WESTMINSTER

INTERNATIONAL UNIVERSITY IN TASHKENT

An Accredited Institution of the University of Westminster (UK)

LECTURE 5
Percentage & Interest rates

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Lecture outline:

- Calculate a percentage of a given quantity;

- Increase or decrease a quantity by a given percentage;

- Find the original value of a quantity when it has been increased or decreased by a given percentage;

- Express one quantity as a percentage of another

- Interest rates (simple vs compound)

- Perpetuity and the rule of 72

- Nominal vs Real interest rate

- Effective annual interest rate and Annual Percentage Rate (APR)

Introduction

- The word 'percentage' is used regularly to describe anything from changes in the interest rate, to the number of people taking holidays abroad, to the success rate of the latest medical procedures or exam results.

- Percentages are a useful way of making comparisons, apart from being used to calculate the many taxes that we pay such as VAT, income tax, domestic fuel tax and insurance tax

- 'per cent' means 'out of 100'; which means 'divide by

Example: If you score 85% (using the symbol '%' for percentage) on a test then, if there were a possible 100 marks altogether, you would have achieved 85 marks. So 85% = 85/100 .

Let us look at some other common percentage amounts, and their fraction and decimal equivalents.

$$75\% = \frac{75}{100} = \frac{3}{4} = 0.75$$

$$25\% = \frac{25}{100} = \frac{1}{4} = 0.25$$

$$10\% = \frac{10}{100} = \frac{1}{10} = 0.1$$

$$5\% = \frac{5}{100} = \frac{1}{20} = 0.05$$

Writing fractions as percentages

Now let us look at writing fractions as percentages. For example, say you get 18 marks out of 20 in a test. What percentage is this?

First, write the information as a fraction. You gained 18 out of 20 marks, so the fraction is $\frac{18}{20}$. Since a percentage requires a denominator of 100, we can turn $\frac{18}{20}$ into a fraction out of 100 by multiplying both numerator and denominator by 5:

$$\frac{18}{20} = \frac{18 \times 5}{20 \times 5} = \frac{90}{100} = 0.9 = 90\%.$$

What if you scored 53 out of 68?

Then to change a fraction to a percentage, divide the numerator by the denominator and multiply by 100%

$$\frac{53}{68} \times 100\% = 77.94\%$$

Finding percentage amounts

You want tip a waiter in a restaurant 10% of your meal,

Find 10% of $24.5:

$$10\% \text{ of } \$24.5 = \frac{10}{100} \times \$24.5 = 0.1 \times \$24.5 = \$2.45$$

Find the total amount you spent on food and a tip without calculating the tip amount:

$$\$24.5 \times (1 + 0.1) = \$24.5 \times 1.1 = \$26.95$$

What will be the total amount be if your tip is 15%?

The cost of a computer is £699 including VAT. Calculate the cost before VAT if VAT is 17.5%.

£699 represents the cost including VAT, so that must equal the cost before VAT, plus the VAT itself, which is 17.5% of the cost before VAT. So, the total must be 100% + 17.5% = 117.5% of the cost before VAT. Thus, to find 1% we divide by 117.5.

117.5% of the price excluding VAT = £699,

1% of the price excluding VAT = $\dfrac{£699}{117.5}$.

To find the cost before VAT we want 100%, so now we need to multiply by 100.

Then the price excluding VAT = $\dfrac{£699}{117.5} \times 100 =$ £594.89.

We might wish to calculate the percentage by which something has increased or decreased. To do this we use the rule

$$\frac{actual\ increase\ or\ decrease}{original\ cost} \times 100\%$$

Example 1: Four years ago, you paid $30,000 for your flat. It is now valued at $42,000. Calculate the percentage increase in the value of the house.

Percentage increase$=\frac{\$12,000}{\$30,000} \times 100\% = 40\%$

Example 2: A car cost $12,000. After 3 years it is worth $8,000. What is the percentage decrease?

Percentage decrease$=\frac{\$4,000}{\$12,000} \times 100\% = 33\%$

Interest

Interest is the feeling of wanting to know or learn about something or someone.

Interest is money paid regularly at a particular rate for the use of money lent, or for delaying the repayment of a debt.

https://www.youtube.com/watch?v=Yut9qPyT9jE

The interest rate is the amount a lender charges a borrower and is a percentage of the principal—the amount loaned. The interest rate on a loan is typically noted on an annual basis known as the annual percentage rate (APR).

Simple vs Compound interest rate

- Future Value – Amount to which an investment will grow after earning interest.
- Simple interest – Interest earned only on the original investment; no interest is earned on interest.
- Compound interest – Interest earned on interest.

$$\text{Future value of } \$100 = \$100 \times (1 + r)^t$$

Present Value – Value today of a future cashflow.

$$\text{Present value} = \frac{\text{future value after } t \text{ periods}}{(1 + r)^t}$$

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Perpetuity – Stream of cash payments that never ends.

Present value of perpetuity = $\frac{C}{r} = \frac{cash\ payment}{interest\ rate}$

WHAT IS THE RULE OF 72?

And how does it work?

$$\frac{72}{RATE\ OF\ RETURN} = \text{NO. OF YEARS NEEDED TO DOUBLE YOUR MONEY}$$

Real interest rate

Nominal interest rate – Rate at which money invested grows.

Real interest rate – Rate at which the purchasing power of an investment increases.

$$1 + real\ interest\ rate = \frac{1 + nominal\ interest\ rate}{1 + inflation\ rate}$$

$$Real\ interest\ rate \approx nominal\ interest\ rate - inflation\ rate$$

Effective annual interest rate

- **Effective annual interest rate** – Interest rate that is annualized using compound interest.

$$(1+\text{annual rate})=(1 + \text{monthly rate})^{12}$$

Annual Percentage Rate (APR) – Interest rate that is annualized using simple interest.

$$\text{Annual Percentage Rate}=\text{Monthly rate} \times 12$$

Want to discover how long it will take to save $1,000,000? This "millionaire calculator" will help.

https://investinganswers.com/calculators/saving/million-dollar-savings-calculator-how-long-do-i-need-save-become-millionaire-3678#:~:text=If%20you%20start%20with%20%2420%2C000,million%20dollars%20in%2039.83%20years.

Concluding remarks

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- Math Centre,Percentages
  https://www.mathcentre.ac.uk/resources/uploaded/mc-ty-percent-2009-1.pdf

- Brealey, Myers and Marcus, Fundamentals of Corporate Finance, Chapter 3