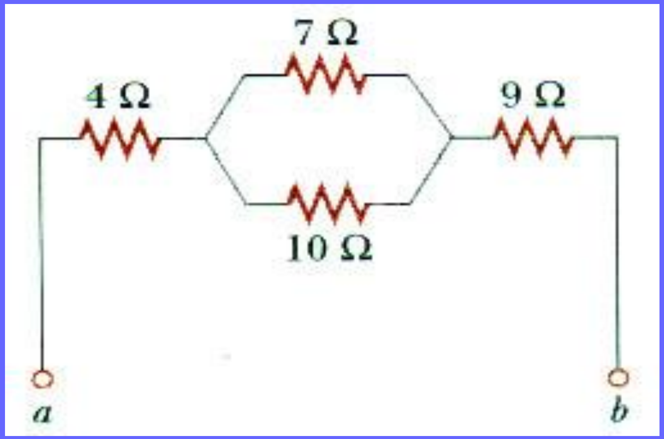
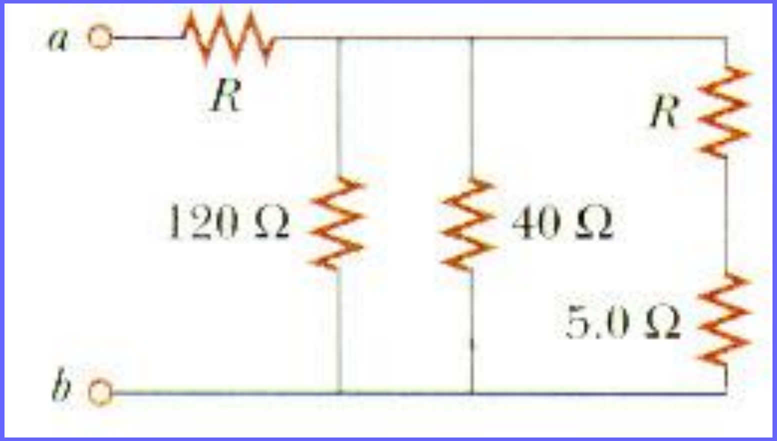
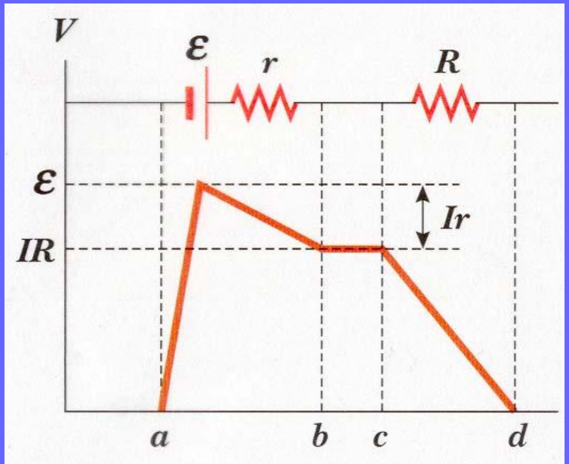
Lecture 12
Current & Resistance (2)



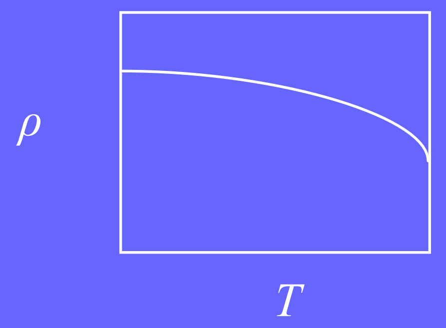




Temperature Variation of Resistance

- The resistivity of a metal depends on many (environmental) factors.
- The most important factor is the temperature.
- For most metals, the resistivity increases with increasing temperature.
- The increased resistivity arises because of larger friction caused by the more violent motion of the atoms of the metal.

- For most metals, resistivity increases approx. linearly with temperature.



Metallic Conductor

$$\rho = \rho_o\left[1+\alpha\left(T-T_o\right)\right]$$

- ρ is the resistivity at temperature T (measured in Celsius).
- ρ_o is the reference resistivity at the reference temperature T_o (usually taken to be 20 ºC).
- α is a parameter called temperature coefficient of resistivity.

- For a conductor with fixed cross section.

$$R = R_o\left[1+\alpha\left(T-T_o\right)\right]$$

Example:

A resistance thermometer, which measures temperature by measuring the change in the resistance of a conductor, is made of platinum and has a resistance of 50.0 Ω at 20°C. When the device is immersed in a vessel containing melting indium, its resistance increases to 76.8 Ω. Find the melting point of Indium.

