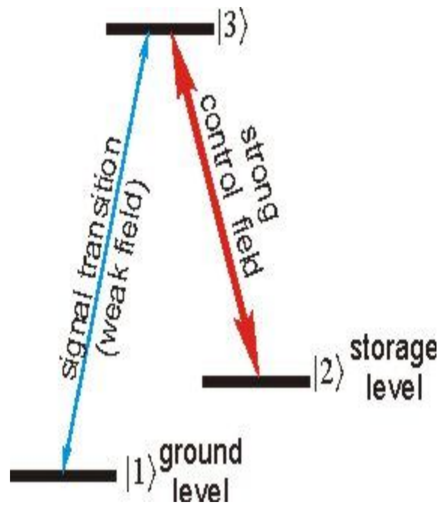
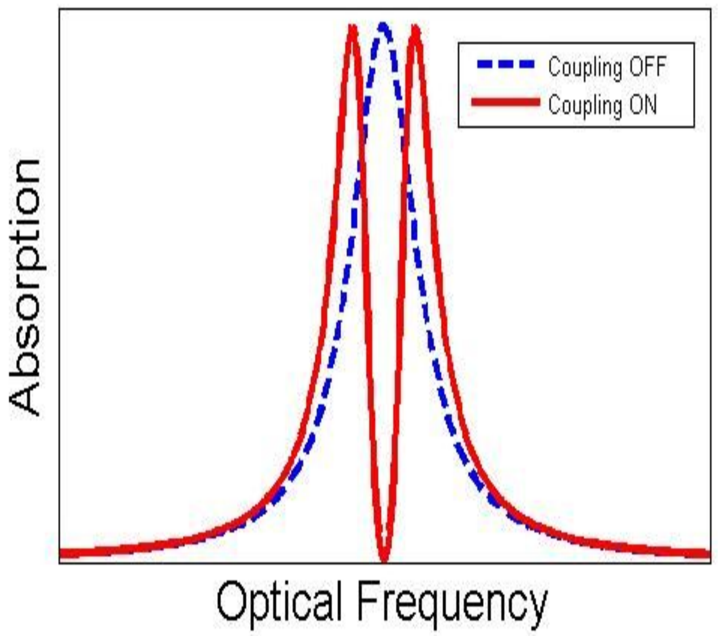
# Some unusual subwavelength resonances and effects: EIT, Fano-resonance, Anapoles. Review

# Electromagnetically Induced Transparency

**Electromagnetically induced transparency** - is a effect of a coherent optical nonlinearity which renders a medium transparent window over a narrow spectral range within an absorption line.

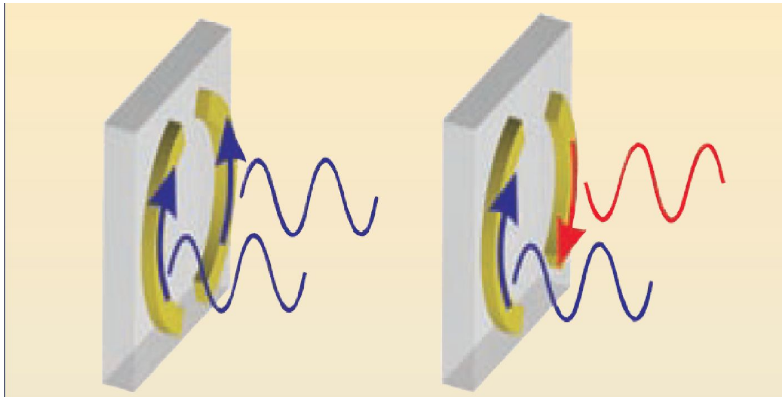


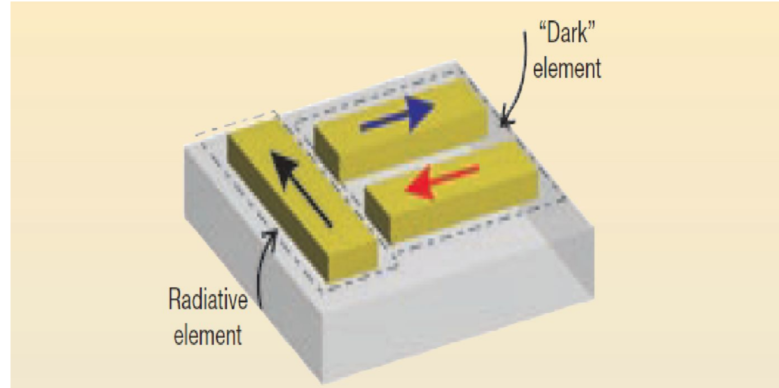



# How to see EIT in metamaterials

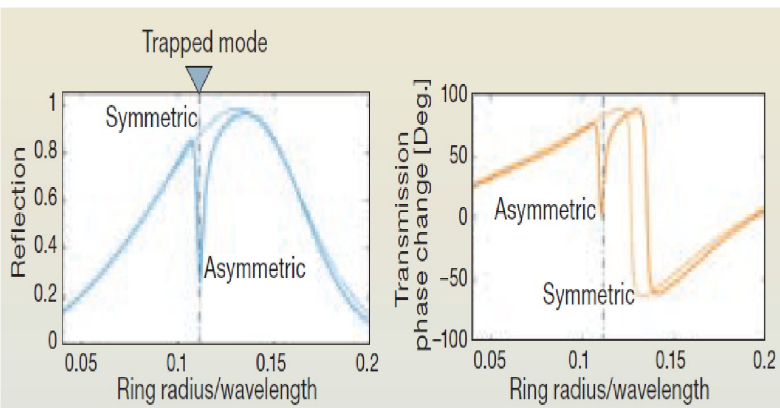
1. Split rings with asymmetry (Metamaterial Induced Transparency)

2. Plasmonic molecule with Fano-resonance (Plasmon Induced Transparency)

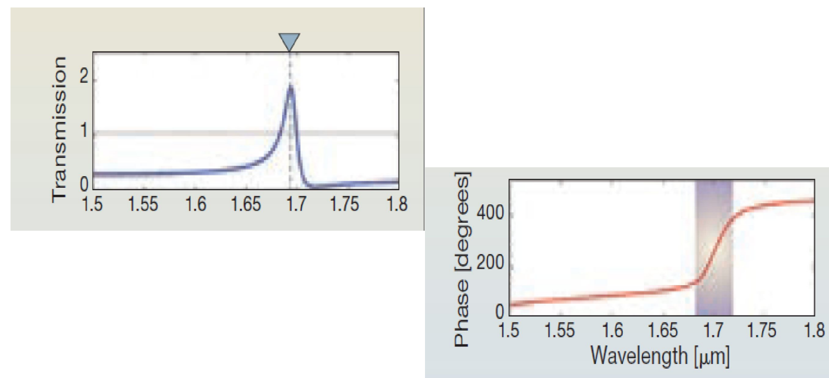





## "Trapped mode" resonance




## Bright/dark mode resonance




Metamaterial-Induced Transparency: Sharp Fano Resonances and Slow Light //Nikitas Papasimakis and Nikolay I. Zheludev //Optics and Photonics News Vol. 20, Issue 10, pp. 22-27 (2009),

Plasmon-Induced Transparency in Metamaterials // Shuang Zhang, Dentcho A. Genov, Yuan Wang, Ming Liu, and Xiang Zhang // PRL 101, 047401 (2008)

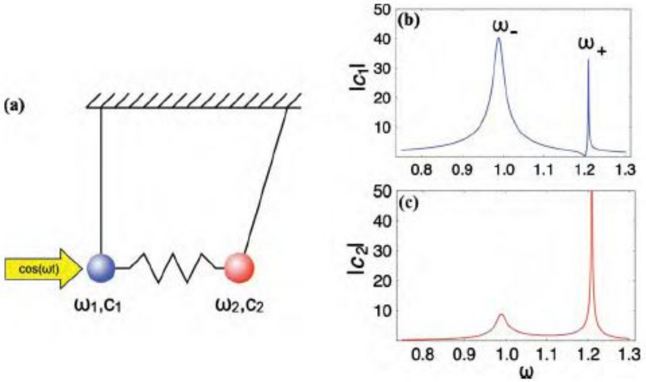
Sharp Trapped-Mode Resonances in Planar Metamaterials with a Broken Structural Symmetry // V. A. Fedotov, M. Rose, S. L. Prosvirnin, N. Papasimakis and N. I. Zheludev // PRL 99, 147401 (2007)

# Fano- resonance

**Fano resonance** is a type of resonant scattering phenomenon that gives rise to an asymmetric line-shape. Interference between a background and a resonant scattering process produces the asymmetric line-shape.

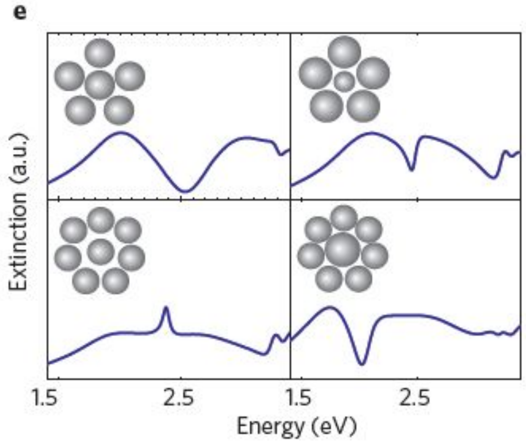
$$I \propto \frac{(F\gamma + \omega - \omega_0)^2}{(\omega - \omega_0)^2 + \gamma^2}$$

$$Q_{sca} = \frac{2}{q^2} \sum_{\lambda=1}^{\infty} (2\lambda + 1)\left\{|a_\lambda|^2 + |b_\lambda|^2\right\}$$



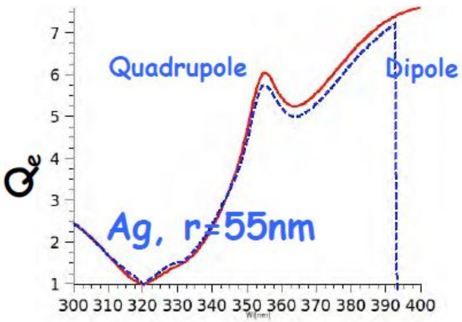
# Fano- resonance

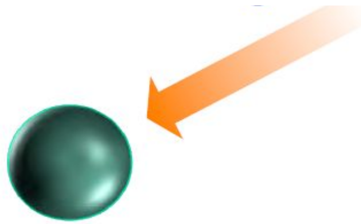
- The Fano resonance in metamaterials associated with mutual excitation of at least two scattering channels - modes occurring in the inclusions of metamaterials. This is possible due to the collective excitation of **dark mode**, which interferes with a resonances **bright mode**. As a result of such interference, it occurs **asymmetrical peak** of the transmission of electromagnetic waves through the layer of the metamaterial. Usually, **bright mode** has a strong connection with the incident plane wave. In contrast, **dark mode** weakly coupled with the incident plane wave and can not be directly excited it. Thus, in the vicinity of the resonance frequency, constructive and destructive interference between these modes are occured, which manifests itself as acute asymmetrical peak Fano-resonance in the scattering metamolecules




$$Q_{sca} = \frac{2}{q^2}\sum_{\lambda=1}^{\infty}(2\lambda+1)\left\{|a_\lambda|^2 + |b_\lambda|^2\right\}$$

