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Physics 1

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Lecture 8

- Electrostatics
- Electric charge.
- Coulomb’s law.
- Electric field.
- Gauss’ law.
- Electric potential.

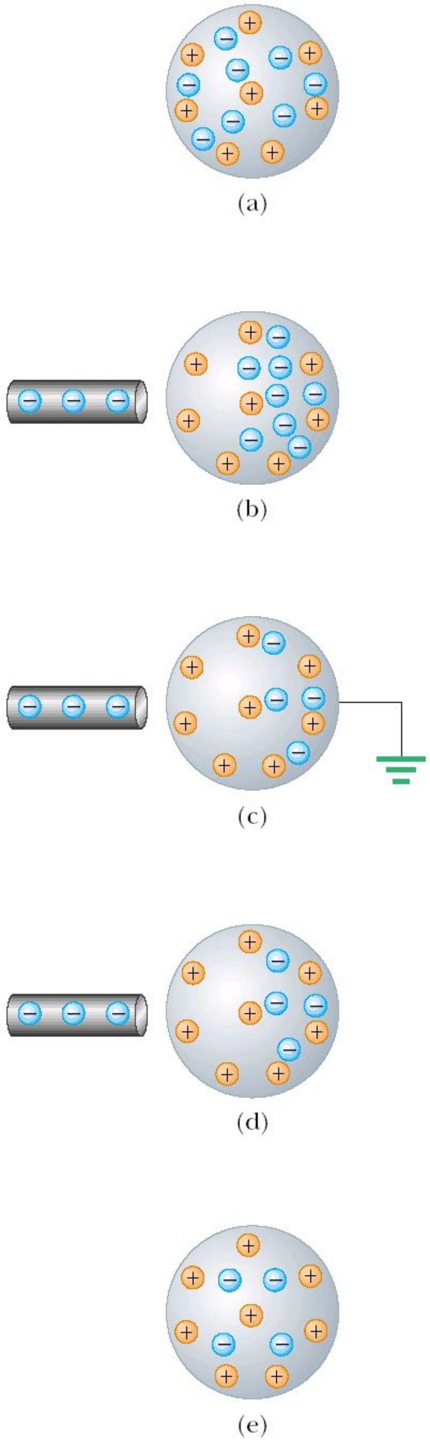
Electric Forces

Electric forces are dominant in the behavior of matter. The electric forces are responsible for:

- Electrons, binding to a positive nucleus, forming a stable atom;
- Atoms, binding together into molecules;
- Molecules binding together into liquids and solids;
- All chemical reactions;
- All biological processes.
- **Friction** and other **contact forces**.

Electrostatics

- Electrostatics is the science of stationary charges.
- There exists two types of charges – positive and negative.
- If an object has an excess of electrons, it is *negatively* charged; if it has a deficiency of electrons, it is *positively* charged.
- Like charges repel, and unlike charges attract.



Charging by induction

(a) We have a neutrally charged conductor.

(b) Negatively charged rod polarizes the sphere. The charge in the rod repels electrons to the opposite side of the sphere.

(c) Then we ground the sphere and some part of electrons is repelled into the Earth. There is **induced** positive charge near the rod.

(d) Then ground connection is removed.

(e) Eventually, we get positively charged sphere.

The Law of Conservation of Charge

- Charge of an isolated system is conserved.
- This law is a fundamental physical law: net charge is the same before and after any interaction.

Elementary charges

	Mass (kg)	Charge (C)
Neutron, n	1.675×10^{-27}	0
Proton, p	1.673×10^{-27}	1.602×10^{-19}
Electron, e^-	9.11×10^{-31}	-1.602×10^{-19}

- Elementary charges are electrons and protons. Usually only electrons can be free and take part in electrical processes.
- Excess of electrons causes negative charge and deficiency of electrons causes positive charge of a body.

Coulomb's law

- From Coulomb's experiments, we can generalize the following properties of the electric force between two stationary point charges:
  - is inversely proportional to the square of the separation r between the particles and directed along the line joining them;
  - is proportional to the product of the charges q_1 and q_2 on the two particles;
  - is attractive if the charges are of opposite sign and repulsive if the charges have the same sign;
  - is a conservative force.

