (3 inclusive) defective USB-sticks? $P(x \geq 3)$

Solving Problems with Binomial Tables

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Solving Problems with Binomial

Cumulative probability



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We find the three probabilities in the table for $n = 5$, $p = 0.15$, and $r = 3, 4$, and 5 and add them together



Solving Problems with Binomial Tables

Cumulative probabilities

TABLE 2.9 (partial) – Table for Binomial Distribution

$$\begin{aligned} P(3 \text{ or more defects}) &= P(3) + P(4) + P(5) \\ &= 0.0244 + 0.0022 + 0.0001 \\ &= 0.0267 \end{aligned}$$

4	0.0000	0.0005	0.0022
5	0.0000	0.0000	0.0001

We find the three probabilities in the table for $n = 5$, $p = 0.15$, and $r = 3, 4$, and 5 and add them together



Suppose that Ali, a real estate agent, has 10 people interested in buying a house. He believes that for each of the 10 people the probability of selling a house is 0.40. What is the probability that he will sell 4 houses, $P(x = 4)$?

N	x	...	p=.20	p=.25	p=.30	p=.35	p=.40	p=.45	p=.50
10	0	...	0.1074	0.0563	0.0282	0.0135	0.0060	0.0025	0.0010
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	10	...	0.0000	0.0000	0.0000	0.0000	0.0001	0.0003	0.0010



Suppose that Ali, a real estate agent, has 10 people interested in buying a house. He believes that for each of the 10 people the probability of selling a house is 0.20. What is the probability that he will sell 7 houses,

$$P(x = 7)?$$

N	x	...	p=.20	p=.25	p=.30	p=.35	p=.40	p=.45	p=.50
10	0	...	0.1074	0.0563	0.0282	0.0135	0.0060	0.0025	0.0010
	1	...	0.2684	0.1877	0.1211	0.0725	0.0403	0.0207	0.0098
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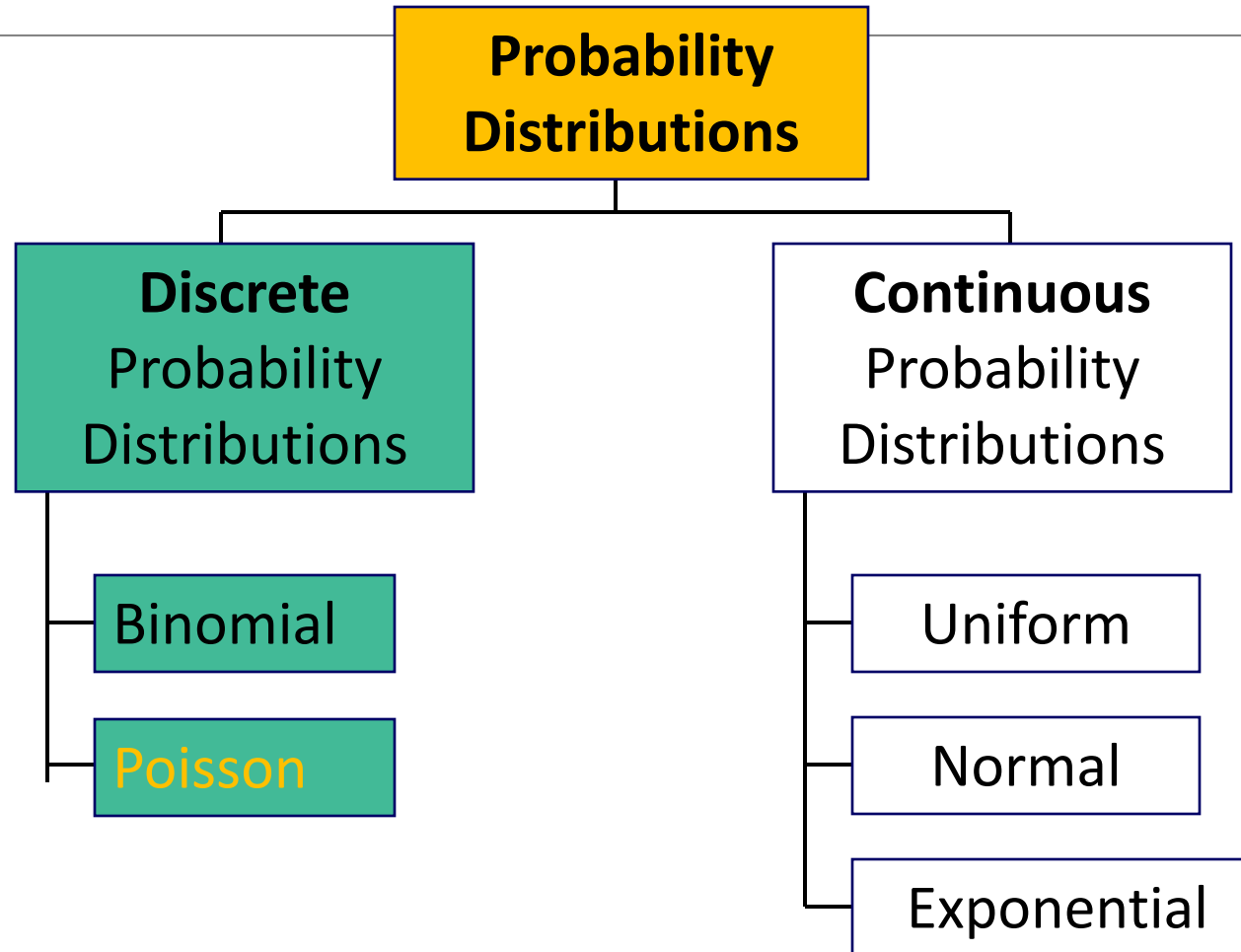


Heidi believes that for each of the 10 people the probability of selling a house is 0.35. What is the probability that he will sell more than 7 houses, 7 houses included, $P(x \geq 7)$?

N	x	...	p=.20	p=.25	p=.30	p=.35	p=.40	p=.45	p=.50
10	0	...	0.1074	0.0563	0.0282	0.0135	0.0060	0.0025	0.0010
	1	...	0.2684	0.1877	0.1211	0.0725	0.0403	0.0207	0.0098
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10	...	0.0000	0.0000	0.0000	0.0000	0.0001	0.0003	0.0010	



Probability Distributions



Poisson random variable,

first proposed by Frenchman Simeon Poisson (1781-1840)

A Poisson distributed random variable is often useful in estimating the number of occurrences over a specified interval of time or space.

It is a discrete random variable that may assume an infinite sequence of values ($x = 0, 1, 2, \dots$).

Poisson Random Variable - three requirements

1. The number of expected outcomes in one interval of time or unit space is unaffected (independent) by the number of expected outcomes in any other non-overlapping time interval.

Example: What took place between 3:00 and 3:20 p.m. is not affected by what took place between 9:00 and 9:20 a.m.



Poisson Random Variable - requirements (continued)

2. The expected (or mean) number of outcomes over any time period or unit space is proportional to the size of this time interval.

Example:

We expect half as many outcomes between 3:00 and 3:30 P.M. as between 3:00 and 4:00 P.M.

3. This requirement also implies that the probability of an occurrence must be constant over any intervals of the same length.

Example:

The expected outcome between 3:00 and 3:30 P.M. is equal to the expected occurrence between 4:00 and 4:30 P.M..

Examples of a Poisson Random variable

- The number of cars arriving at a toll booth in 1 hour (the time interval is 1 hour)
- The number of failures in a large computer system during a given day (the given day is the interval)
- The number of delivery trucks to arrive at a central warehouse in an hour.
- The number of customers to arrive for flights at an airport during each 10-minute time interval from 3:00 p.m. to 6:00 p.m. on weekdays



Situations where the Poisson distribution is widely used: Capacity planning – time interval

Areas of capacity planning observed in a sample:

A bank wants to know how many customers arrive at the bank in a given time period during the day, so that they can anticipate the waiting lines and plan for the number of employees to hire.

At peak hours they might want to open more guichets (employ more personnel) to reduce waiting lines and during slower hours, have a few guichets open (need for less personnel).



Poisson Probability Distribution

Poisson Probability Function

$$f(x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

where:

$f(x)$ = probability of x occurrences in an interval

λ = mean number of occurrences in an interval

$e = 2.71828$



Example: Drive-up ATM Window

Poisson Probability Function: Time Interval

Suppose that we are interested in the number of arrivals at the drive-up ATM window of a bank during a 15-minute period on weekday mornings.

If we assume that the probability of a car arriving is the same for any two time periods of equal length and that the arrival or non-arrival of a car in any time period is independent of the arrival or non-arrival in any other time period, **the Poisson probability function is applicable.**

Then if we assume that an analysis of historical data shows that the **average number of cars** arriving during a 15-minute interval of time is 10, the Poisson probability function with $\lambda = 10$ applies.