# Fano- resonance in the metallic nano-sphere



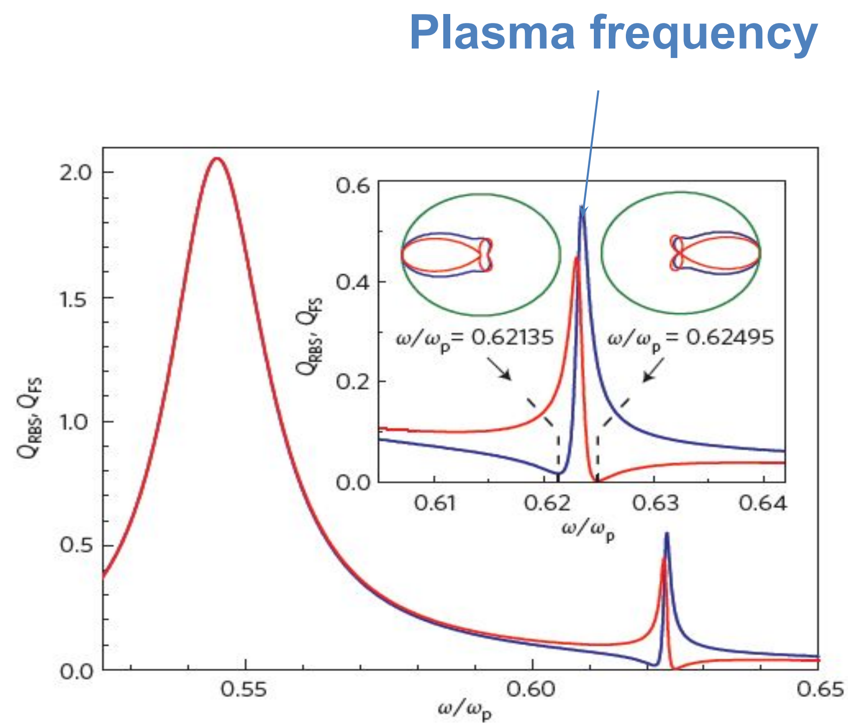



Figure 2 | **Mie scattering against a solid metallic sphere.** Radar back

BROAD mode (Lowest-order, E-Dipole)
DARK mode (Higher-order, EM Multipole)

$$Q_{sca} = \frac{2}{q^2}\sum_n (2n+1)\left[|a_n|^2 + |b_n|^2\right]$$

Nature Mat. **9** 707

# Fano- resonance

- A narrow spectral line
- Frequency scanning
- High Q-factor
- Strong field localization
- Sensing

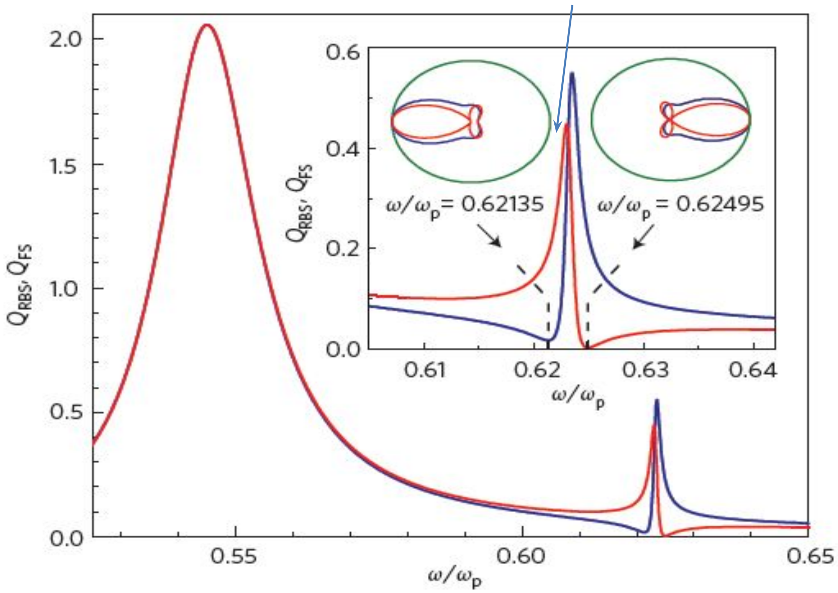
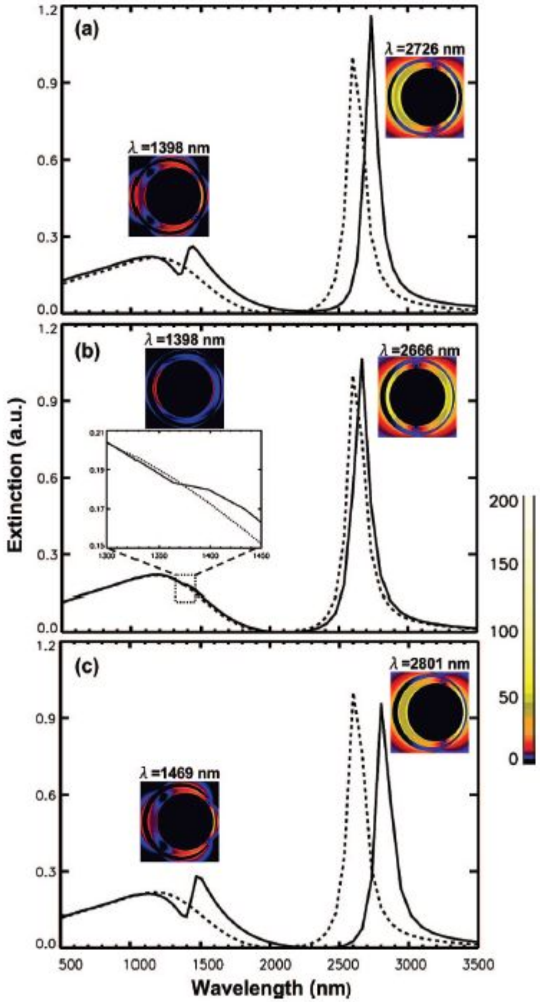
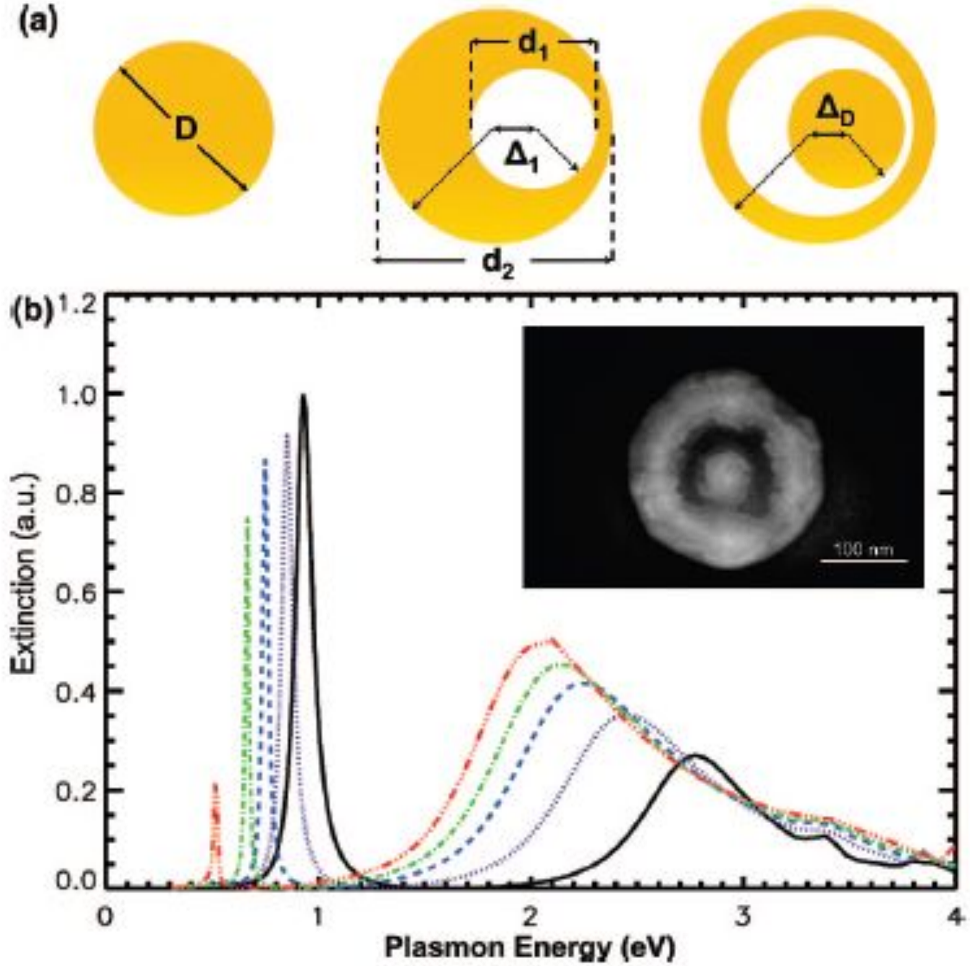



**Figure 2 | Mie scattering against a solid metallic sphere.** Radar back

Nature Mat. **9** 707

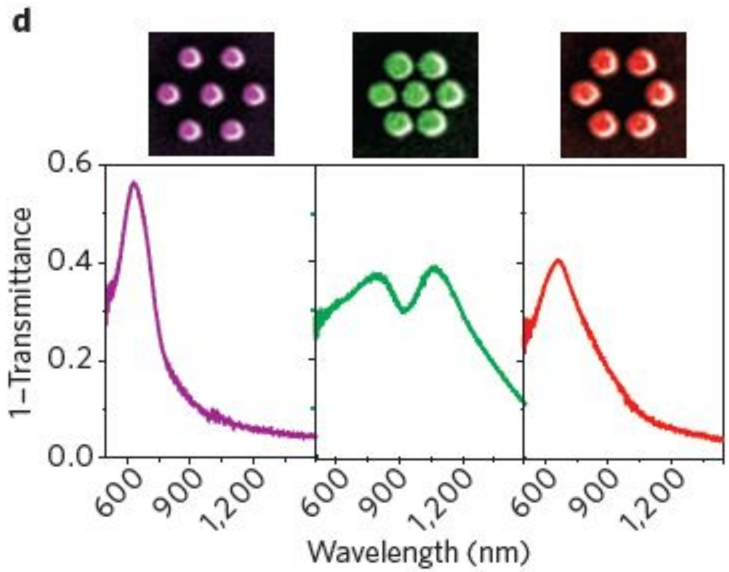
# Fano- resonance. Other types of the particles. System of the nano-disks

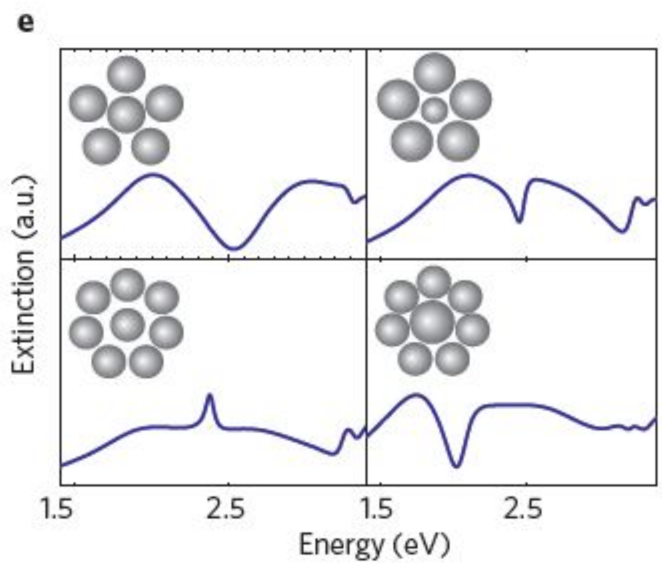


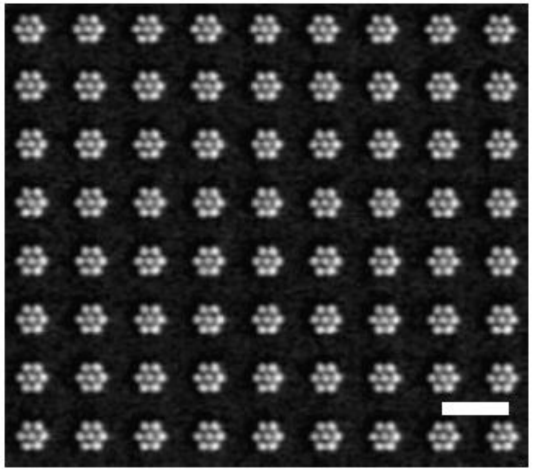




Nano Lett. **8** 3983

# Fano- resonance. Other types of the particles. Nano-clusters of Ag, Au



Nano Lett. **10** 2721
Science **328** 1135

# 2. Vortex/whirpool resonances in nano-particles

**Vortex resonance**- occurs in plasmonic particles and accompanied by **vortex** distribution of the Poynting vector close to the nano-particle and penetrated the fields inside particle. Strong retardation, absorbtion.