Coulomb's Law

- The magnitude of the electric force is

$$F_e = k_e \frac{|q_1||q_2|}{r^2}$$

- $k_e = 8.9875 \times 10^9 \ \mathrm{N \cdot m^2/C^2}$ the Coulomb constant, it can be written in the following form:

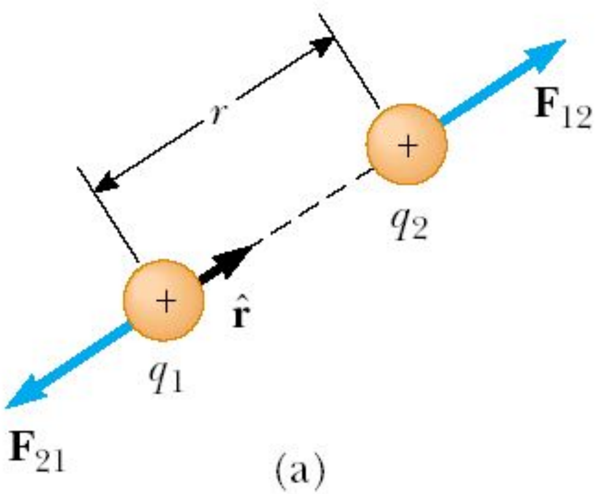
$$k_e = \frac{1}{4\pi\epsilon_0}$$

- where $\epsilon_0 = 8.8542 \times 10^{-12} \ \mathrm{C^2/N \cdot m^2}$ ric permittivity of free space.

In a vector form, the force exerted by charge q_1 on q_2 is:

$$\mathbf{F}_{12} = k_e \frac{q_1 q_2}{r^2} \hat{\mathbf{r}}$$

Where $\hat{r}$ is a unit vector directed from q_1 to q_2.



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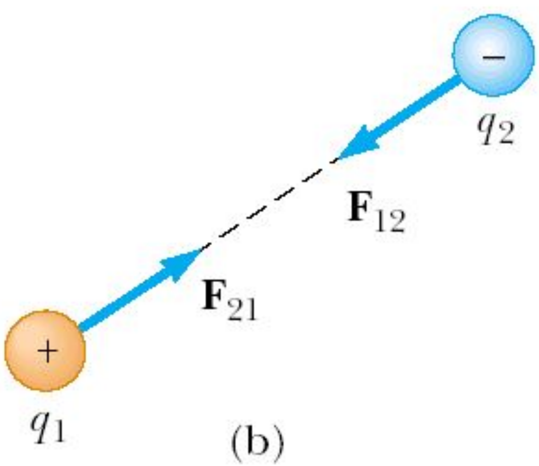
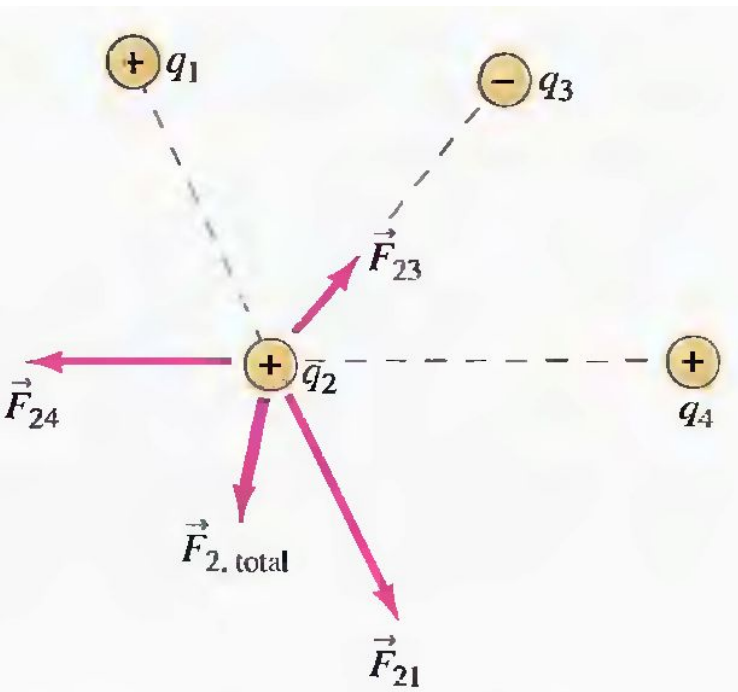