Poisson Probability Function: Time Interval

We want to know the probability of five arrivals in 15 minutes.

$$f(x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

$$\lambda = 10/15\text{-minutes}, \quad x = 5$$

$$f(5) = \frac{10^5 (2.71828)^{-10}}{5!} = .0378$$

Using Poisson Probability Tables

x	λ								
	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90
0	0.9048	0.8187	0.7408	0.6703	0.6065	0.5488	0.4966	0.4493	0.4066
1	0.0905	0.1637	0.2222	0.2681	0.3033	0.3293	0.3476	0.3595	0.3659
2	0.0045	0.0164	0.0333	0.0536	0.0758	0.0988	0.1217	0.1438	0.1647
3	0.0002	0.0011	0.0033	0.0072	0.0126	0.0198	0.0284	0.0383	0.0494
4	0.0000	0.0001	0.0003	0.0007	0.0016	0.0030	0.0050	0.0077	0.0111
5	0.0000	0.0000	0.0000	0.0001	0.0002	0.0004	0.0007	0.0012	0.0020
6	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0002	0.0003
7	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Example: Find $P(X = 2)$ if $\lambda = .50$

$$P(X = 2) = \frac{e^{-\lambda} \lambda^x}{x!} = \frac{e^{-0.50} (0.50)^2}{2!} = .0758$$

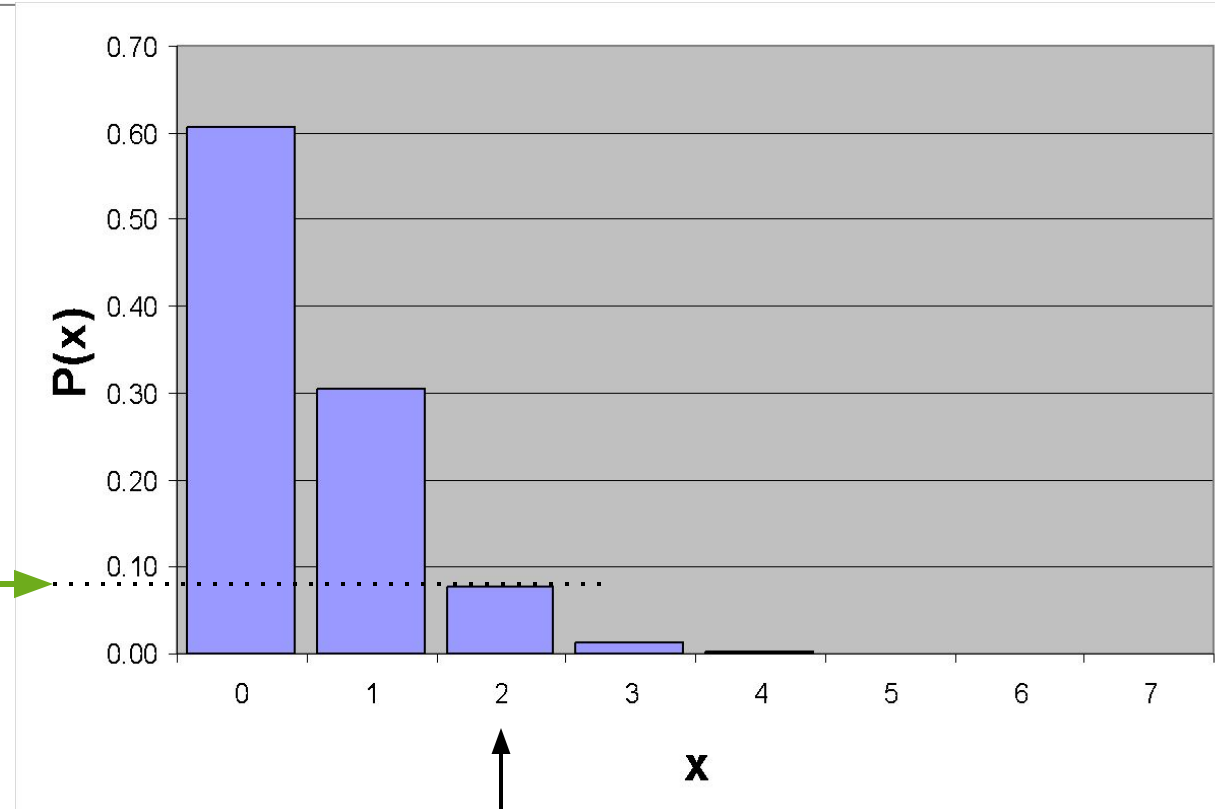


The shape of a Poisson Probabilities Distribution

Graphically:

$\lambda = .50$

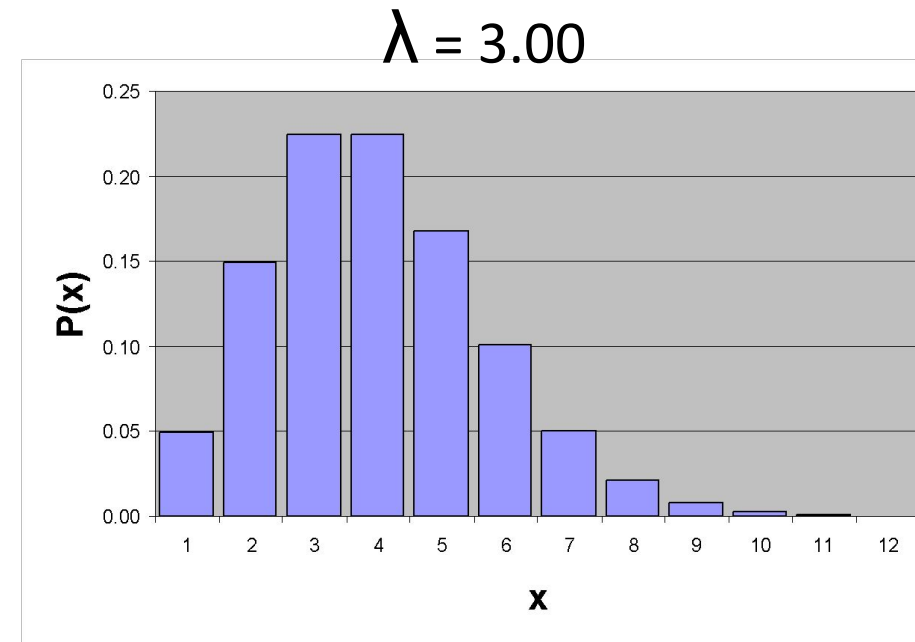
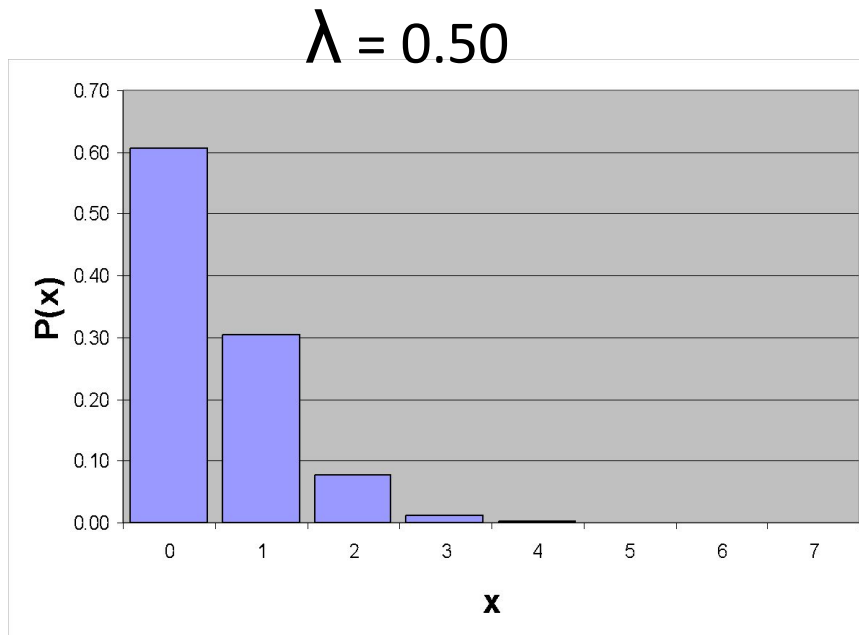
X	$\lambda = 0.50$
0	0.6065
1	0.3033
2	0.0758
3	0.0126
4	0.0016
5	0.0002
6	0.0000
7	0.0000