# Vortex/whirpool resonances in nano-sphere. Extinction's coefficients

$$c_n^{TM} = -\frac{\epsilon j_n(ka)[(k_0a)j_n(k_0a)]' - \epsilon_0[(ka)j_n(ka)]'j_n(k_0a)}{\epsilon j_n(ka)[(k_0a)h_n(k_0a)]' - \epsilon_0[(ka)j_n(ka)]'h_n(k_0a)},$$

$$c_n^{TE} = -\frac{j_n(ka)[(k_0a)j_n(k_0a)]' - [(ka)j_n(ka)]'j_n(k_0a)}{j_n(ka)[(k_0a)h_n(k_0a)]' - [(ka)j_n(ka)]'h_n(k_0a)},$$

где $k = \omega\sqrt{\epsilon\mu_0}$

$k_0 = \omega\sqrt{\epsilon_0\mu_0}$

**The strong field conditions:**

***c<<1- week scattering, strong concentration of the field inside particle***

# Vortex/whirpool resonances in nano-sphere. Extinction's coefficients

**The strong field localization conditions**

$$\epsilon j_n(ka)[(k_0a)y_n(k_0a)]' = \epsilon_0[(ka)j_n(ka)]'y_n(k_0a) \quad \text{(TM)},$$

$$j_n(ka)[(k_0a)y_n(k_0a)]' = [(ka)j_n(ka)]'y_n(k_0a) \quad \text{(TE)},$$

Y- Neiman function

$$\mathrm{Im}[\epsilon/\epsilon_0] \ll (k_0a)^{2n+1}\frac{n+1}{n^2[(2n-1)!!]^2}.$$

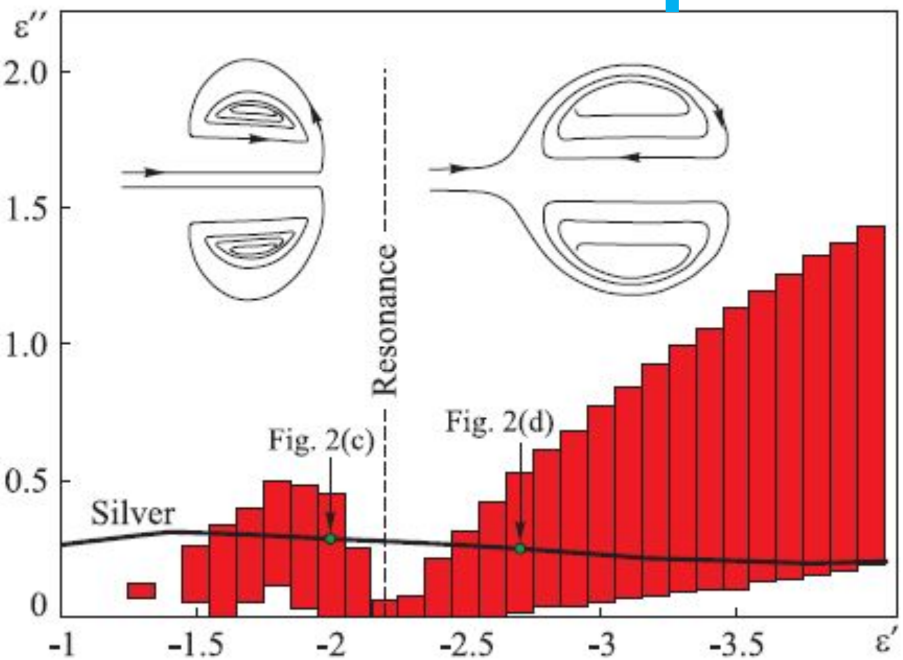
J. Opt. Soc. Am. B **24** A89
Opt. Express **13** 8372
Phys. Rev. Lett. **97** 263902

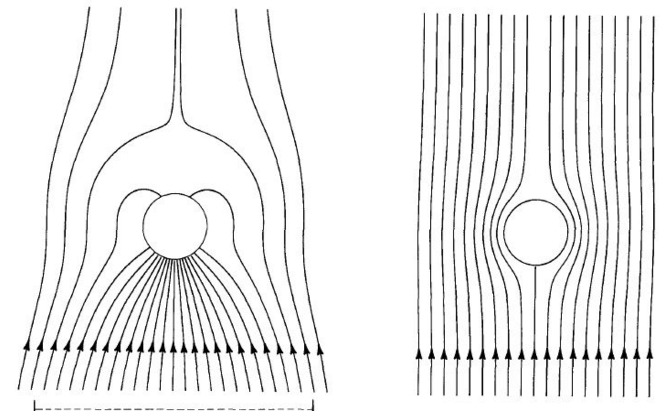
**How can a particle absorb more than the light incident on it?**

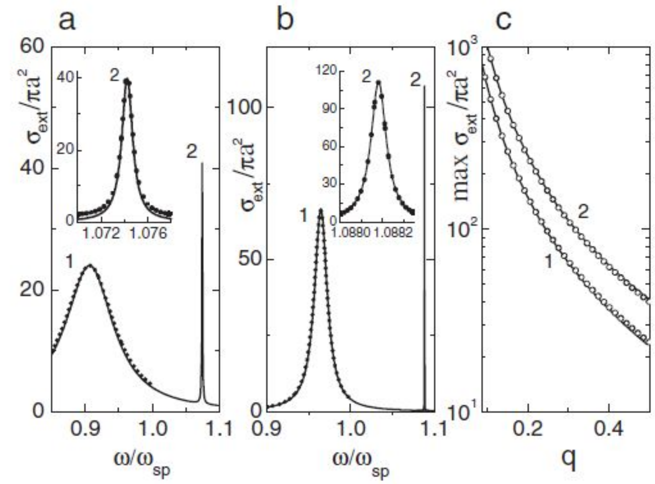
Am. J. Phys. 51 (4), April 1983

Craig F. Bohren
*Department of Meteorology, Pennsylvania State University, University Park, Pennsylvania 16802*

# Perfect absorption






Opt. Express **13** 8372
Phys. Rev. Lett. **97** 263902

# Vortex/whirpool resonances in nano-sphere. Examples. Poynting Vector

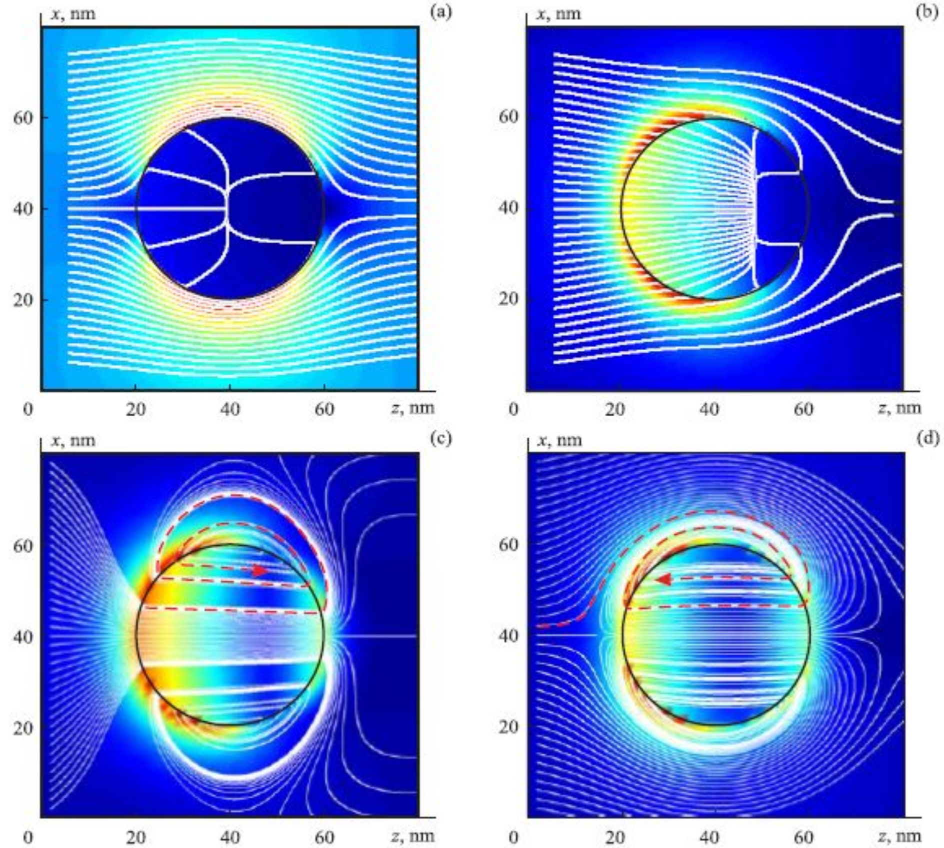
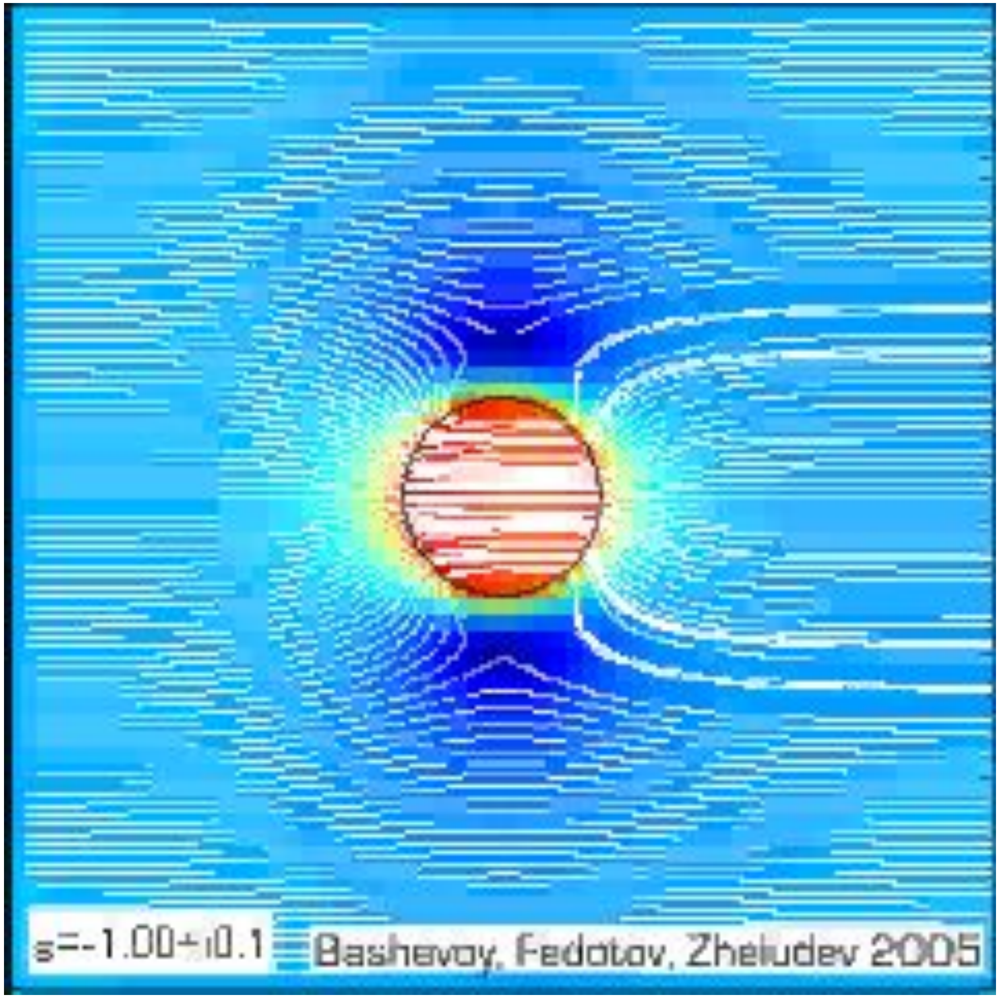



Fig. 2. [Movie 2.5 MB, 10.5 MB version] Mie Theory: powerflow distribution around a spherical nanoparticle with a radius of approximately 20 nm ($\lambda/r = 20$) in the plane containing the directions of propagation (from left to right) and polarization of the incident light. The colors indicate the absolute value of the Poynting vector, the white lines show the direction of powerflow. (a) $\varepsilon = -2.0 + i10.0$, $\lambda = 400$ nm; (b) $\varepsilon = -2.0 + i1.0$, $\lambda = 400$ nm; (c) $\varepsilon = -2.0 + i0.28$ — the dielectric coefficient of silver at $\lambda = 354$ nm. Red dashed lines indicate outward vortex structure; (d) $\varepsilon = -2.71 + i0.25$ — the dielectric coefficient of silver at $\lambda = 367$ nm. Red dashed lines indicate inward vortex structure [15].