Solution:

Using $\alpha = 3.92 \times 10^{-3} (^{o}C)^{-1}$ from table.

$R_o = 50.0\ \Omega$.

$T_o = 20^{o}C$.

$R = 76.8\ \Omega$.

$$T - T_o = \frac{R - R_o}{\alpha R_o} = \frac{76.8\Omega - 50.0\Omega}{\left[3.92 \times 10^{-3} \left(^{o}C\right)^{-1}\right]\left[50.0\Omega\right]}$$

$$= 137^{o}C$$

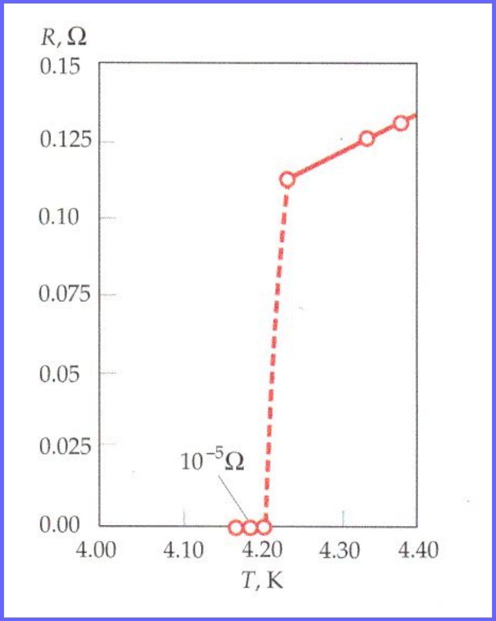
$$T = 157^{o}C$$

[Q] A resistance thermometer using a platinum wire is used to measure the temperature of a liquid. The resistance is 2.42 ohms at 0ºC, and when immersed in the liquid it is 2.98 ohms. The temperature coefficient of resistivity of platinum is 0.0038 . What is the temperature of the liquid?

Superconductivity

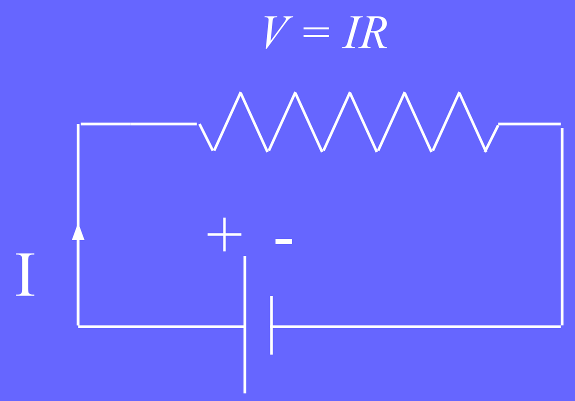
- **1911: H. K. Onnes, who had figured out how to make liquid helium, used it to cool mercury to 4.2 K and looked at its resistance:**

- **At low temperatures the resistance of some metals□0, measured to be less than** $10^{-16} \cdot \rho_{conductor}$ **(i.e.,** $\rho < 10^{-24}\ \Omega m$**)!**



Electrical energy and power

- In any circuit, battery is used to induce electrical current
  - chemical energy of the battery is transformed into kinetic energy of mobile charge carriers (electrical energy gain)
- Any device that possesses resistance (resistor) present in the circuit will transform electrical energy into heat
  - kinetic energy of charge carriers is transformed into heat via collisions with atoms in a conductor (electrical energy loss)



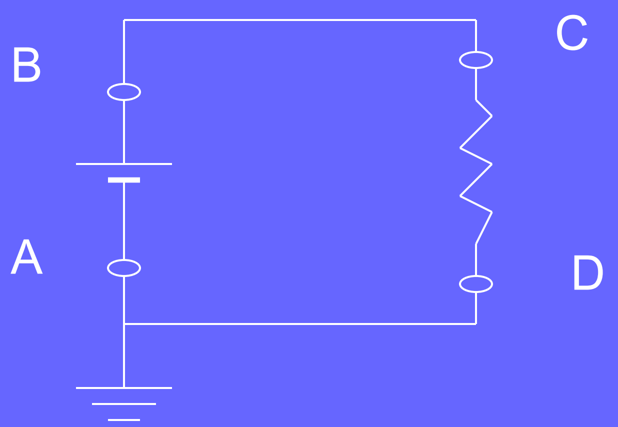
Electrical energy

- Consider circuit on the right in detail
- AB: charge gains electrical energy form the battery

$$\Delta E = \Delta Q \cdot \Delta V$$

(battery looses chemical energy)