$P(X = 2) = .0758$

Poisson Distribution Shape

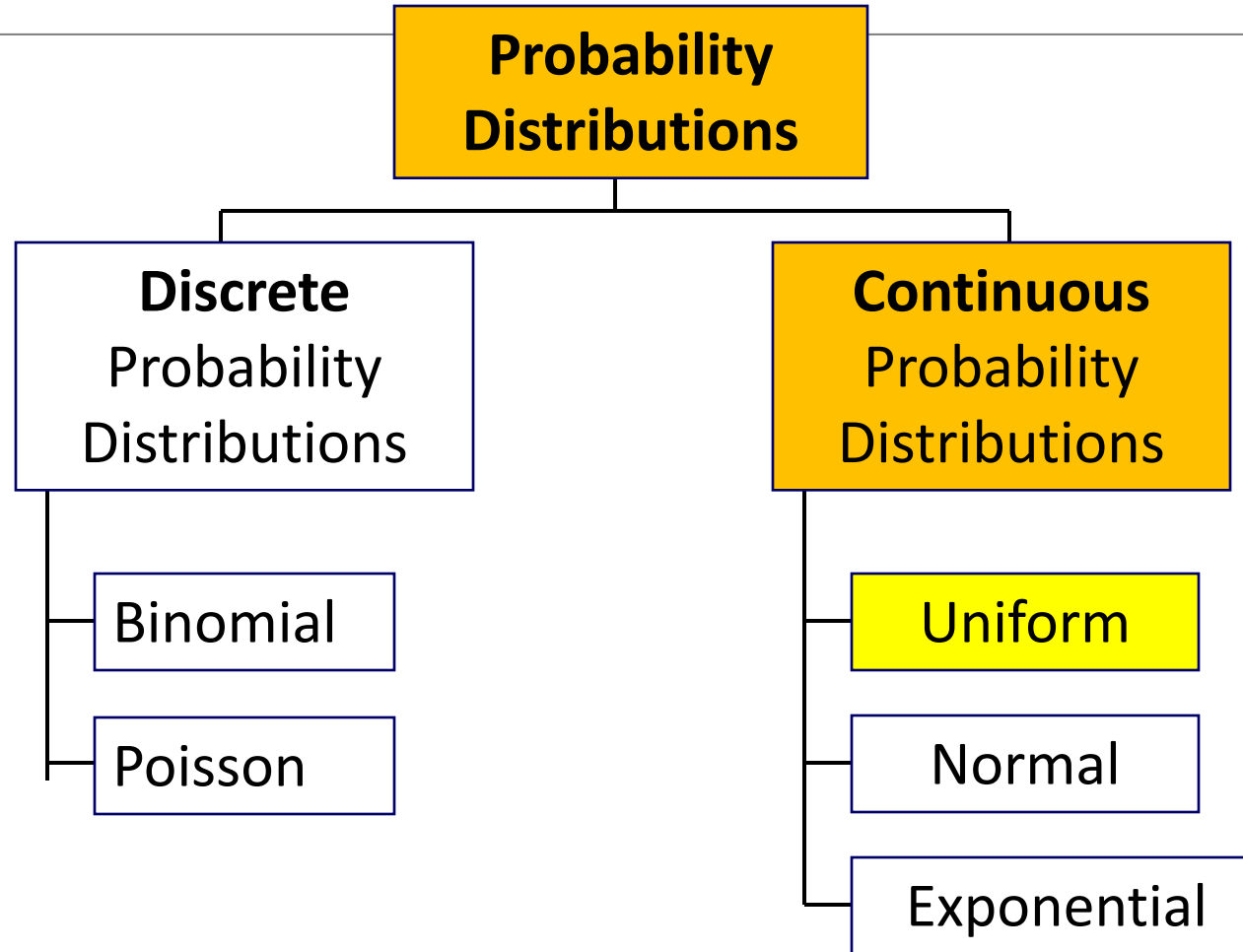
The shape of the Poisson Distribution depends on the parameter λ :





Probability Distributions

Continuous probability distributions

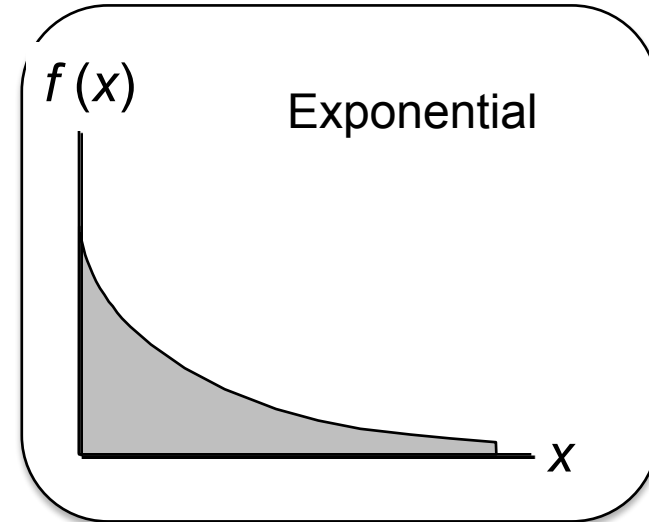
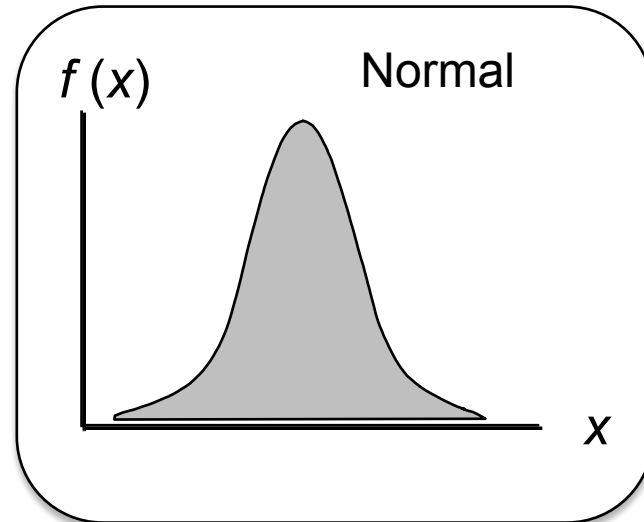
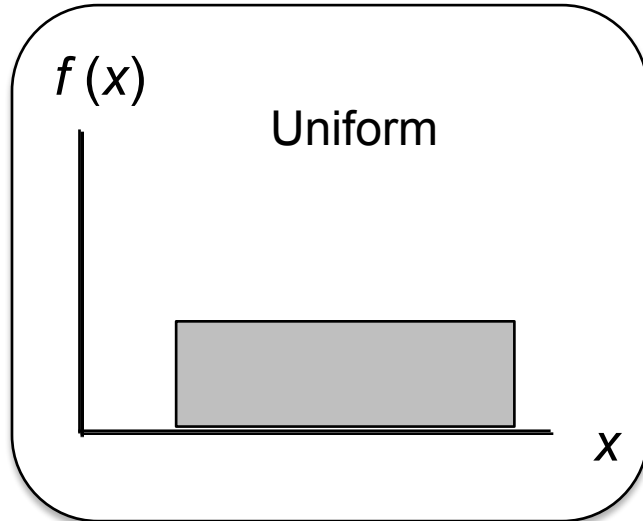


Continuous Probability Distributions

Uniform Probability Distribution

Normal Probability Distribution

Exponential Probability Distribution





Continuous random variables

Examples of continuous random variables include the following:

- The *number of deciliters (dl)* coca cola poured into a glass labeled “3 dl”.
- The *flight time* of an airplane traveling from Chicago to New York
- The *lifetime* of a batterie
- The *drilling depth* required to reach oil in an offshore drilling operation

Continuous random variable

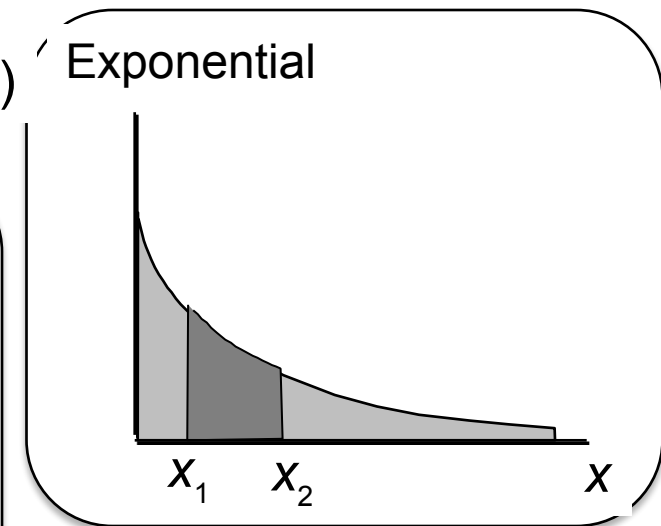
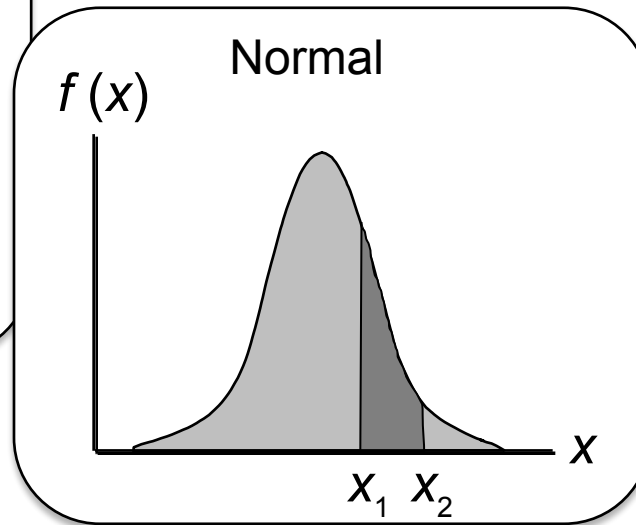
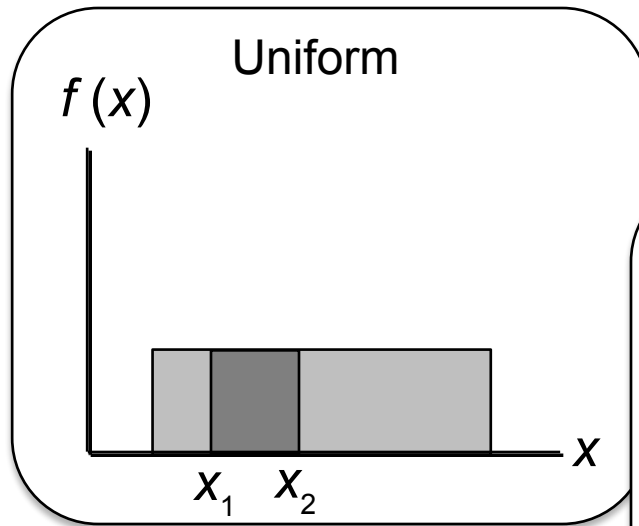
A continuous random variable can assume any value in an interval on the real line or in a collection of intervals.

It is not possible to talk about the probability of the random variable assuming a particular value, because the probability will **be close to 0**.

Instead, we talk about the probability of the random variable assuming a value within a given interval.



The probability of the random variable assuming a value within some given interval from x_1 to x_2 is defined to be the area under the graph of the probability density function between x_1 and x_2 .