# Vortex/whirpool resonances in nano-sphere. Examples. Poynting Vector



# Vortex/whirpool resonances. Example of nano- ellipsoid . Poynting Vector

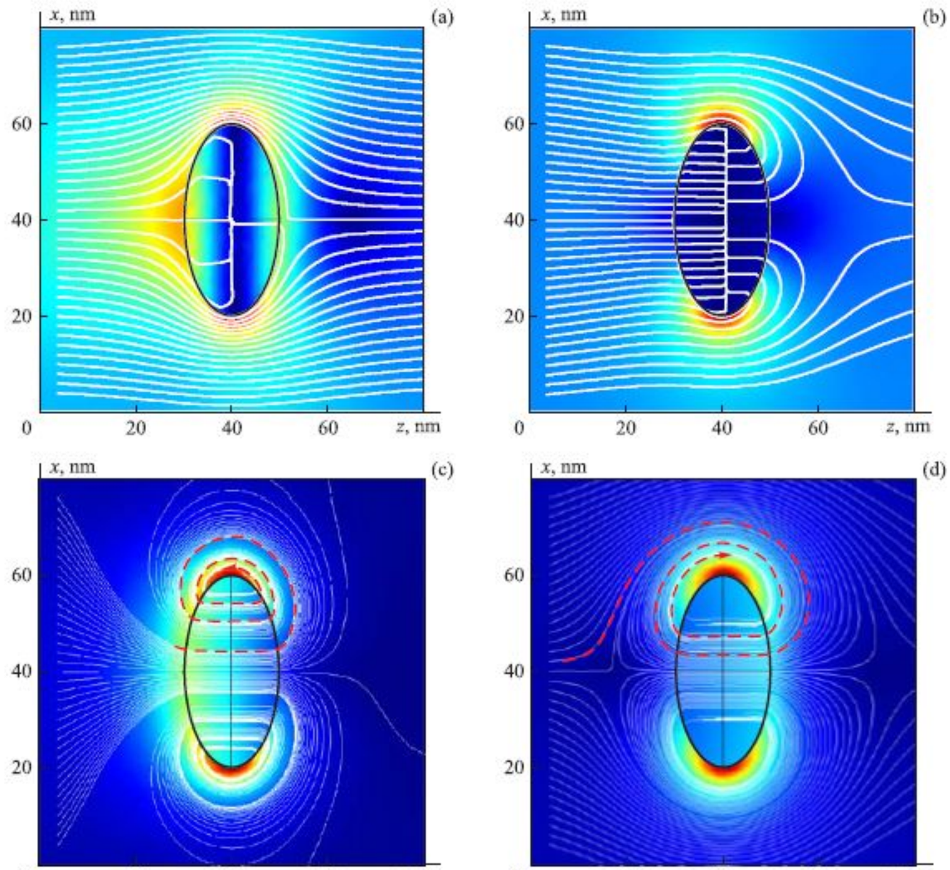



Fig. 3. 3D finite element modelling: powerflow distribution around an oblate spheroidal nanoparticle (with a semi-major axial radius of approximately 20 nm ($\lambda/r = 20$) and an aspect ratio of 2) in the plane containing the directions of propagation (from left to right) and polarization of the incident light. The colors indicate the absolute value of the Poynting vector, the white lines show the direction of powerflow. (a) $\varepsilon = -3.52 + i10.0$, $\lambda = 400$ nm; (b) $\varepsilon = -3.52 + i1.0$, $\lambda = 400$ nm; (c) $\varepsilon = -3.37 + i0.2$ — the dielectric coefficient of silver at $\lambda = 380$ nm. Red dashed lines indicate outward vortex structure; (d) $\varepsilon = -4.0 + i0.2$ — the dielectric coefficient of silver at $\lambda = 392$ nm. Red dashed lines indicate inward vortex structure [15].

# Vortex/whirpool resonances. Example of Yin and yang Symbol

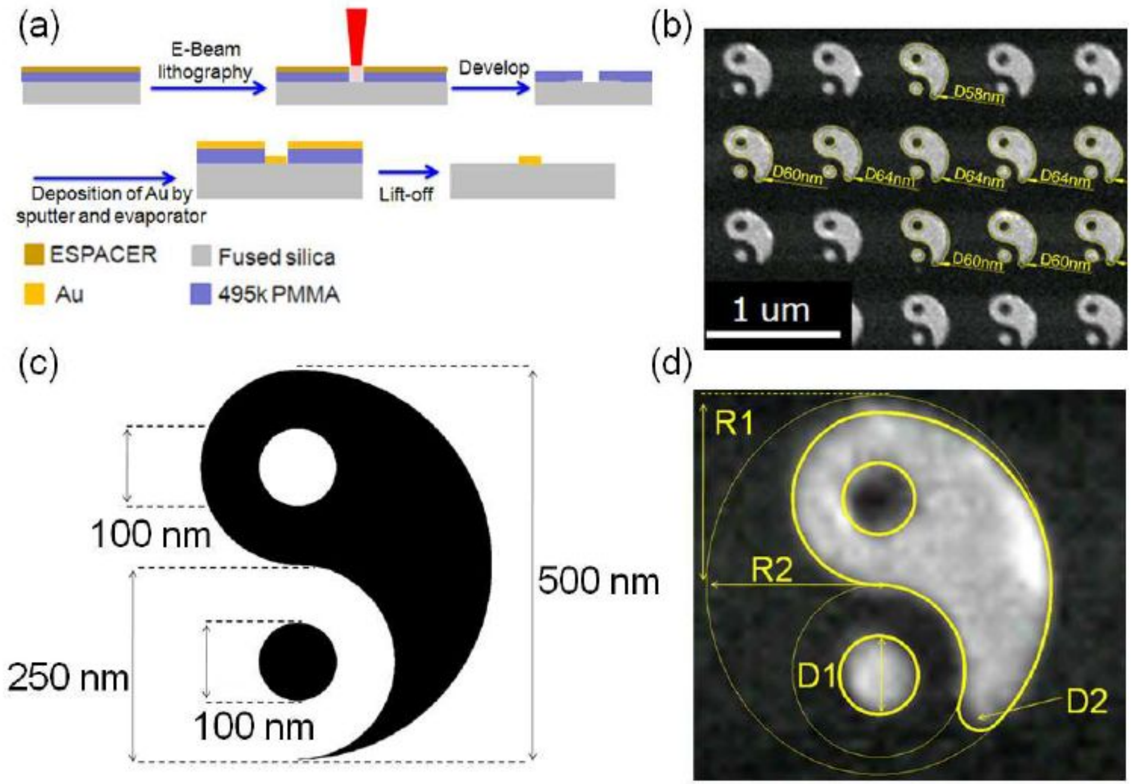
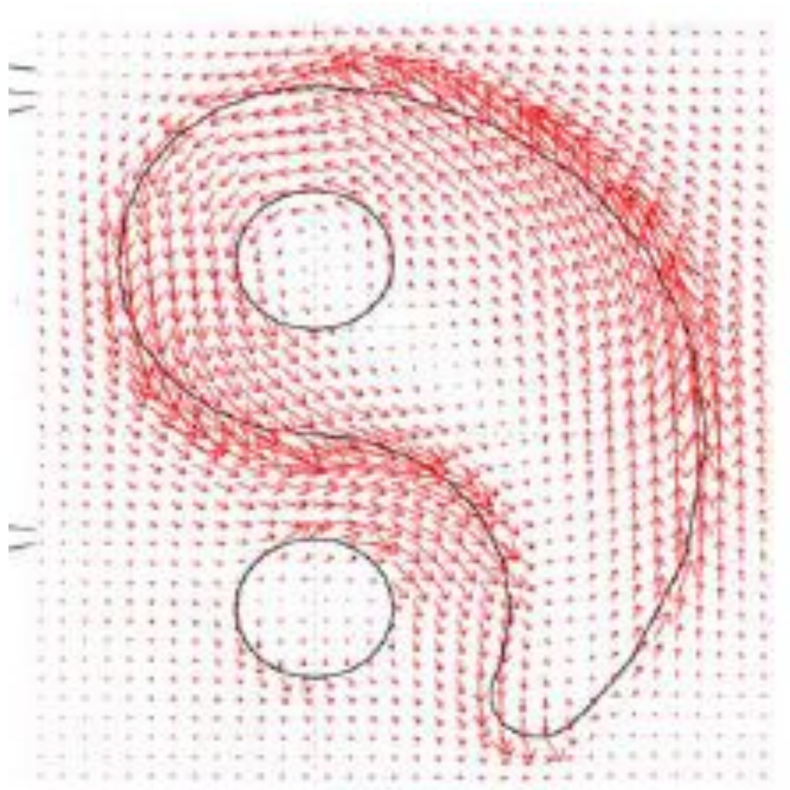



Fig. 1. (a) Schematic diagram showing the process of e-beam lithography. Our samples were fabricated without any adhesion layer (such as Ti or Cr) between the gold film and the glass substrate. A 3 nm film of gold was sputter-deposited on the developed resist layer, followed by the thermal evaporation of another 27 nm of gold. (b) SEM image of a small region from the fabricated sample, showing diameter of tails. (c) The ideal Taiji mark designed for the studies reported in this paper. (d) Magnified view of a single cell in the SEM image of frame (b), showing the fitting curves of a fabricated Taiji mark ($R_1$ = 275 nm, $R_2$ = 250 nm, $D_1$ = 100 nm, diameter of the tangent circle $D_2$ = 60 nm).