- CD: electrical energy lost (transferred into heat)
- Back to A: same potential energy (zero) as before
- Gained electrical energy = lost electrical energy on the resistor



Power

- Compute rate of energy loss (power dissipated on the resistor)

$$P = \frac{\Delta E}{\Delta t} = \frac{\Delta Q}{\Delta t}\Delta V = I\Delta V$$

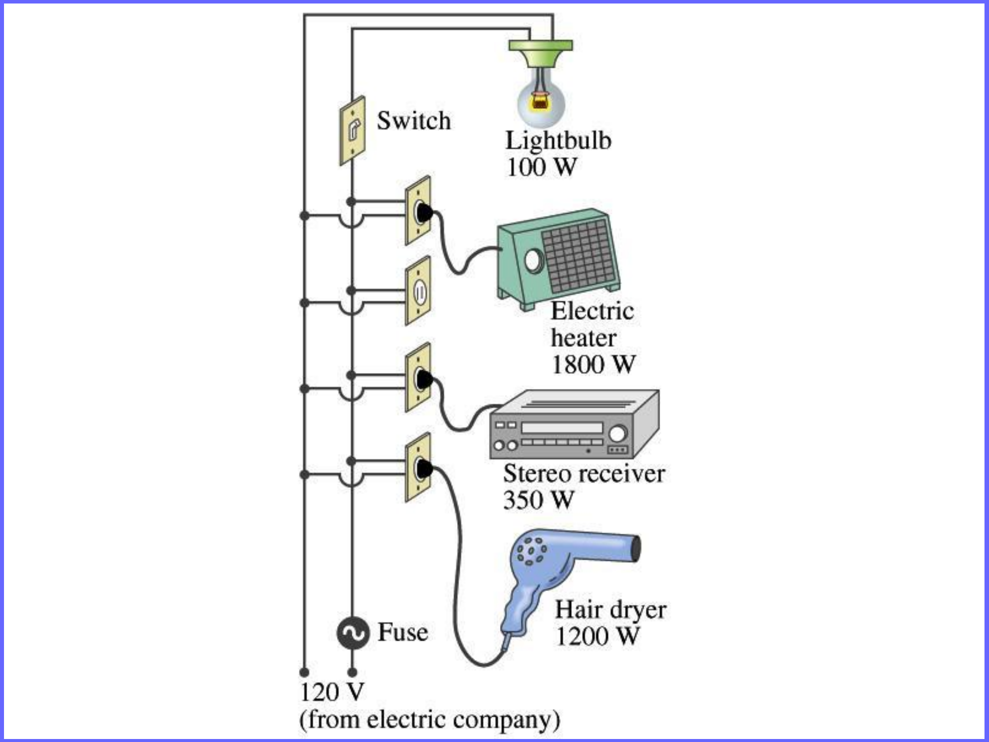
- Use Ohm's law

$$P = I\Delta V = I^2 R = \frac{\left(\Delta V\right)^2}{R}$$

- Units of power: SI: watt
  delivered energy: kilowatt-hours

$$1\,\text{kWh} = \left(10^3 W\right)\left(3600\ s\right) = 3.60\times 10^6 J$$

[Q] Calculate the total current drawn by all the devices in the circuit in the figure.



Example

A high-voltage transmission line with resistance of 0.31 Ω/km carries 1000A , starting at 700 kV, for a distance of 160 km. What is the power loss due to resistance in the wire?

Observations:

1. Given resistance/length, compute total resistance
2. Given resistance and current, compute power loss

$$R = \rho L = \left(0.31\,\Omega/km\right)\left(160\,km\right) = 49.6\,\Omega$$

Now compute power

$$P = I^2 R = \left(1000\ A\right)^2\left(49.6\,\Omega\right) = 49.6\times10^6\ W$$

(1) An aluminum wire carrying a current has a diameter 0.800 mm. The electric field in the wire is 0.640 V/m. What is: a) the current carried by the wire? b) the potential difference between two points in the wire 12.0 m apart? C) the resistance of a 12.0 m length of the wire?

(2) A copper wire has resistance 5 Ohms. Given that the resistivity of silver is 85 percent of the resistivity of copper, what is the resistance of a silver wire three times as long with twice the diameter?

(3) A current of 5A exists in a 10 W resistor for 4min. (a) How many coulombs, and (b) how many electrons pass through any cross section of the resistor in this time?

(4) What is the resistance of a device that operates with a current of 7A when the applied voltage is 110V?