Probability Density Function

The **probability density function**, $f(x)$, of a continuous random variable X has the following properties:

1. $f(x) > 0$ for all values of x
2. The area under the probability density function $f(x)$ over all values of the random variable X within its range, is equal to 1.0
3. The probability that X lies between two values is the area under the density function graph between the two values

$$P(a < X < b) = \int_a^b f(x)dx$$



Probability Density Function

(continued)

The **probability density function**, $f(x)$, of random variable X has the following properties:

4. The **cumulative density function** $F(x_0)$ is the area under the probability density function $f(x)$ from the minimum x value up to x_0

$$F(x_0) = \int_{x_m}^{x_0} f(x) dx$$

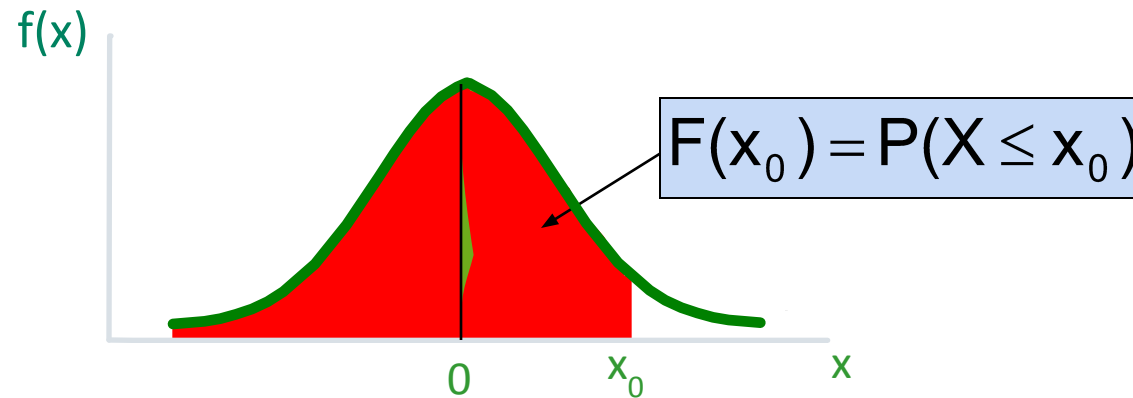
where x_m is the minimum value of the random variable x



Probability as an Area

(continued)

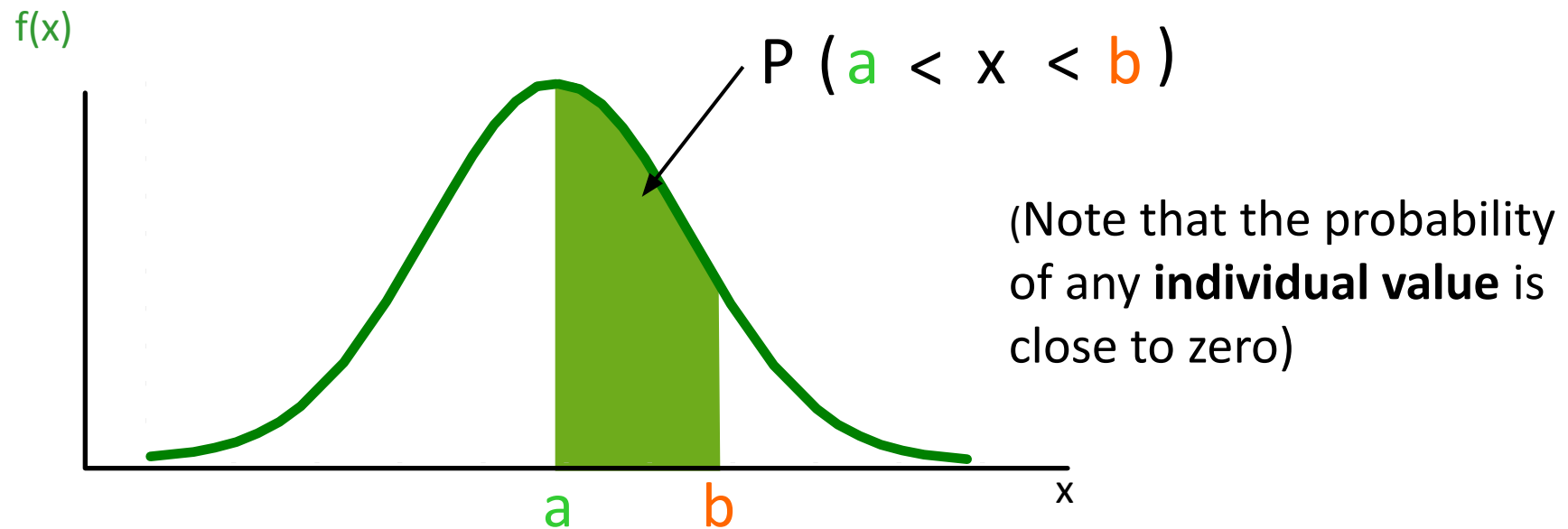
1. The total area under the curve $f(x)$ is 1.0
2. The area under the curve $f(x)$ to the left of x_0 is $F(x_0)$, where x_0 is any value that the random variable can take.





Probability as an Area

Shaded area under the curve is the **probability** that X is between a and b





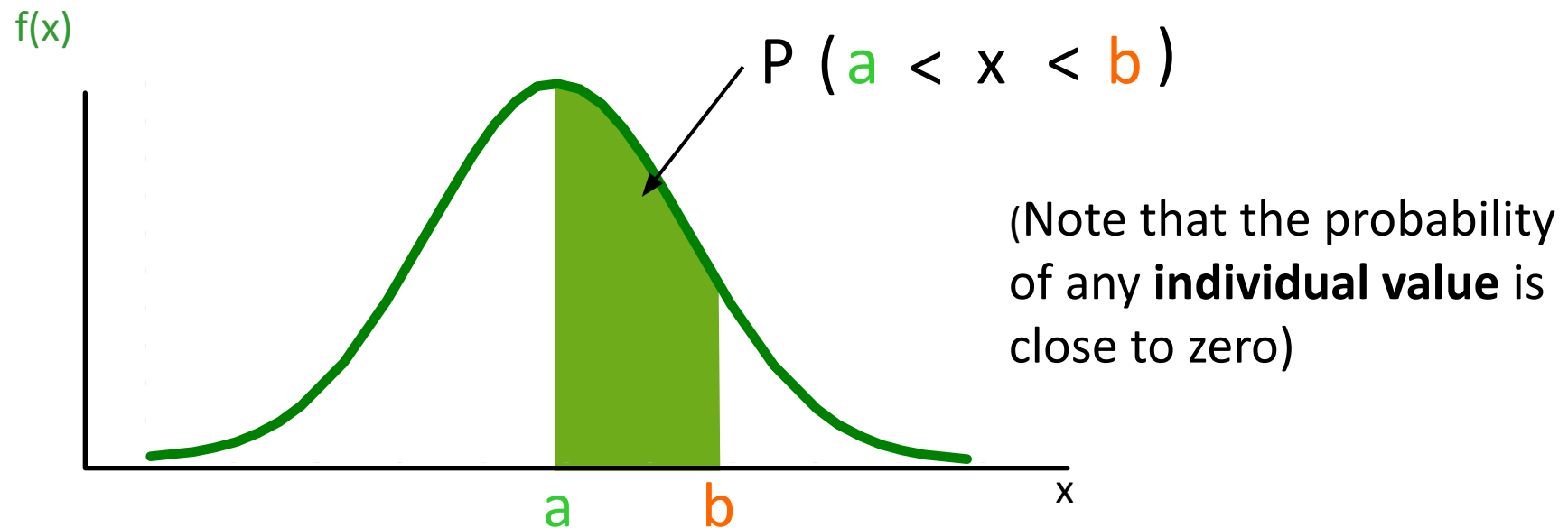
Cumulative Distribution Function, $F(x)$

Let a and b be two possible values of X , with $a < b$. The **probability that X lies between a and b** is:

$$P(a < X < b) = F(b) - F(a)$$

Cumulative probability as an Area

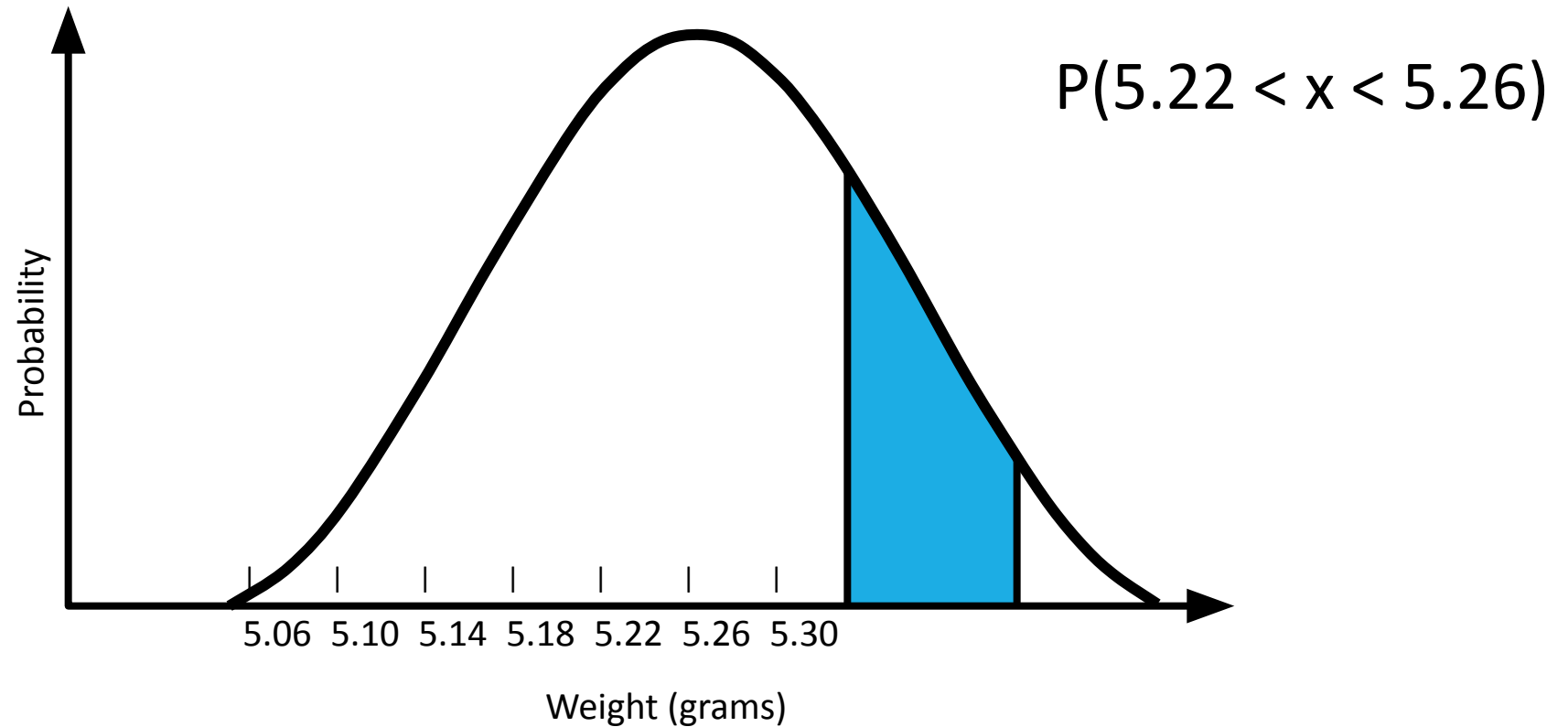
Shaded area under the curve is the **probability** that X is between a and b





Probability Distribution of a Continuous Random Variable

FIGURE 2.5 – Sample Density Function





The Normal Distribution

The Normal Distribution is one of the most popular and useful continuous probability distributions

- The probability density function:

$$f(X) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

- The shape and location of the normal distribution is described by the mean, μ , and the standard deviation, σ



The Normal Distribution

Bell Shaped

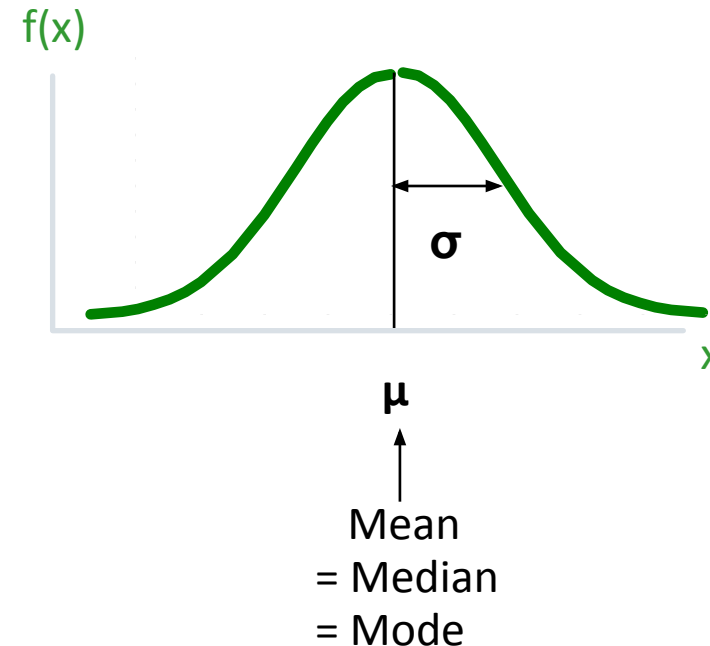
Symmetrical

Mean, Median and Mode are Equal

Location of the curve is determined by the mean, μ

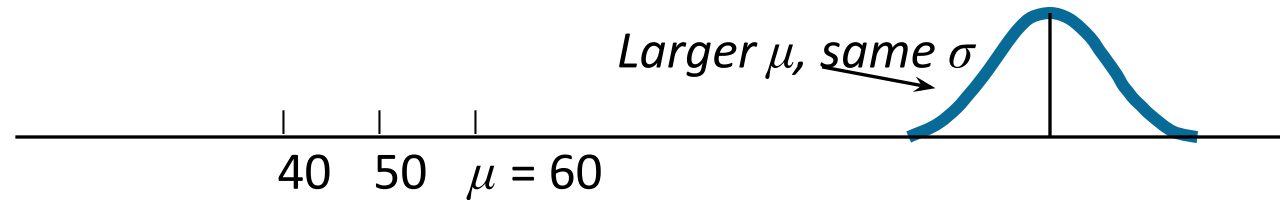
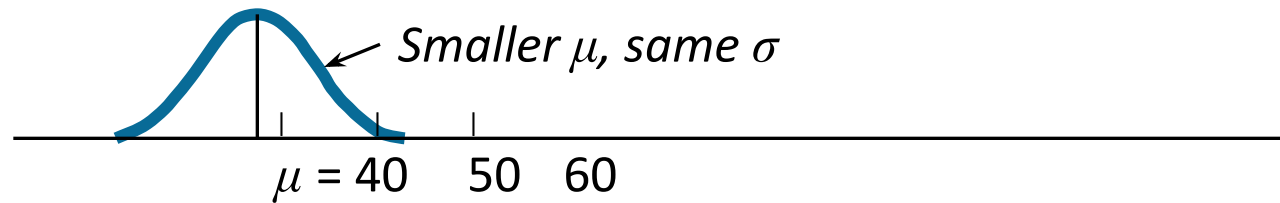
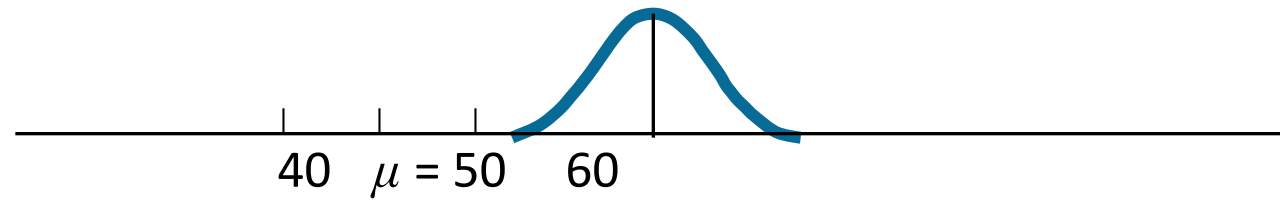
Spread is determined by the standard deviation, σ

The random variable has an infinite theoretical range: $+\infty$ to $-\infty$



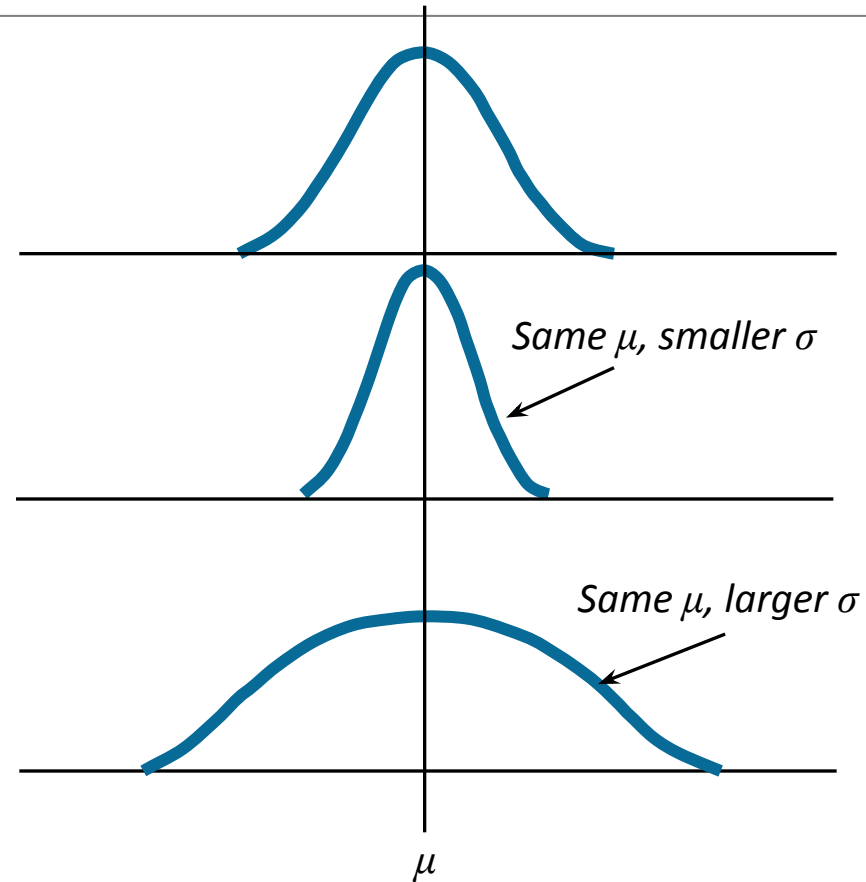


The location of the normal distribution on the x-axis is described by the mean, μ .