Opt. Express **18** 19665

# Vortex/whirpool resonances. Example of Yin and Yang Symbol. Vector Poynting



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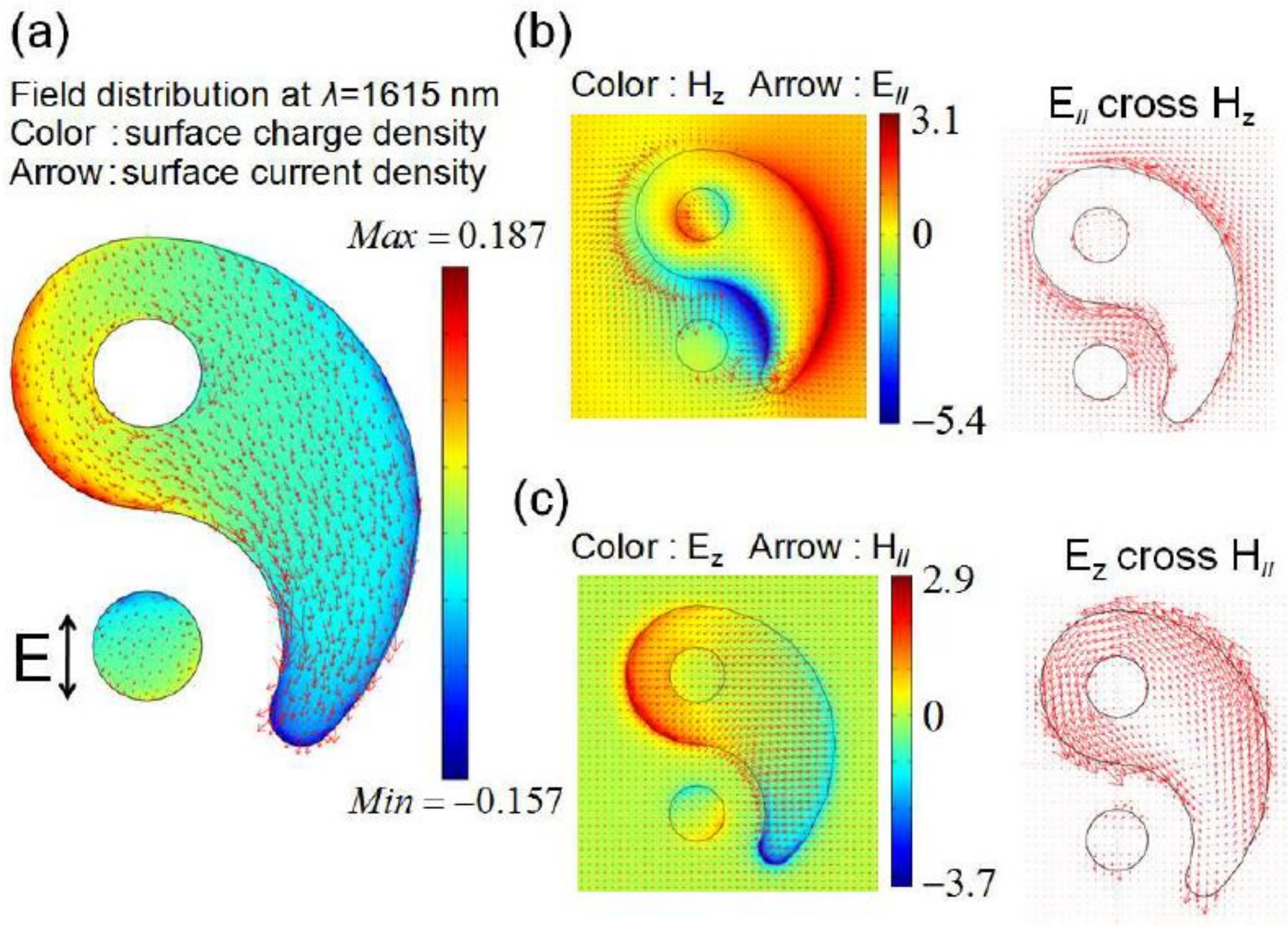
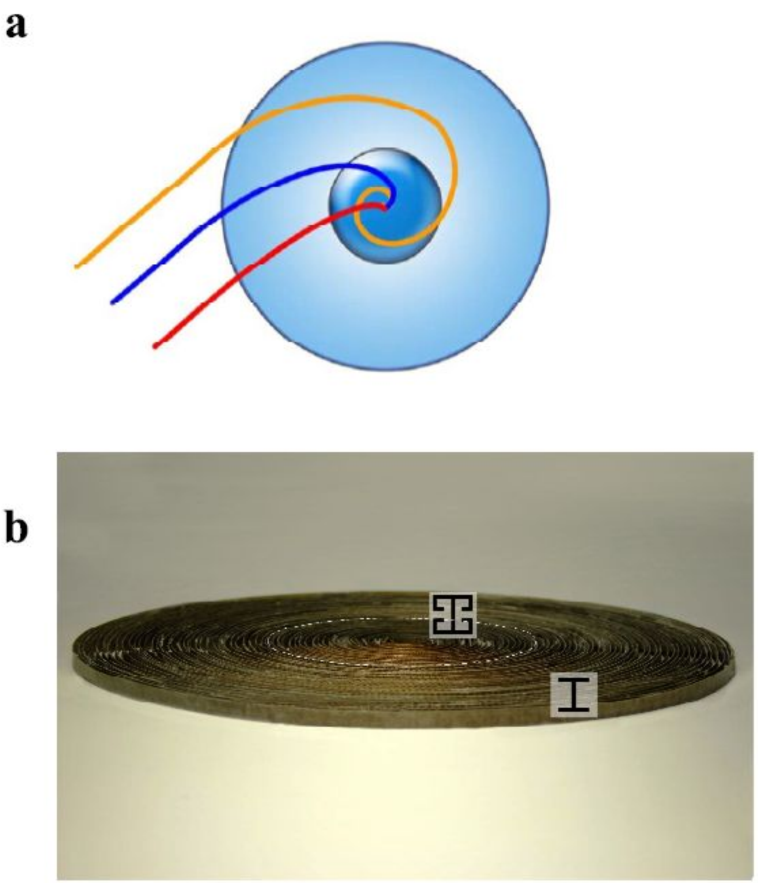
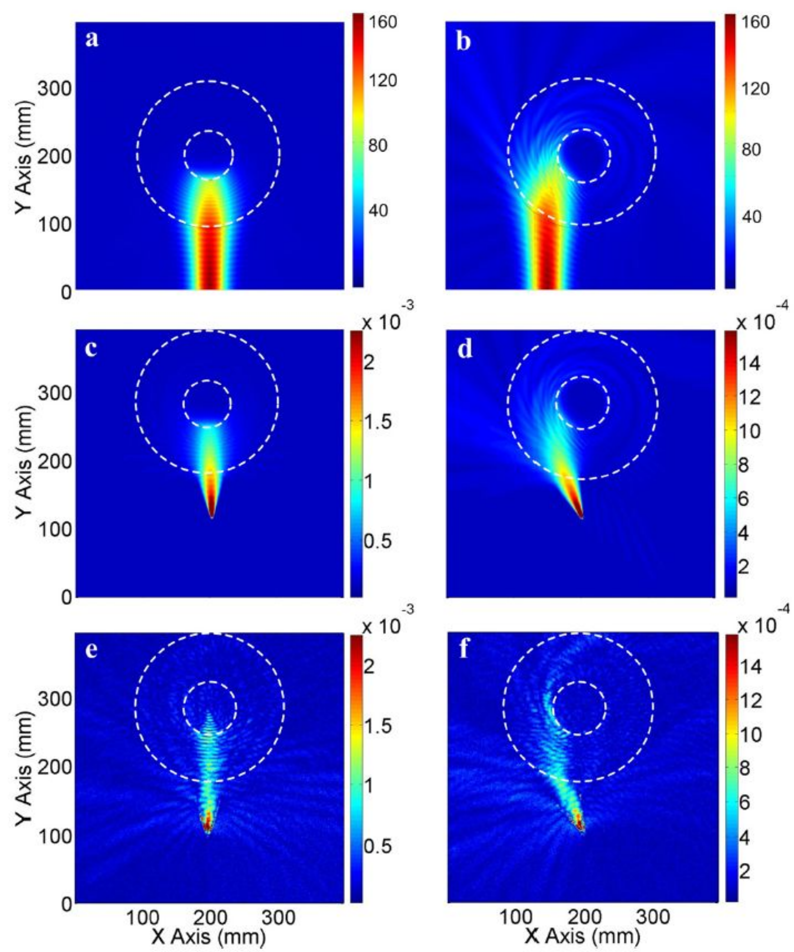
# Vortex/whirpool resonances. Example of Yin and yang Symbol. Fields distributions



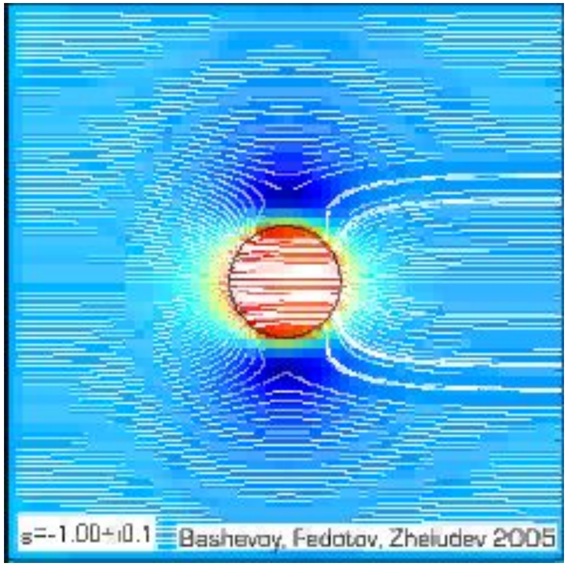

Opt. Express **18** 19665

# Vortex/whirpool resonances. Example of a model of a Black hole. Vector Poynting



# Vortex/whirpool resonances

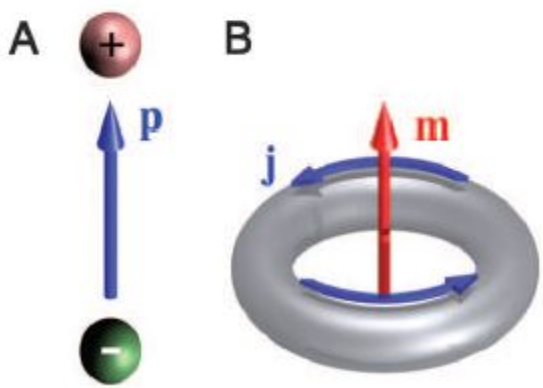


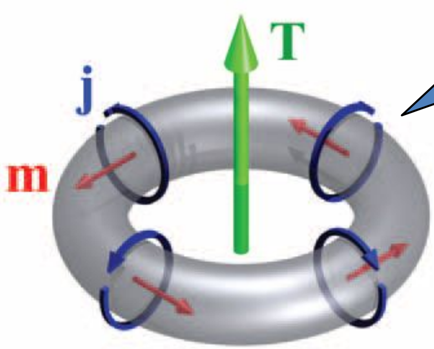

**Applications:**

- **Strong field localization**
- **As element of delay line**
- **High Q-factor resonator**
- **Element of the nano-antennas?**

# 3. Toroidal Dipole in Metamaterials

**What is toroidal dipole**




**Toroidal dipole**

*Curl* ***B***

**Unusual electromagnetic phenomena**

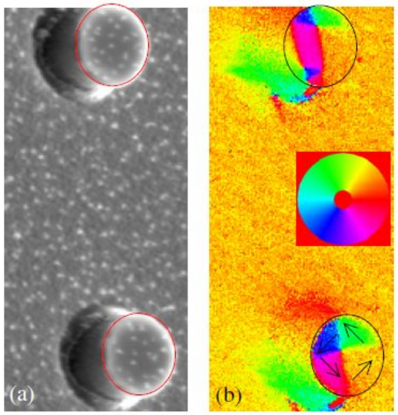
- **Extremely High Q-factor**
- **Optical activity & circular dichroism**
- **Negative refraction and backward waves**
- **Generation of waves with gauge irreducible vector potentials with no EM fields**
- **Aharonov-Bohm Effect**

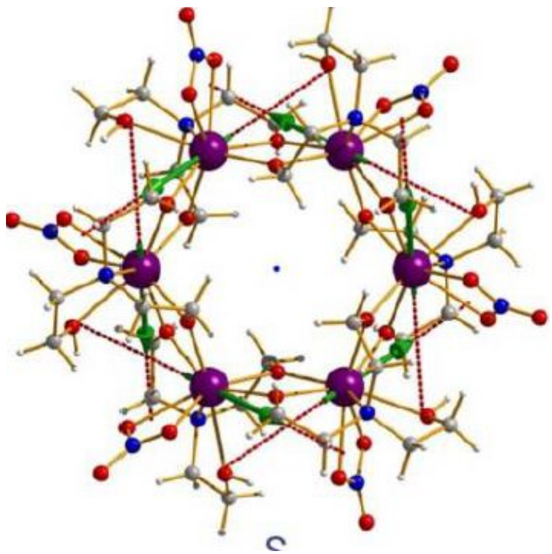
- **The Radiation pattern of P=T**
- **Quantum Effects in metamaterials due to manipulation with Vector Potentials**

T. Kaelberer *et al, Science* 330, 1510 (2010)

*B. Zel'dovich, Sov. Phys. JETP, 6,1184 (1958)*

# Toroidal dipole in nature

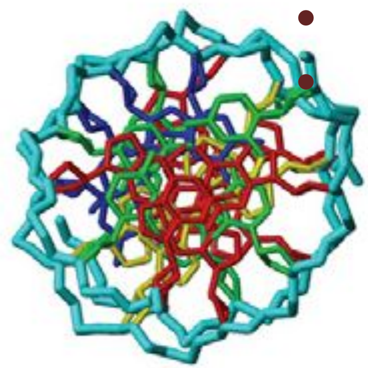


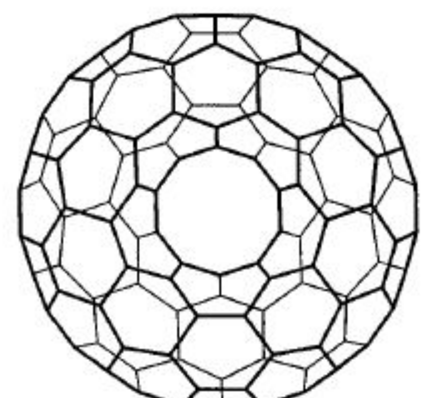
## Why toroidal dipole?