(5) Thermal energy is developed in a resistor at a rate of 100W when the current is 3.0A. What is the resistance in ohms?

(6) A 1250W radiant heater is constructed to operate at 115V. (a) What will be the current in the heater? (b) What is the resistance of the heating coil?

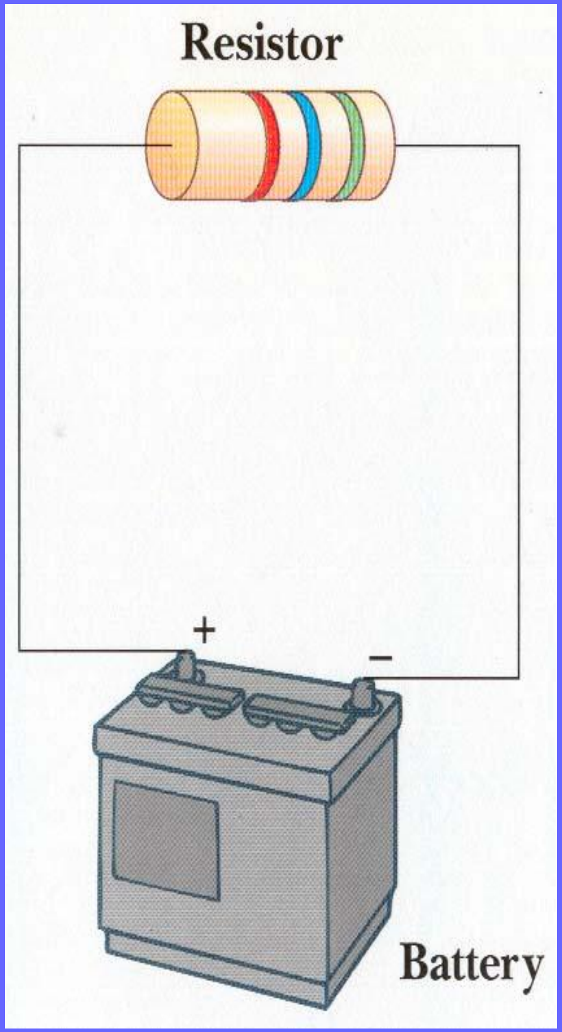
What is emf?

- A current is maintained in a closed circuit by a source of emf.
  The term emf was originally an abbreviation for *electromotive force* but emf is NOT really a force, so the long term is discouraged.

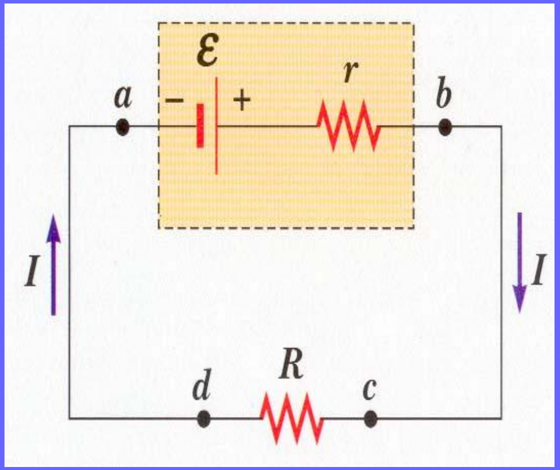
- A source of emf works as "charge pump" that forces electrons to move in a direction opposite the electrostatic field inside the source.

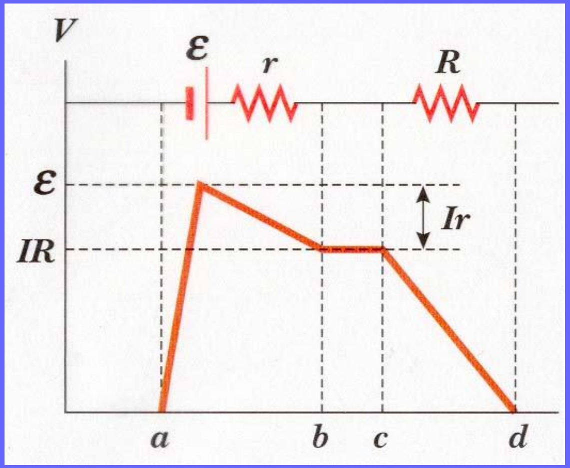
Examples of such sources are:

batteries
generators
thermocouples
photo-voltaic cells



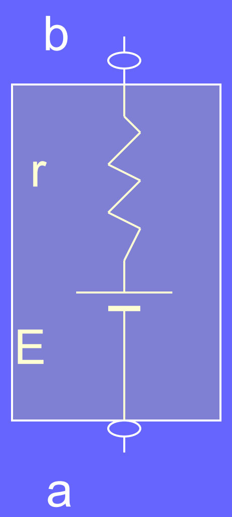
$\Longrightarrow$





- Each real battery has some internal resistance
- AB: potential increases by on the source of EMF, then decreases by Ir (because of the internal resistance)
- Thus, terminal voltage on the battery ΔV is

$$\Delta V = \varepsilon - Ir$$



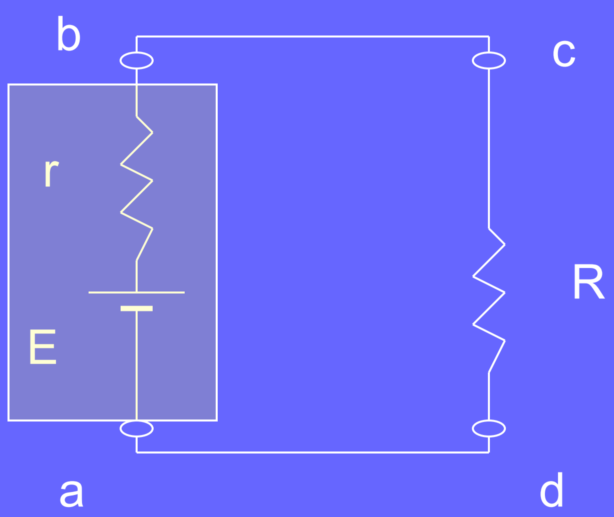
- Note: EMF is the same as the terminal voltage when the current is zero (open circuit)

- Now add a load resistance R
- Since it is connected by a conducting wire to the battery → terminal voltage is the same as the potential difference across the load resistance

$$\Delta V = \varepsilon - Ir - IR = 0$$

$$or$$

$$\varepsilon = IR - Ir$$

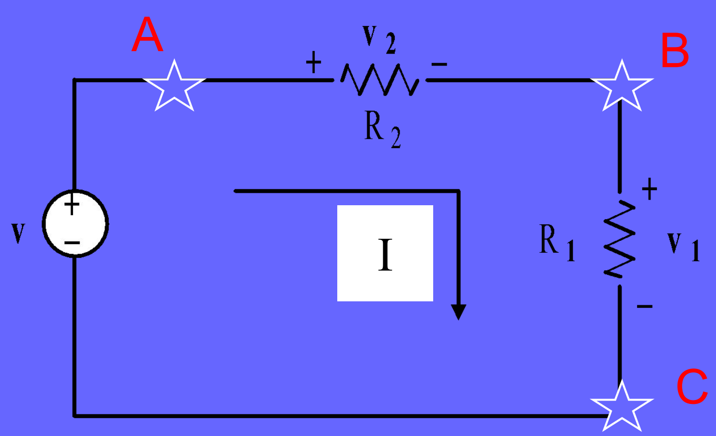


- Thus, the current in the circuit is

$$I = \frac{\varepsilon}{R + r}$$

[Q] Under what condition does the potential difference across the terminals of a battery equal its emf?

Resistors in series



1. Because of the charge conservation, all charges going through the resistor R_2 will also go through resistor R_1. Thus, currents in R_1 and R_2 are the same,

$$I_1 = I_2 = I$$

2. Because of the energy conservation, total potential drop (between A and C) equals to the sum of potential drops between A and B and B and C,

$$\Delta V = IR_1 + IR_2$$

By definition,

$$\Delta V = IR_{eq}$$

Thus, R_{eq} would be

$$R_{eq} \equiv \frac{\Delta V}{I} = \frac{IR_1 + IR_2}{I} = R_1 + R_2$$

$$\boxed{R_{eq} = R_1 + R_2}$$

- Analogous formula is true for any number of resistors,

$$R_{eq} = R_1 + R_2 + R_3 + \ldots$$ (series combination)

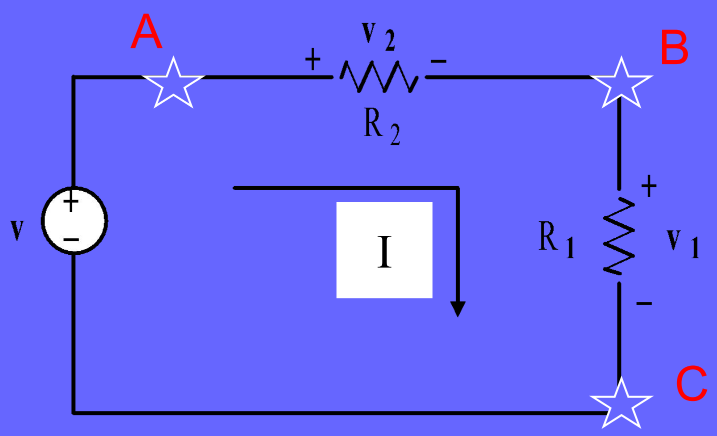
- It follows that the equivalent resistance of a series combination of resistors is greater than any of the individual resistors

[Q] How would you connect resistors so that the equivalent resistance is **larger** than the individual resistance?