erent charges attracts

Forces of Multiple Charges

Electrostatic force is a vector quantity, so in the case of multiple charges the principle of superposition is applicable:

$$\vec{F} = \sum_{i=1}^{N} \vec{F}_i = \frac{q}{4\pi\varepsilon_0} \sum_{i=1}^{N} \frac{q_i}{r_i^2} \hat{r}_i$$



rce on charge q_2 is the

rces:

$$\vec{F}_{2,\text{total}} = \vec{F}_{21} + \vec{F}_{23} + \vec{F}_{24}.$$

Electric Field

- In general: field forces can act through space, producing an effect even when no physical contact occurs between interacting objects.
- Charges gives rise to an *electric field.*
- The electric field can be detected at any particular point by a small test positive charge q_o and observing if it experiences a force. Then the electric field vector is:

$$\mathbf{E} \equiv \frac{\mathbf{F}_e}{q_0}$$

- Note: force F_e and field E are not produced by the test charge q_o.

Typical Electric Field Values

Source	E (N/C)
Fluorescent lighting tube	10
Atmosphere (fair weather)	100
Balloon rubbed on hair	1 000
Atmosphere (under thundercloud)	10 000
Photocopier	100 000
Spark in air	>3 000 000
Near electron in hydrogen atom	5×10^{11}

Electric Field Vector

- The force exerted by q on the test charge q_0 is:

$$\mathbf{F}_e = k_e \frac{q q_0}{r^2} \hat{\mathbf{r}}$$

- Then dividing it by q_0 we get the electric field vector:

$$\mathbf{E} = k_e \frac{q}{r^2} \hat{\mathbf{r}}$$

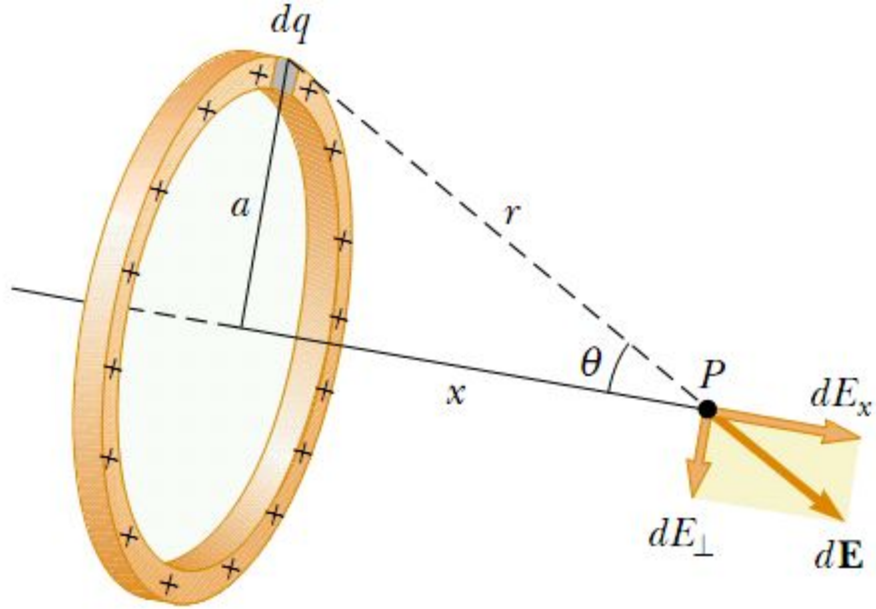
- Electric field is created by a charge.
- If a charge is positive then the electric field vector is directed away from the source charge.
- If a charge is negative then the electric field vector is directed to the source charge.