- Correct characterization of toroidal objects
- Interaction with electromagnetic fields
- Sensitive sensors of toroidal objects
- Fundamental interest
- Quantum effects like Aharonom-Bohm effect

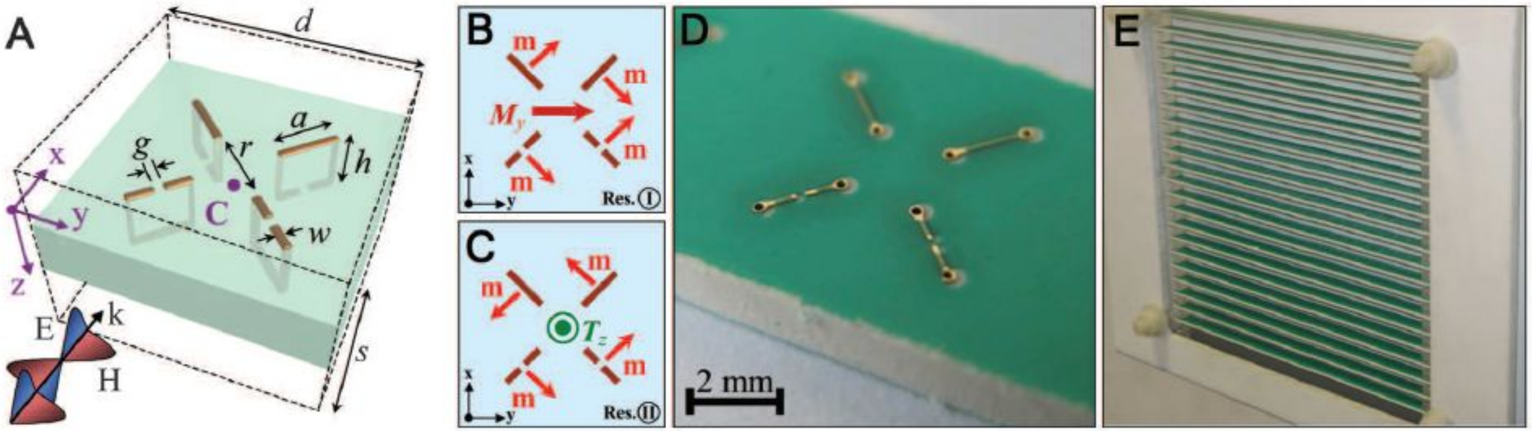
Y. B. Zel'dovich, 1958
Naumov I, at al., 2004
M. Kläui at al., 2003
Y. F. Popov at al., 1998
Y. V. Kopaev at al., 2009
L. Ungur at al., 2012
A. Ceulemans at al., 1998
A. Karsisiotis at al., 2013

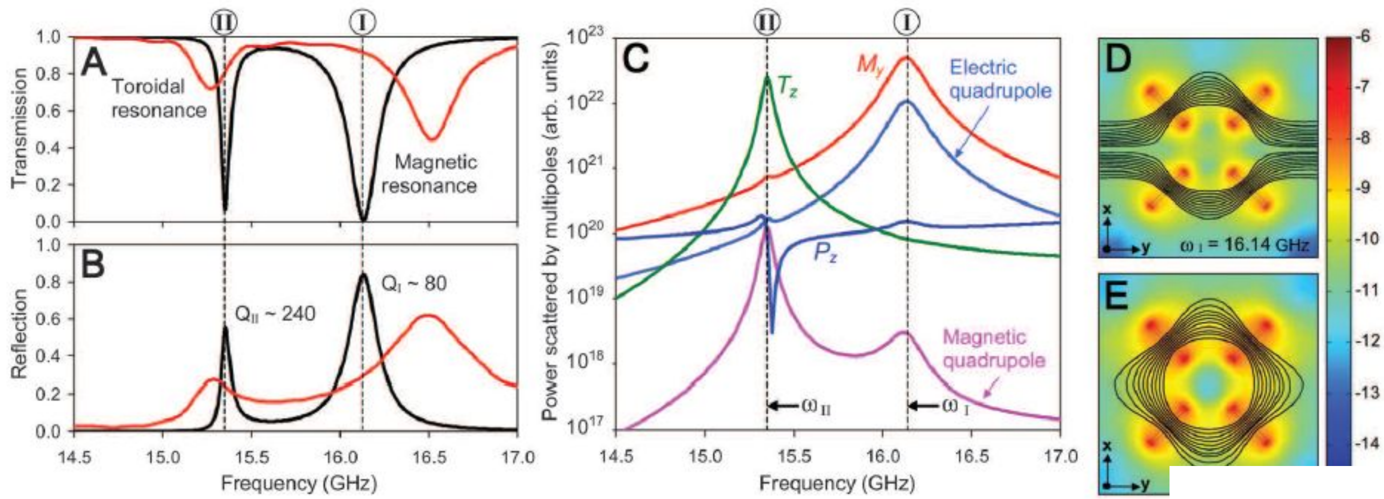


B-DNA



# First demonstration of toroidal response by metamaterials




UNIVERSITY OF Southampton

T. Kaelberer *et al*, *Science* 330, 1510 (2010)

# Toroidal response in multipoles expansion. Radiating power of multipoles.

$$I = \frac{2\omega^4}{3c^3}|\mathbf{P}|^2 + \frac{2\omega^4}{3c^3}|\mathbf{M}|^2 + \frac{4\omega^5}{3c^4}(\mathbf{P}\cdot\mathbf{T}) + \frac{2\omega^6}{3c^5}|\mathbf{T}|^2 + \frac{\omega^6}{5c^5}Q_{\alpha\beta}Q_{\alpha\beta} + \frac{\omega^6}{20c^5}M_{\alpha\beta}M_{\alpha\beta} +$$

$$+\frac{2\omega^6}{15c^5}\left(\mathbf{M}\cdot\langle\mathbf{R}_{\mathrm{M}}^2\rangle\right) + \mathrm{o}\left(\frac{1}{c^5}\right)$$

**We need to consider this term in order to correctly describe the characteristics of toroidal objects.**

P- Electric

M- Magne

T- toroidal

Q- Electric

M – Magnetic quadrupole moment

j- current density

$\mathbf{P} = \frac{1}{\ }\int \mathbf{j}\, d^3r$

$\mathbf{j})\, d^3r$

$\frac{\ }{10c}\int [\ \mathbf{j})\mathbf{r} - 2r^2\mathbf{j}]\, d^3r$

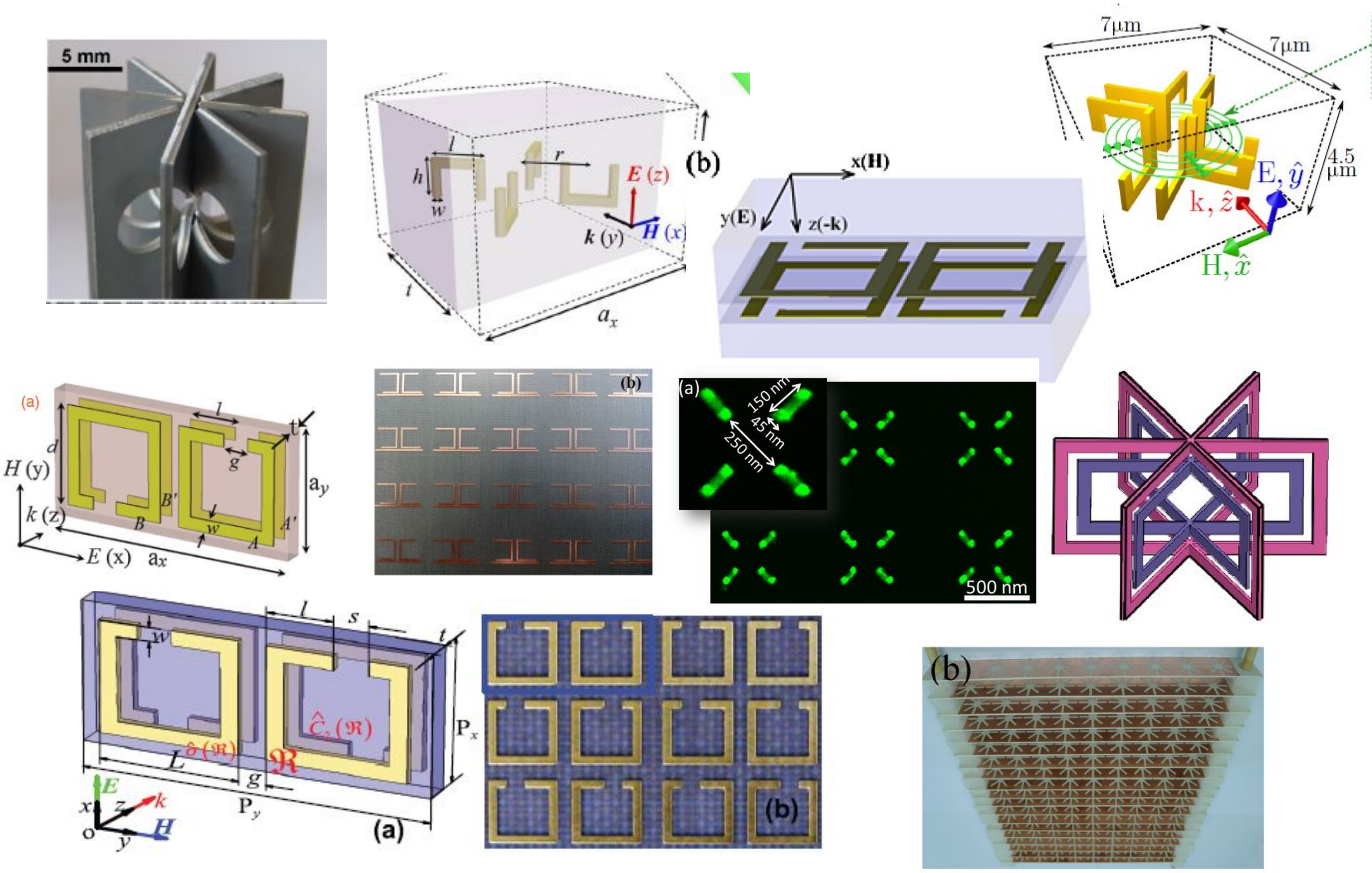
$$Q_{\alpha\beta} = \frac{1}{i\omega}\int\left[r_\alpha j_\beta + r_\beta j_\alpha - \frac{2}{3}(\mathbf{r}\cdot\mathbf{j})\right]d^3r$$

$$M_{\alpha\beta} = \frac{1}{3c}\int\left[(\mathbf{r}\times\mathbf{j})_\alpha r_\beta + (\mathbf{r}\times\mathbf{j})_\beta r_\alpha\right]d^3r$$

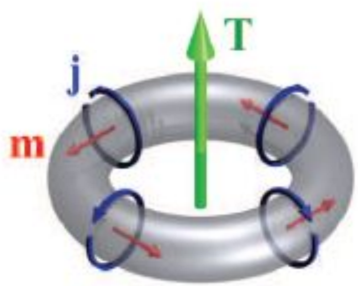
T. Kaelberer *et al, Science* (2010)

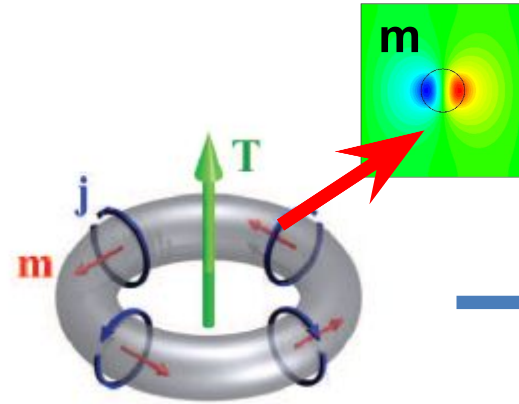
E. E. Radescu and G. Vaman, PRE (2002)

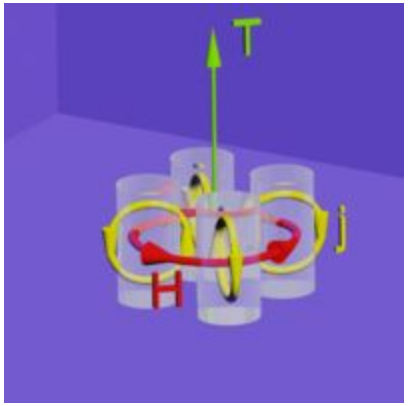
# Family of the toroidal metamolecules. Complicated disign?