[Q] When resistors are connected in **series,** which of the following would be the **same** for each resistor: potential difference, current, power?

example

In the electrical circuit below, find voltage across the resistor R_1 in terms of the resistances R_1, R_2 and potential difference between the battery's terminals V.



Energy conservation implies:

$$V = V_1 + V_2$$

with $V_1 = IR_1$ and $V_2 = IR_2$

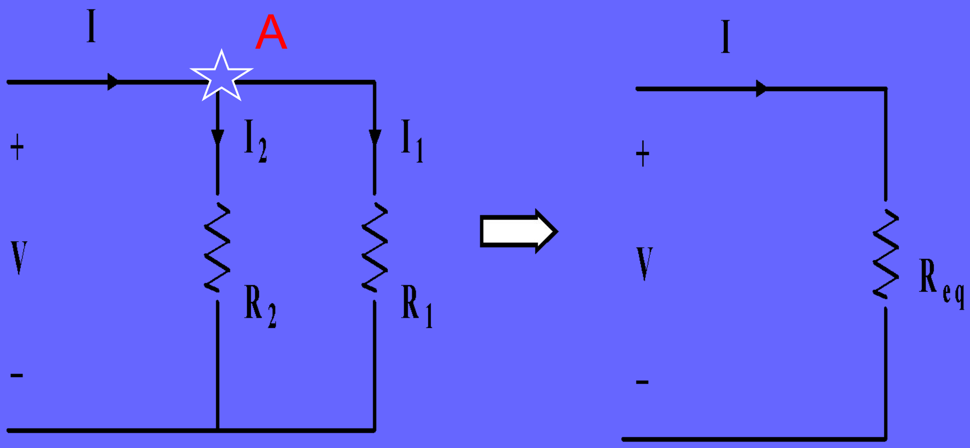
Then, $V = I\left(R_1 + R_2\right)$, so $I = \dfrac{V}{R_1 + R_2}$

Thus,

$$V_1 = V\frac{R_1}{R_1 + R_2}$$

This circuit is known as **voltage divider**.

Resistors in parallel



1. Since both R_1 and R_2 are connected to the same battery, potential differences across R_1 and R_2 are the same,

$$V_1 = V_2 = V$$

2. Because of the <u>charge conservation</u>, current, entering the junction A, must equal the current leaving this junction,

$$I = I_1 + I_2$$

By definition, $$I = \frac{V}{R_{eq}}$$

Thus, R_{eq} would be $$I = \frac{V}{R_{eq}} = \frac{V_1}{R_1} + \frac{V_2}{R_2} = \frac{V}{R_1} + \frac{V}{R_2}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$ or $$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

- Analogous formula is true for any number of resistors,

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots \quad \text{(parallel combination)}$$

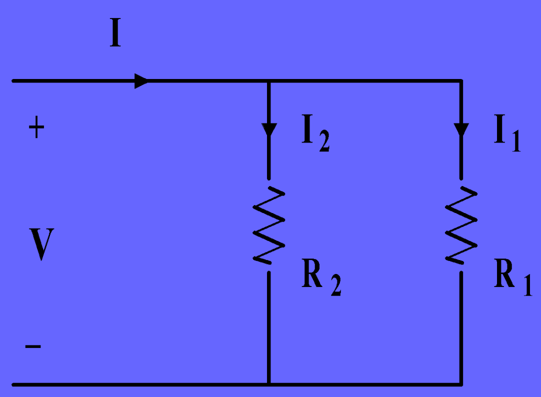
- It follows that the equivalent resistance of a parallel combination of resistors is always less than any of the individual resistors

[Q] How would you connect resistors so that the equivalent resistance is **smaller** than the individual resistance?

[Q] When resistors are connected in **parallel,** which of the following would be the **same** for each resistor: potential difference, current, power?

example

In the electrical circuit below, find current through the resistor R_1 in terms of the resistances R_1, R_2 and total current I induced by the battery.