Continuous Charge Distribution

- Volume charge density $\rho \equiv \frac{Q}{V}$ $dq = \rho \, dV$
- Surface charge density $\sigma \equiv \frac{Q}{A}$ $dq = \sigma \, dA$
- Linear charge density $\lambda \equiv \frac{Q}{\ell}$ $dq = \lambda \, d\ell$

Electric Field of a Uniformly Charged ring

- A ring of radius *a* carries a uniformly distributed positive total charge *Q*. Let's find the electric field due to the ring along the central axis perpendicular to the plane of the ring.



eld at point P on the x

t of charge dq. dE has

pendicular components:

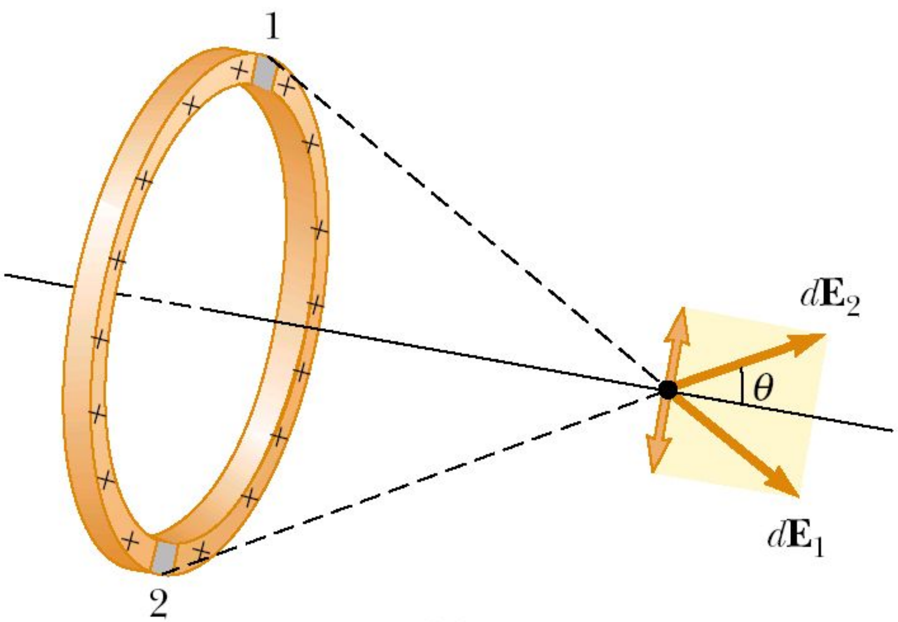
Using the symmetry: The

ent of the field at *P* due

is canceled by the perpendicular

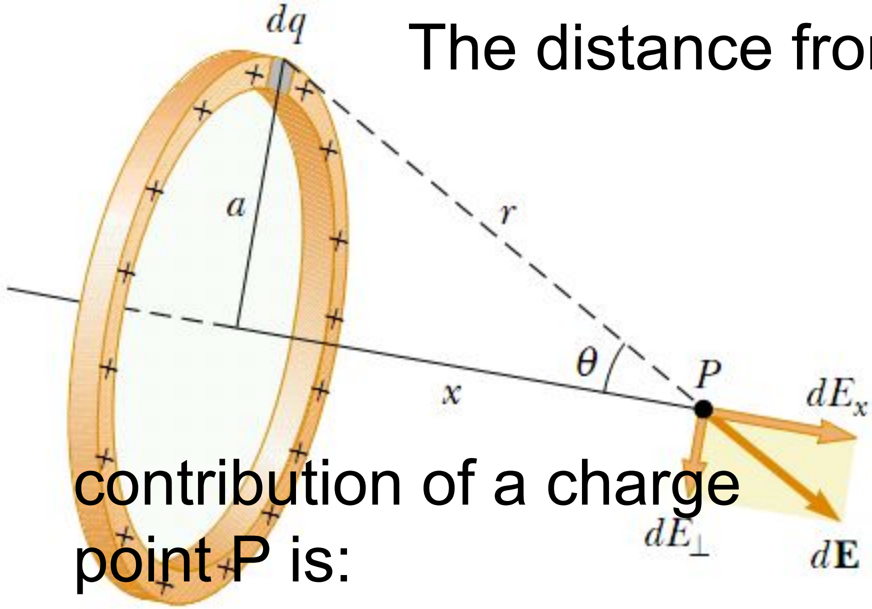
due to segment 2.