# 2. Toroidal response in dielectric metamaterials- without Joule losses

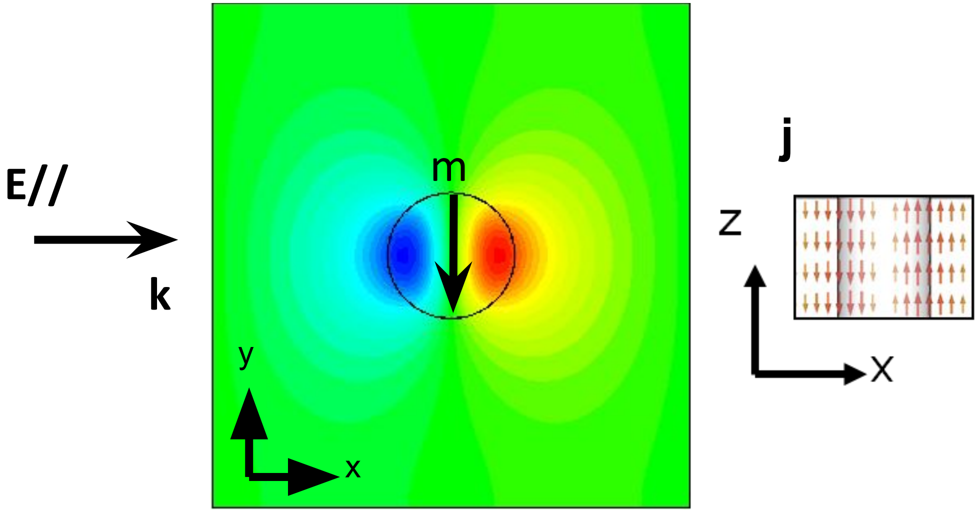



**T. Kaelberer *et al, Science* 330, 1510 (2010)**

**Magnetic moments due to Mie- resonance in high-index dielectric particles**



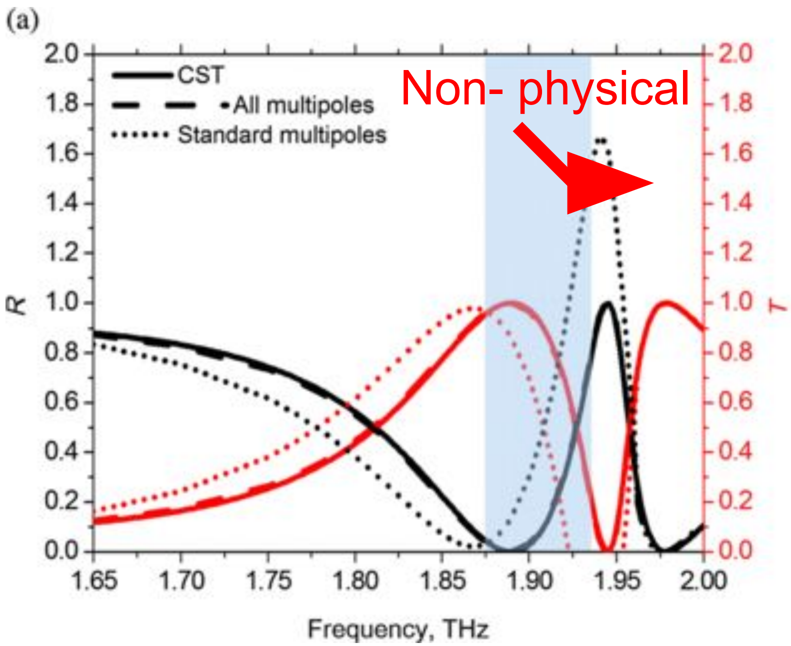

## High index dielectrics :

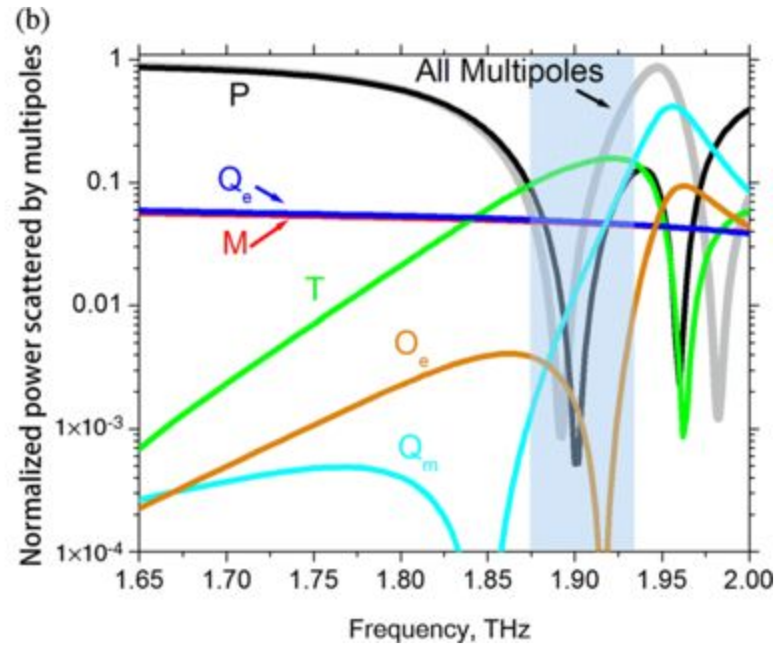
**In microwave- BSTO ceramics**

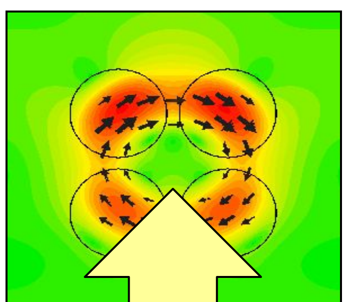
S. O'Brien and J. B. Pendry, 2002

L. Peng and al., 2007

# LiTaO3 cluster: Reflection and Transmission; Radiated power of multipoles

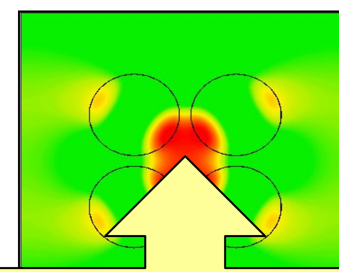


**Closed magnetic field- Toroidal response.**

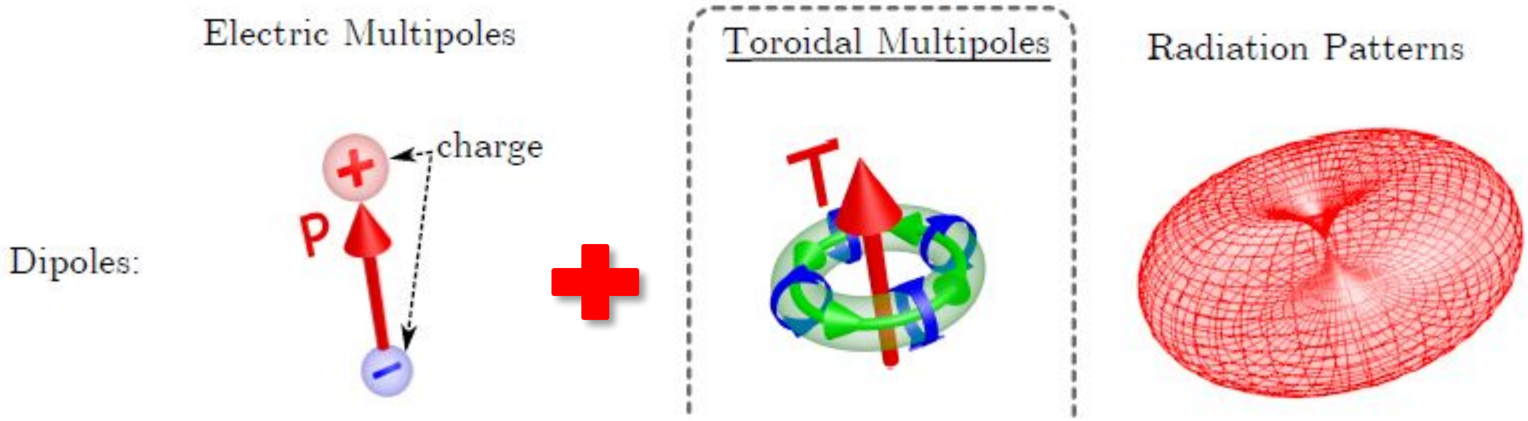
**No fileds between cylinders**



**Strong localization of E- field between cylinders- Exitation of Nonlinearities**

0 $8.22 \times 10^6 \, V/m$

# Non-Trivial Excitation: P=ikT and analog of Electromagnetically induced transparency




$$\mathbf{P} = \frac{1}{i\omega}\int \mathbf{j}\, d^3 r$$

$$\mathbf{T} = \frac{1}{10c}\int \left[ (\mathbf{r}\cdot\mathbf{j})\mathbf{r} - 2r^2 \mathbf{j} \right] d^3 r$$

**Interference of P and T gives EIT and symmetrical Peak of transmission**

Fedotov et al., *Scientific Reports* 3, 2967
Afanasiev, G. N. & Stepanovsky, Y. P., *J. Phys. A Math. Gen.* 28, 4565

# Non-Trivial non-radiating toroidal source

**For Electric dipole**

$$\varphi_P = \frac{(\mathbf{r}\cdot\mathbf{P})D(\omega,r)}{cr}\,\frac{\exp(-ikr+i\omega t)}{r}, \qquad \mathbf{A}_P = ik\,\mathbf{P}\,\frac{\exp(-ikr+i\omega t)}{r}$$

$$\mathbf{E}_P = \left[\frac{(\mathbf{r}\cdot\mathbf{P})F(\omega,r)}{c^2r^2}\mathbf{r} - \frac{G(\omega,r)}{c^2}\mathbf{P}\right]\frac{\exp(-ikr+i\omega t)}{r}$$

$$\mathbf{H}_P = -\frac{ikD(\omega,r)}{cr}\,[\mathbf{r}\times\mathbf{P}]\,\frac{\exp(-ikr+i\omega t)}{r}$$

**For Toroidal dipole**

$$\varphi_T = 0, \qquad \mathbf{A}_T = \left[\frac{(\mathbf{r}\cdot\mathbf{T})F(\omega,r)}{c^2r^2}\mathbf{r} - \frac{G(\omega,r)}{c^2}\mathbf{T}\right]\frac{\exp(-ikr+i\omega t)}{r}$$

$$\mathbf{E}_T = \left[\frac{ikG(\omega,r)}{c^2}\mathbf{T} - \frac{ik(\mathbf{r}\cdot\mathbf{T})F(\omega,r)}{c^2r^2}\mathbf{r}\right]\frac{\exp(-ikr+i\omega t)}{r}$$

$$\mathbf{H}_T = -\frac{k^2D(\omega,r)}{cr}\,[\mathbf{r}\times\mathbf{T}]\,\frac{\exp(-ikr+i\omega t)}{r}$$

Fedotov et al., *Scientific Reports* 3, 2967
Afanasiev, G. N. & Stepanovsky, Y. P., *J. Phys. A Math. Gen.* 28, 4565

# Non-Trivial Excitation: P=ikT

$$\mathbf{A}_{tot} = \mathbf{A}_P + \mathbf{A}_T = \left[\frac{(\mathbf{r}\cdot\mathbf{T})F(\omega,r)}{c^2r^2}\mathbf{r} - \frac{G(\omega,r)}{c^2}\mathbf{T} + ik\mathbf{P}\right]\frac{\exp(-ikr+i\omega t)}{r}$$

$$\mathbf{E}_{tot} = \mathbf{E}_P + \mathbf{E}_T = \left[\frac{(\mathbf{r}\cdot(\mathbf{P}-ik\mathbf{T}))F(\omega,r)}{c^2r^2}\mathbf{r} - \frac{G(\omega,r)}{c^2r}(\mathbf{P}-ik\mathbf{T})\right]\frac{\exp(-ikr+i\omega t)}{r}$$

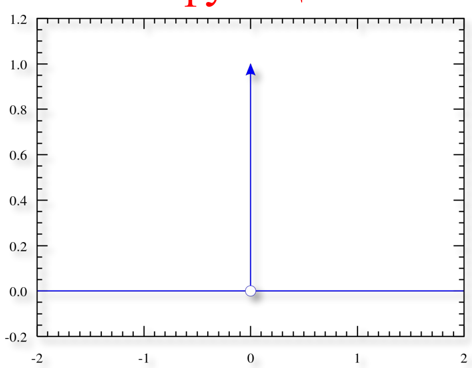
$$\mathbf{H}_{tot} = \mathbf{H}_P + \mathbf{H}_T = -\frac{ikD(\omega,r)}{cr}[\mathbf{r}\times(\mathbf{P}-ik\mathbf{T})]\frac{\exp(-ikr+i\omega t)}{r}$$

**P=ikT**
**E and H vanish in FAR-field zone**


Fedotov et al., *Scientific Reports* 3, 2967
Afanasiev, G. N. & Stepanovsky, Y. P., *J. Phys. A Math. Gen.* 28, 4565