Charge conservation implies:

$$I = I_1 + I_2$$

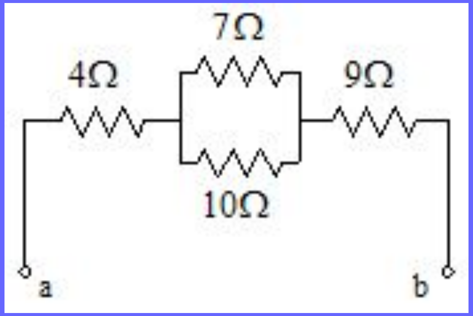
with $I_1 = \frac{V}{R_1}$, and $I_2 = \frac{V}{R_2}$

Then, $I_1 = \frac{IR_{eq}}{R_1}$, with $R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$

Thus, $I_1 = I \frac{R_2}{R_1 + R_2}$

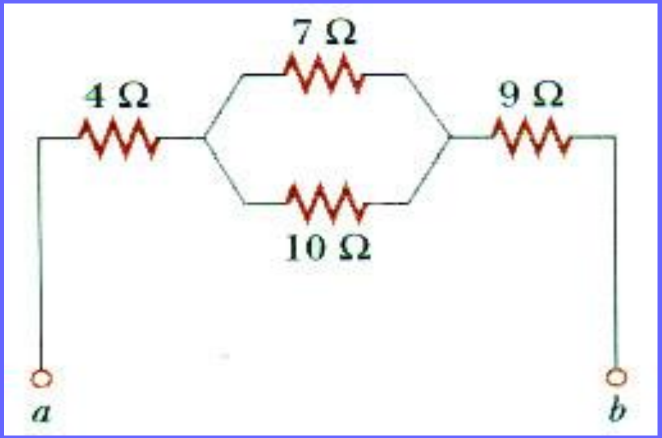
This circuit is known as **current divider**.

(11) (a) Find the equivalent resistance between points a and b in Figure 8.27. (b) A potential difference of 34V is applied between points a and b in Figure 28.28. Calculate the current in each resistor.

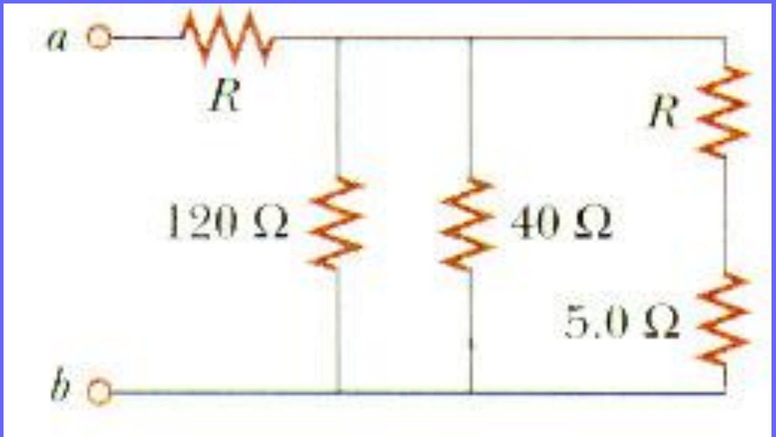


[Q] The resistance between terminals a and b in Figure is 75-ohms. If the resistors labeled R have the same value, determine R.

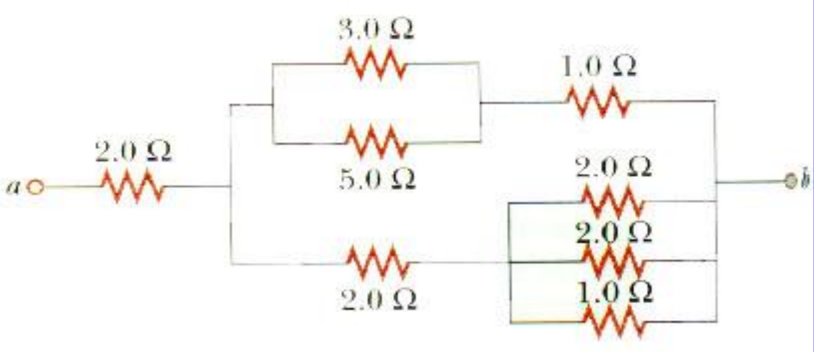
(a) Find the equivalent resistance between points a and b in Figure



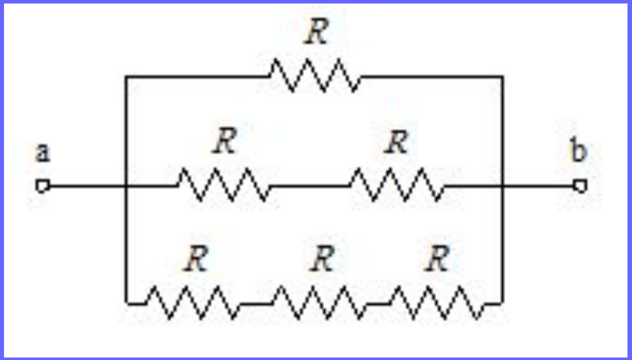
(b) If a potential difference of 34 V is applied between points a and b, calculate the current in **each** resistor.