tal E is directed



(b)

The distance from a charge *dq* to point P:



$$r = (x^2 + a^2)^{1/2}$$

$$\cos\theta = x/r,$$

Then the contribution of a charge *dq* to electric field *E* at point P is:

$$dE_x = dE\cos\theta = \left(k_e\frac{dq}{r^2}\right)\frac{x}{r} = \frac{k_e x}{(x^2+a^2)^{3/2}}\,dq$$

All segments of the ring make the same contribution to the field at P because they are all equidistant from this point. Thus, we can integrate to obtain the total field at P:

$$E_x = \int \frac{k_e x}{(x^2+a^2)^{3/2}}\,dq = \frac{k_e x}{(x^2+a^2)^{3/2}}\int dq = \frac{k_e x}{(x^2+a^2)^{3/2}}\,Q$$

Extreme Case Analysis

- So we found the electric field of a uniformly charged ring along its symmetry axis at distance x from the centre of a ring:

$$E_x = \frac{k_e x}{(x^2 + a^2)^{3/2}} Q$$

k_e is the Coulomb constant, a – the ring's radius, Q – the charge of the ring.

- Let's analyze the obtained result for extreme cases:

1. If x=0, then $E=0$.
2. If x>>a, then we get the Coulomb formula for a point charge:

$$E = k_e \frac{Q}{r^2}$$

- Look more examples of calculating electric field for continuous charge distribution:
  - in Serway p.721-723,
  - Fishbane 642-647.

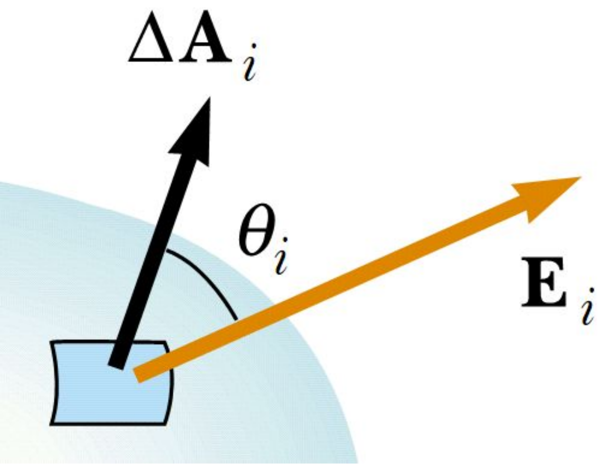
Gauss' Law

- The net flux of electric field through any enclosed surface are equal to the net charge inside that surface divided by permittivity of free space.

$$\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = \frac{q_{\text{in}}}{\epsilon_0}$$

- Here $\mathbf{E} \cdot d\mathbf{A}$ is a scalar product of electric field and differential of area vectors.

Electric Flux

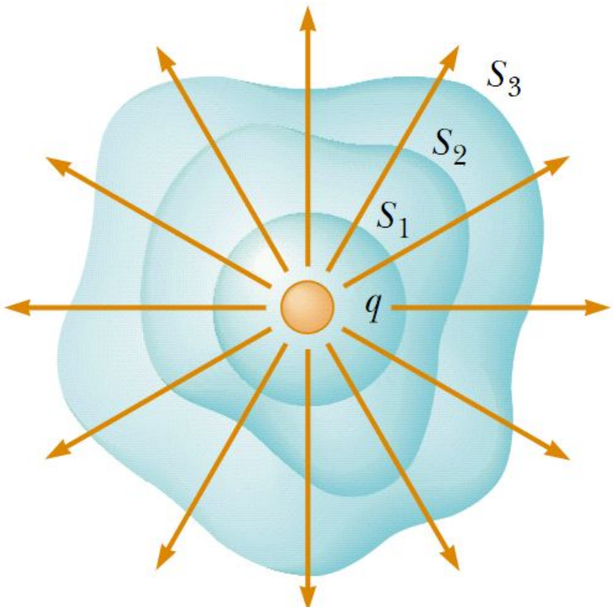


ΔA_i is a vector, which magnitude represents the area of the i-th element of the surface and direction is defined to be perpendicular to the surface element.