# Поля точечного анаполя, ближняя зона: P=ikT

δ- функция




**P=*ik*T**

**E и H исчезают везде, кроме *r*=0:**

$$\boldsymbol{E}_{\boldsymbol{tot}}(r=0) = -ik\boldsymbol{T}\delta(r)\exp(i\omega t)$$
$$\boldsymbol{H}_{\boldsymbol{tot}}(r=0) = -ik\,\mathrm{rot}\,(\boldsymbol{T})\delta(r)\exp(i\omega t)$$
$$\varphi_{\boldsymbol{tot}}(r=0) = \varphi_{\boldsymbol{p}} + \varphi_{\boldsymbol{T}} = 0$$
$$\boldsymbol{A}_{\boldsymbol{tot}}(r=0) = \boldsymbol{T}4\pi\delta(r)\exp(i\omega t)$$

**Бесконечная добротность?**

$Q=\omega_0 W/P$

d

Nemkov *et al*., Non-radiating sources, dynamic anapole and Aharonov-Bohm effect, arxiv 1605.09033
Basharin *et al*., Extremely High Q-factor metamaterials due to Anapole Excitation, arxiv 1608.03233

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ARTICLE

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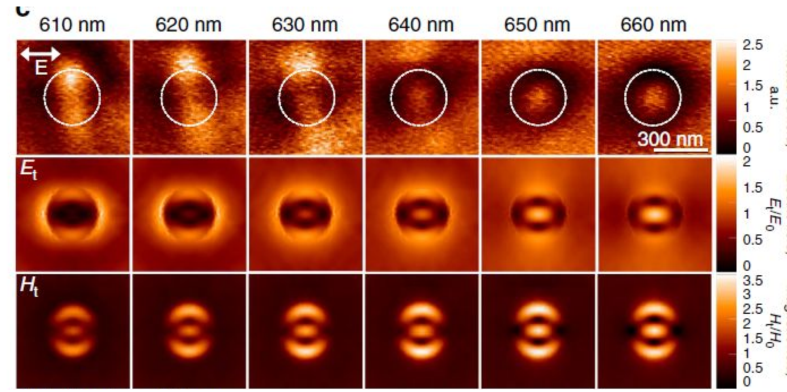
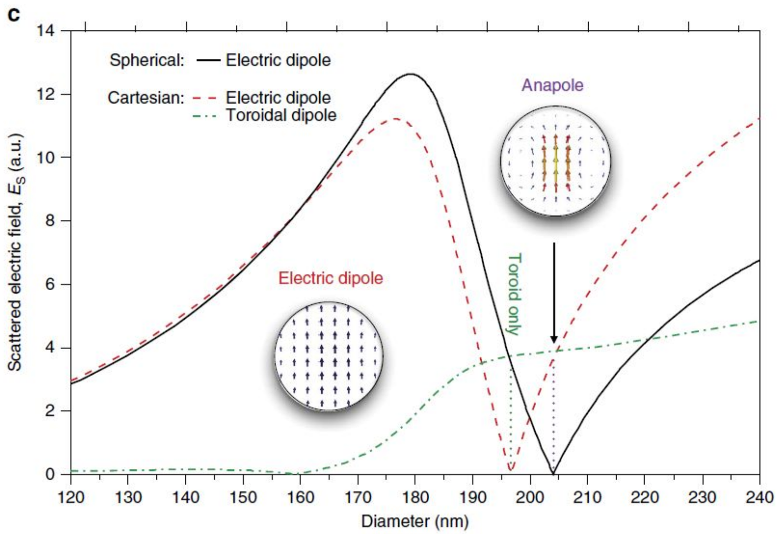
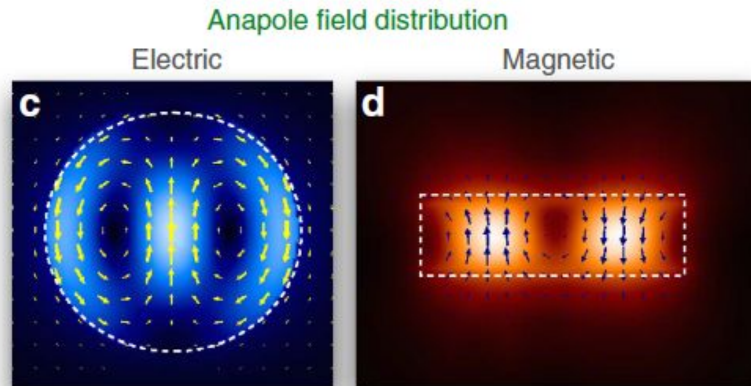
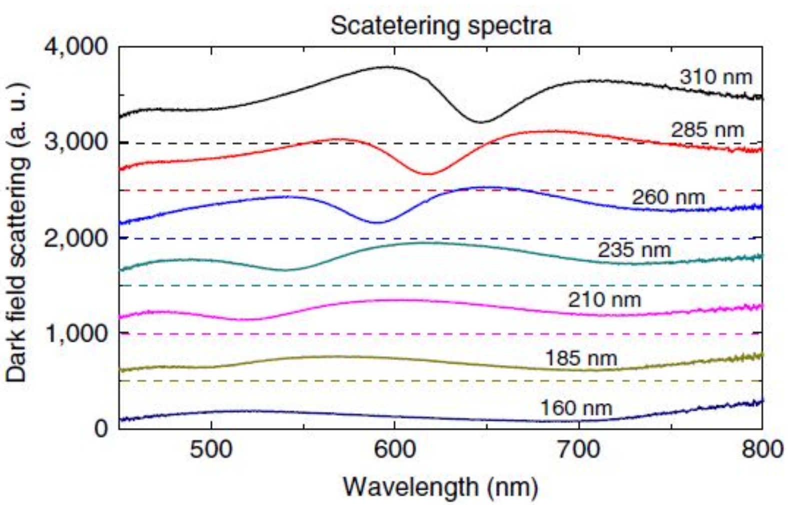
# Nonradiating anapole modes in dielectric nanoparticles

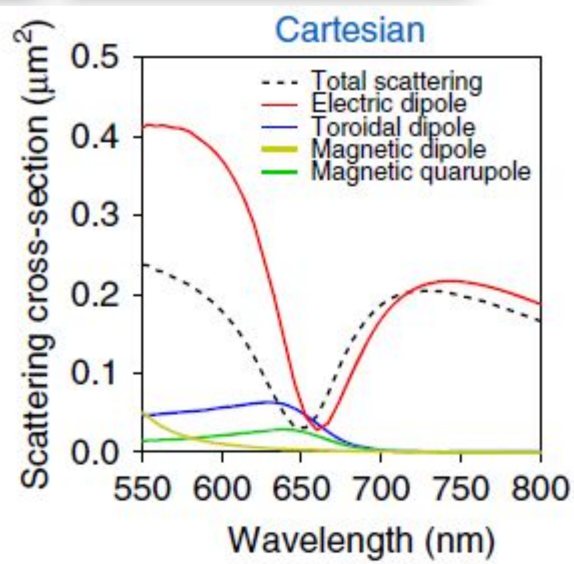
Andrey E. Miroshnichenko[1], Andrey B. Evlyukhin[2,3], Ye Feng Yu[4], Reuben M. Bakker[4], Arkadi Chipouline[5], Arseniy I. Kuznetsov[4], Boris Luk'yanchuk[4], Boris N. Chichkov[2,3,6] & Yuri S. Kivshar[1]



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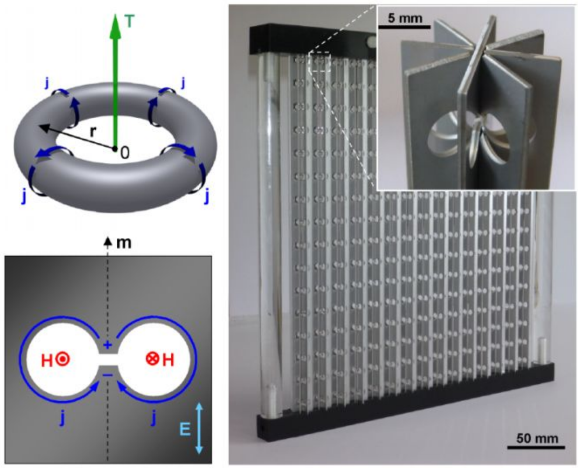
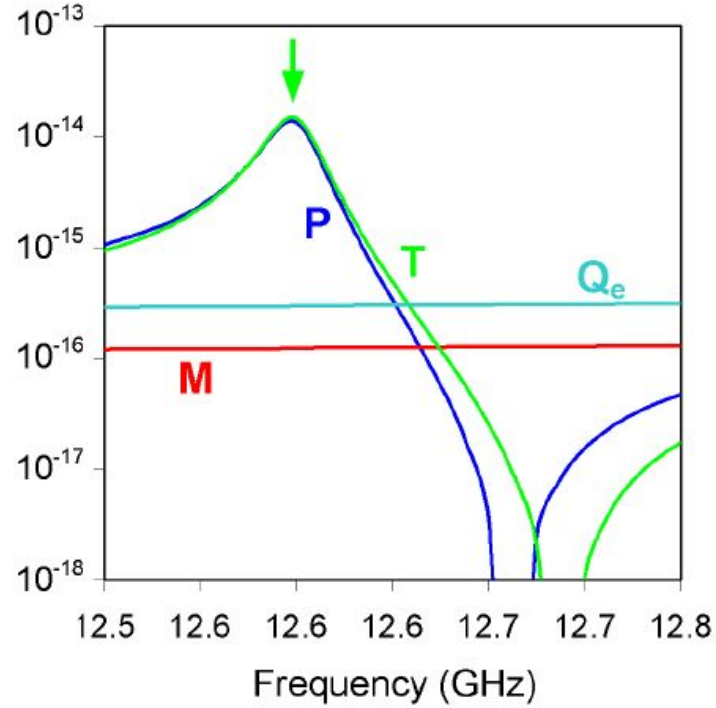
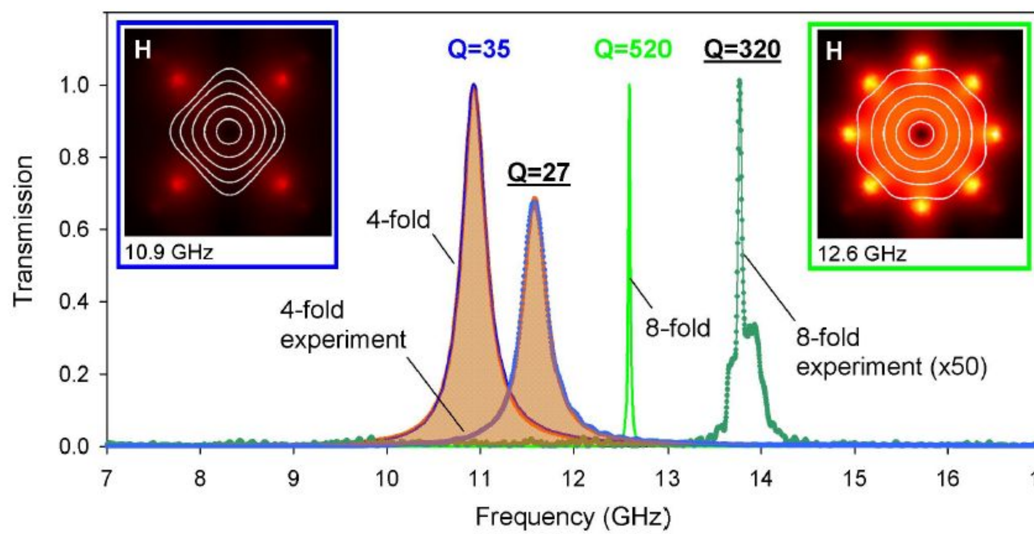
# Resonant Transparency and Non-Trivial Non-Radiating Excitations in Toroidal Metamaterials

SUBJECT AREAS:
ELECTRICAL AND ELECTRONIC ENGINEERING
METAMATERIALS

V. A. Fedotov[1], A. V. Rogacheva[1], V. Savinov[1], D. P. Tsai[2,3] & N. I. Zheludev[1,4]



# Conclusions

Fano – resonance assymetric peak

Vortices in nano-particles give possibilities fo[r]
absorption and localization

Toroidal response in metamaterials i…
very promising candidate for opti…

*Thank you and questions?*

МИСиС