[Q] Determine the equivalent resistance between the terminals a and b for the network illustrated in Figure.

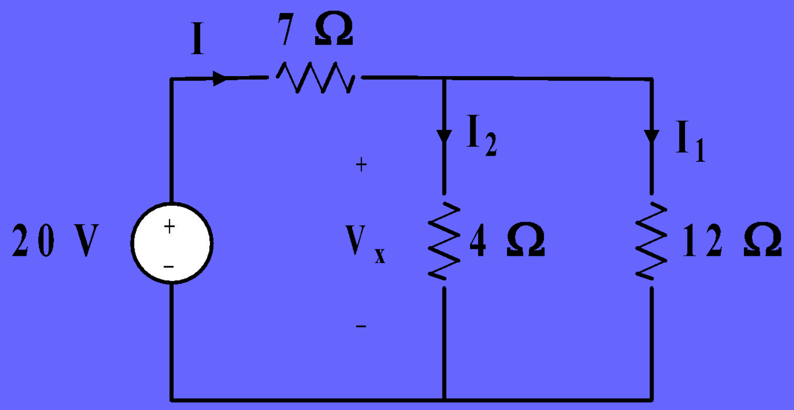


(12) Evaluate the effective resistance of the network of identical resistors, each having resistance *R*, shown in figure



Direct current circuits: example

Find the currents I_1 and I_2 and the voltage V_x in the circuit shown below.



First find the equivalent resistance seen by the 20 V source:

$$R_{eq} = 7\Omega + \frac{4\Omega(12\Omega)}{12\Omega + 4\Omega} = 10\ \Omega$$

Then find current I by,

$$I = \frac{20V}{R_{eq}} = \frac{20V}{10\Omega} = 2\ A$$

We now find I_1 and I_2 directly from the current division rule:

$$I_1 = \frac{2A(4\Omega)}{12\Omega + 4\Omega} = 0.5\ A, \text{ and } I_2 = I - I_1 = 1.5\ A$$

Finally, voltage V_x is $V_x = I_2\left(4\Omega\right) = 1.5A\left(4\Omega\right) = 6V$

(1) A battery with an *emf* of 12V and internal resistance of 0.9W is connected across a load resistor *R*. If the current in the circuit is 1.4A, what is the value of *R*?

(2) What power is dissipated in the internal resistance of the battery in the circuit described in Problem 8.1?

(3) (a) What is the current in a 5.6W resistor connected to a battery with an 0.2W internal resistance if the terminal voltage of the battery is 10V? (b) What is the *emf* of the battery?

(4) If the emf of a battery is 15V and a current of 60A is measured when the battery is shorted, what is the internal resistance of the battery?

(5) The current in a loop circuit that has a resistance of *R*1 is 2A. The current is reduced to 1.6A when an additional resistor *R*2=3W is added in series with *R*1. What is the value of *R*1?

(6) A battery has an emf of 15V. The terminal voltage of the battery is 11.6V when it is delivering 20W of power to an external load resistor R. (a) What is the value of R? (b) What is the internal resistance of the battery?

(7) A certain battery has an open-circuit voltage of 42V. A load resistance of 12Ω reduces the terminal voltage to 35V. What is the value of the internal resistance of the battery?

(8) Two circuit elements with fixed resistances R_1 and R_2 are connected in *series* with a 6V battery and a switch. The battery has an internal resistance of 5Ω, R_1= 32Ω, and R_2=56Ω. (a) What is the current through R_1 when the switch is closed? (b) What is the voltage across R_2 when the switch is closed?

(9) The current in a simple series circuit is 5.0A. When an additional resistance of 2.0Ω is inserted, the current drops to 4.0 A. What was the resistance of the original circuit?

(10) Three resistors (10Ω, 20Ω, and 30Ω) are connected in parallel. The total current through this network is 5A. (a) What is the voltage drop across the network (b) What is the current in each resistor?