The shape of the normal distribution is described by the standard deviation, σ





The Normal Distribution

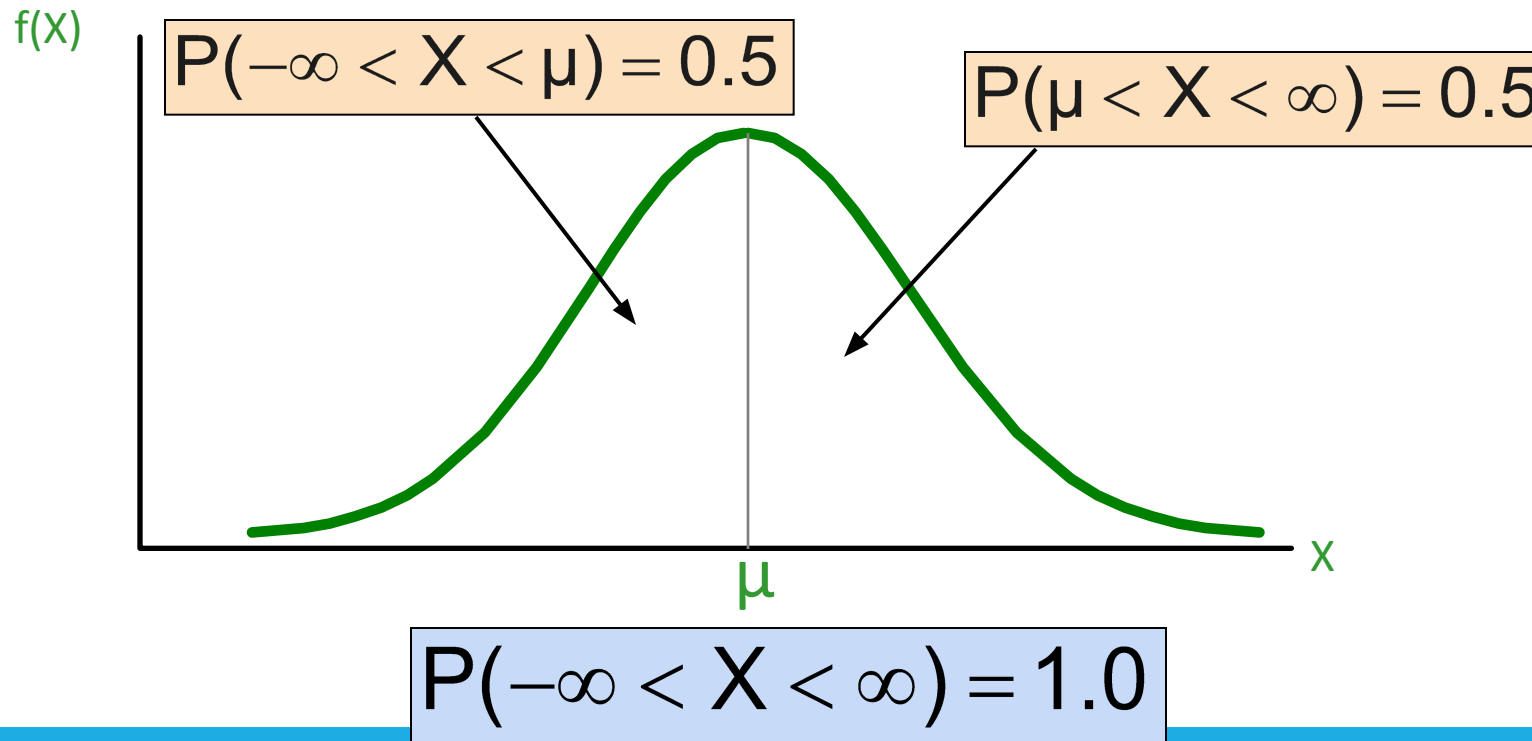
- Symmetrical with the midpoint representing the mean
- Shifting the mean does not change the shape
- Values on the X axis are measured in the number of standard deviations away from the mean $\pm 1\sigma$, $\pm 2\sigma$, $\pm 3\sigma$

- As standard deviation becomes larger, curve flattens
- As standard deviation becomes smaller, curve becomes steeper



Probability as Area Under the Curve

The total area under the curve is 1.0, and the curve is symmetric, so half is above the mean, half is below

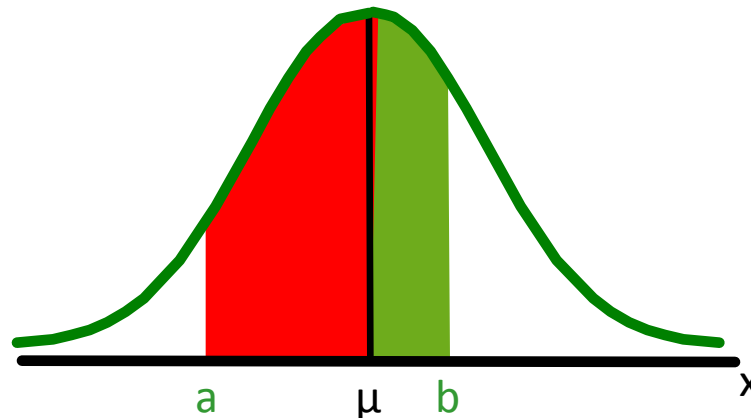




Finding Normal Probabilities

The probability for an **interval of values** is measured by the area under the curve

$$P(a < X < b) = F(b) - F(a)$$

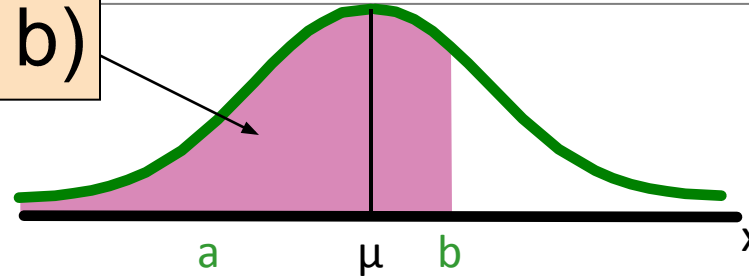




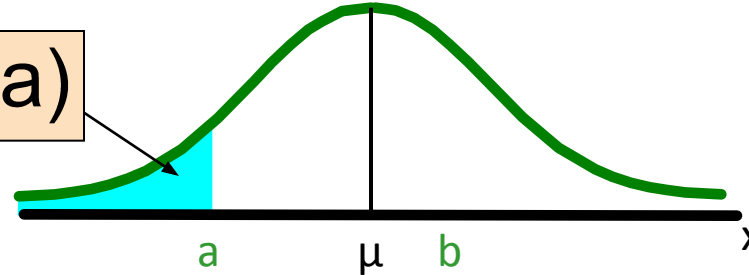
Finding Normal Probabilities

(continued)

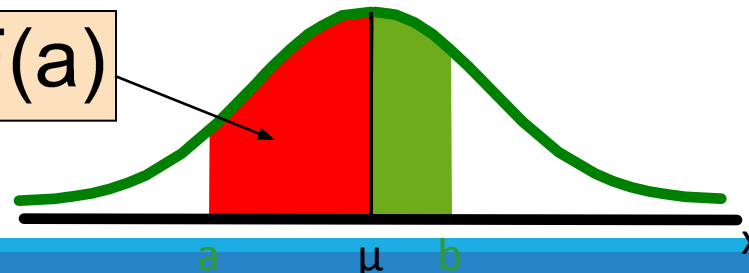
$$F(b) = P(X < b)$$



$$F(a) = P(X < a)$$



$$P(a < X < b) = F(b) - F(a)$$

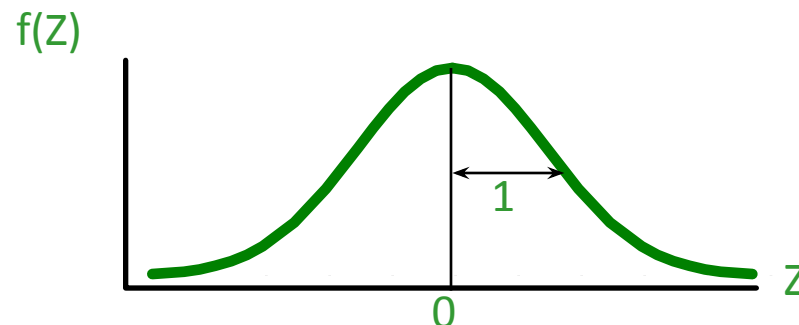




The Standard Normal Distribution – z-values

Any normal distribution (with any mean and standard deviation combination) can be transformed into the **standardized normal distribution (Z)**, with mean **0** and standard deviation **1**

$$Z \sim N(0,1)$$



We say that Z follows the standard normal distribution.