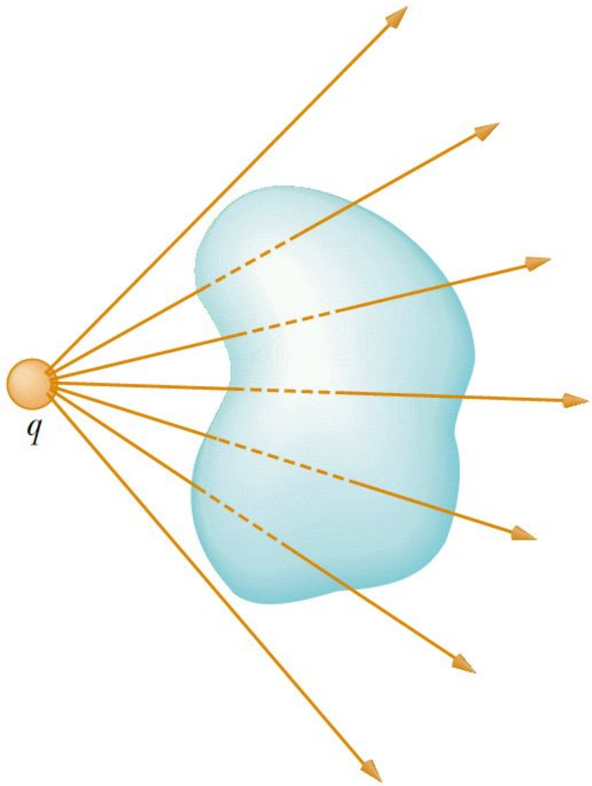
The variation in the electric field over one element of surface can be neglected if the element is sufficiently small.

The electric flux through this element is $\Delta\Phi_E = E_i\,\Delta A_i \cos\theta_i = \mathbf{E}_i \cdot \Delta\mathbf{A}_i$

$$\Phi_E = \lim_{\Delta A_i \to 0} \sum \mathbf{E}_i \cdot \Delta\mathbf{A}_i = \int_{\text{surface}} \mathbf{E} \cdot d\mathbf{A}$$



- According to the Gauss' theorem electric flux through any surface S_1, S_2, S_3 is the same.



- Electric flux from a charge located outside a surface equals zero. The number of lines entering the surface equals the number leaving the surface and the net number equals zero.

Electric Potential Energy

- For infinitesimal displacement *ds* the work done by the electric field on the charge is

  .

$$\mathbf{F} \cdot d\mathbf{s} = q_0 \mathbf{E} \cdot d\mathbf{s}$$

- Then the change in the potential energy of the charge-field system is

$$dU = -q_0 \mathbf{E} \cdot d\mathbf{s}.$$

- Thus for finite displacement from A to B the change in potential energy is

$$\Delta U = -q_0 \int_A^B \mathbf{E} \cdot d\mathbf{s}$$

- This line integral is not path-dependant, as the electric force is conservative.

Electric Potential

- The electric potential at any point in an electric field is

$$V = \frac{U}{q_0}$$

- The potential difference $\Delta V = V_B - V_A$ between two points A and B in an electric field is defined as

$$\Delta V \equiv \frac{\Delta U}{q_0} = -\int_A^B \mathbf{E} \cdot d\mathbf{s}$$

q_0 is a test charge.

Potential Properties

Units in SI

Charge	Q	C (Coulomb)
Electric potential	V	J/C=V (volt)
Electric field	E	N/C=V/m