Using the Standard Normal Table

Step 1

- Convert the normal *distribution into a standard normal distribution*
 - Mean of 0 and a standard deviation of 1
 - The new standard random variable is Z:

$$Z = \frac{X - \mu}{\sigma}$$

where

X = value of the random variable we want to measure

μ = mean of the distribution

σ = standard deviation of the distribution

Z = number of standard deviations from X to the mean, μ



Using the Standard Normal Table

For $\mu = 100$, $\sigma = 15$, find the probability that X is less than 130 = $P(x < 130)$

Transforming x - random variable into a z - standard random variable:

$$Z = \frac{X - \mu}{\sigma} = \frac{130 - 100}{15}$$
$$= \frac{30}{15} = 2 \text{ std dev}$$

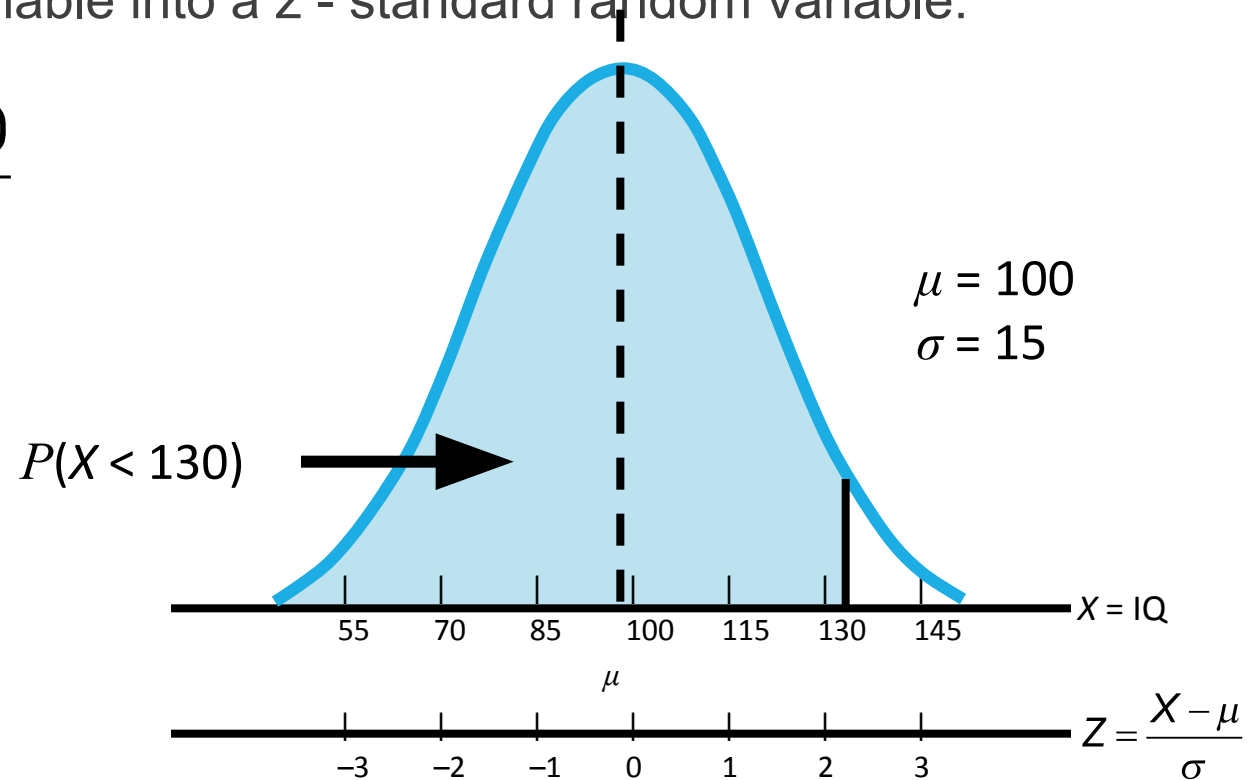


FIGURE 2.9
- Normal Distribution



Using the Standard Normal Table

Step 2

- Look up the probability from the table of normal curve areas
- Column on the left is Z value
- Row at the top has second decimal places for Z values



Using the Standard Normal Table

TABLE 2.10 – Standardized Normal Distribution (partial)

AREA UNDER THE NORMAL CURVE				
Z	0.00	0.01	0.02	0.03
1.8	0.96407	0.96485	0.96562	0.96638
1.9	0.97128	0.97193	0.97257	0.97320
2.0	0.97725	0.97784	0.97831	0.97882
2.1	0.98214	0.98257	0.98300	0.98341
2.2	0.98610	0.98645	0.98679	0.98713

For $Z = 2.00$

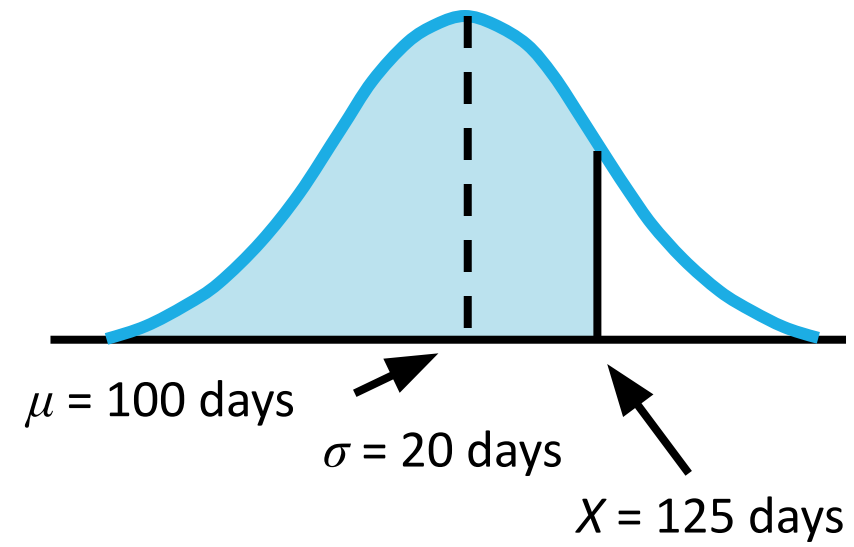
$$P(X < 130) = P(Z < 2.00) = 0.97725$$

$$\begin{aligned} P(X > 130) &= 1 - P(X \leq 130) = 1 - P(Z \leq 2) \\ &= 1 - 0.97725 = 0.02275 \end{aligned}$$



OKAN ÜNİVERSİTESİ Haynes Construction Company

FIGURE 2.10



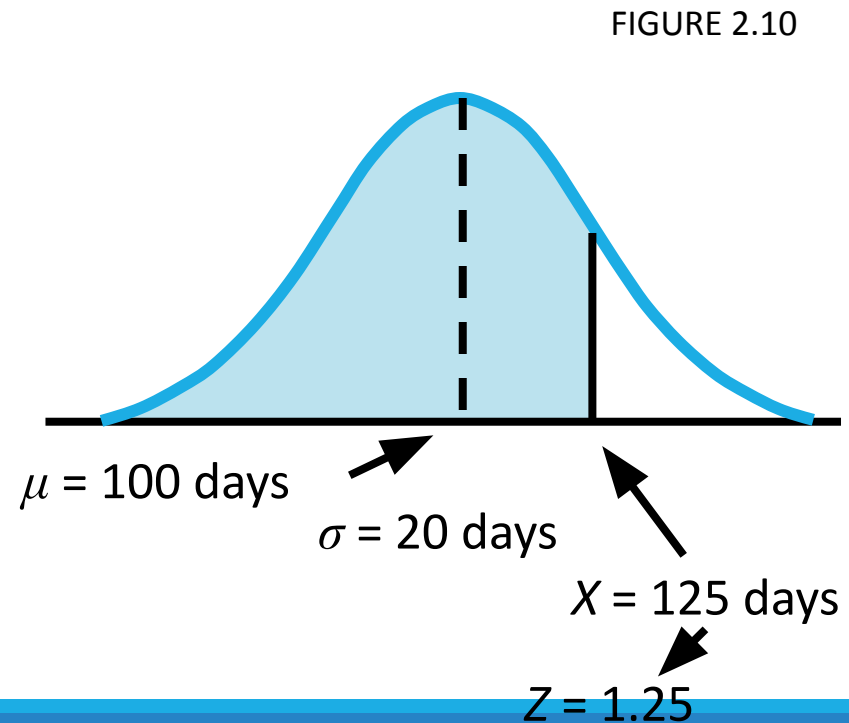


Haynes Construction Company

Compute Z:

$$Z = \frac{X - \mu}{\sigma} = \frac{125 - 100}{20}$$
$$= \frac{25}{20} = 1.25 \quad P(z < 1.25) ?$$

- From the table for $Z = 1.25$
area $P(z < 1.25) = 0.8944$





Haynes Construction Company

• Con

The probability is about 0.89 or 89 %
that Haynes will not violate the contract

Z =

$$= \frac{25}{20} = 1.25$$

— From the table for Z = 1.25
area = 0.89435

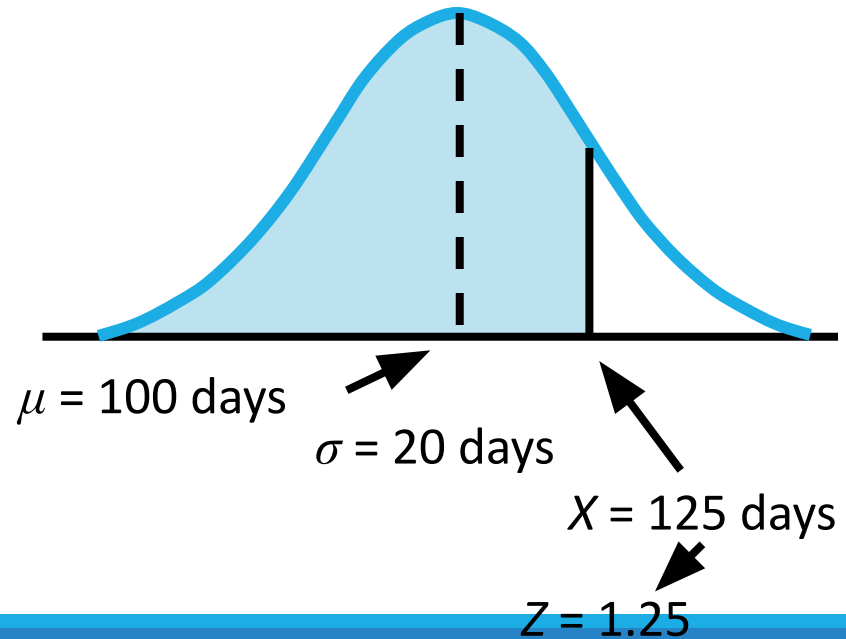


FIGURE 2.10



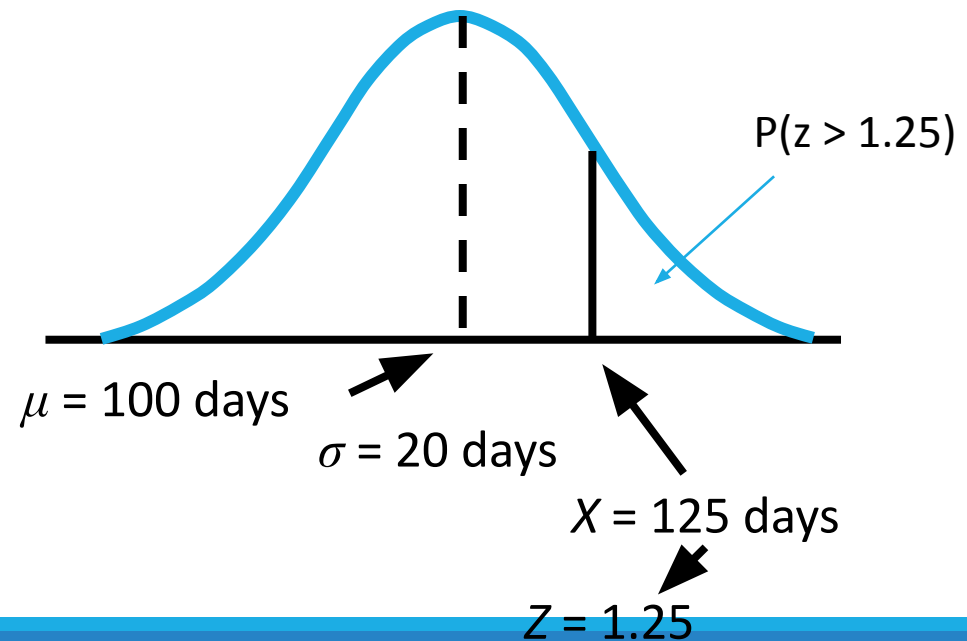
Haynes Construction Company

What is the probability that the company will not finish in 125 days and therefore will have to pay a penalty?

$$Z = \frac{X - \mu}{\sigma} = \frac{125 - 100}{20}$$
$$= \frac{25}{20} = 1.25 \quad P(z > 1.25) ?$$

- From the table for $Z = 1.25$
area $P(z > 1.25) = 1 - P(z < 1.25) = 1 - 0.8944 = 0.1056$ or **10.56 %**

FIGURE 2.10





Haynes Construction Company

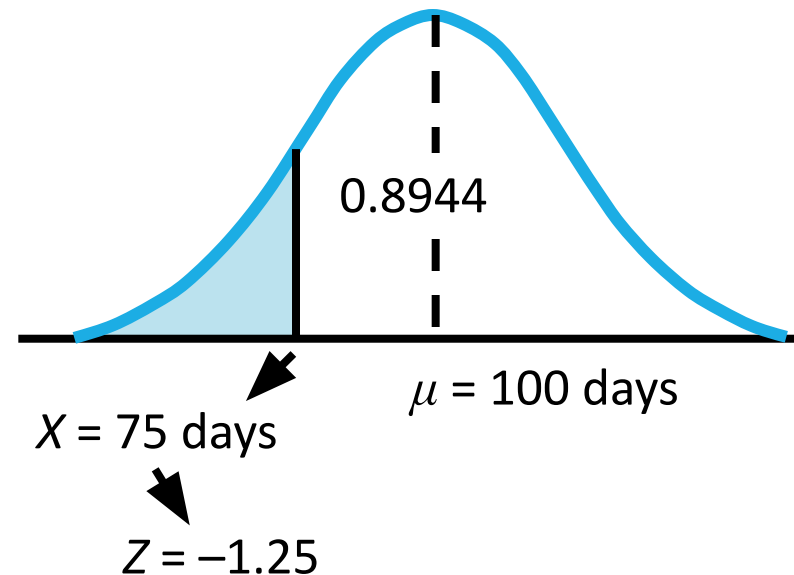
If finished in 75 days or less, bonus = \$5,000

- Probability of bonus?

$$Z = \frac{X - \mu}{\sigma} = \frac{75 - 100}{20}$$
$$= \frac{-25}{20} = -1.25 \quad P(z < -1.25) ?$$

- Because the distribution is symmetrical, equivalent to $Z = 1.25$ so area = 0.8944

FIGURE 2.11





aynes Construction Company

• If fir

– P

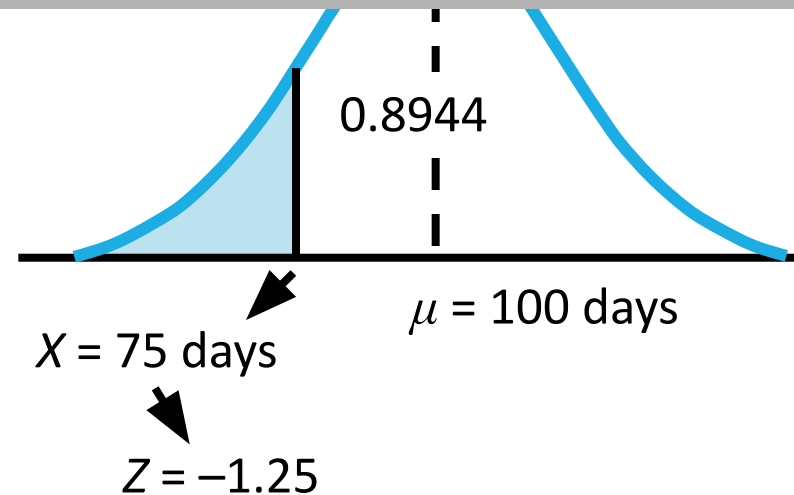
Z =

$$= \frac{75 - 100}{20} = -1.25$$

- Because the distribution is symmetrical, equivalent to $Z = 1.25$ so area = 0.89435

$P(z < -1.25) = 1.0 - P(z < 1.25)$
 $= 1.0 - 0.8944 = 0.1056$

The probability of completing the contract in 75 days or less is about 11%





Haynes Construction Company

Probability of completing between 110 and 125 days?

$$P(110 < X < 125) ?$$

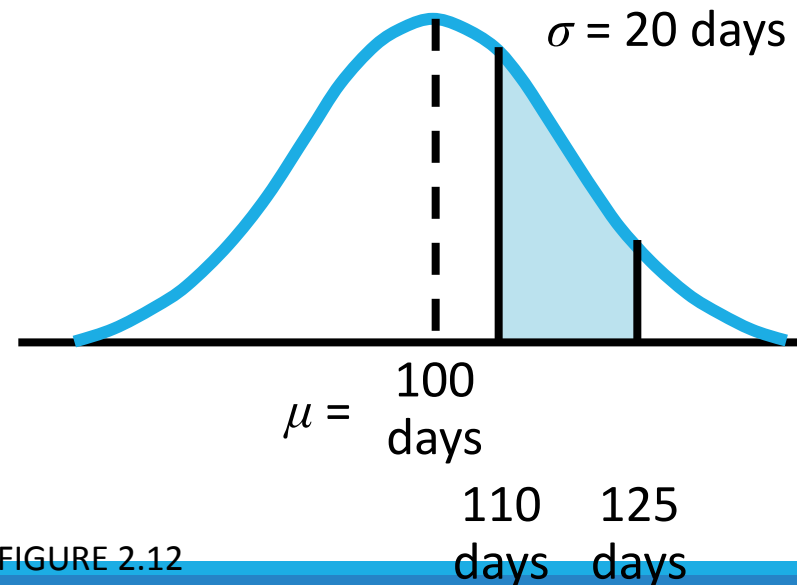


FIGURE 2.12



Probability of

$$P(110 < X < 125)$$

$$P(110 \leq X < 125) = 0.8944 - 0.6915 = 0.2029$$

The probability of completing between 110 and 125 days is about 20%

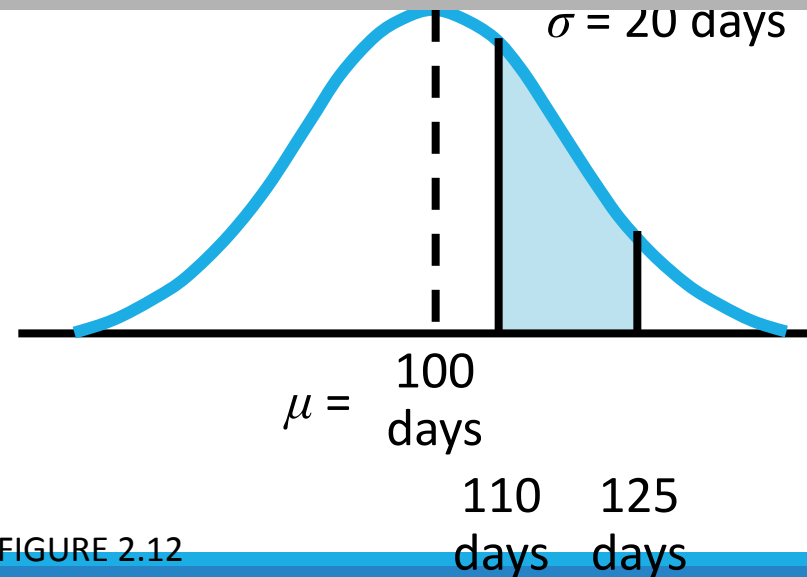


FIGURE 2.12