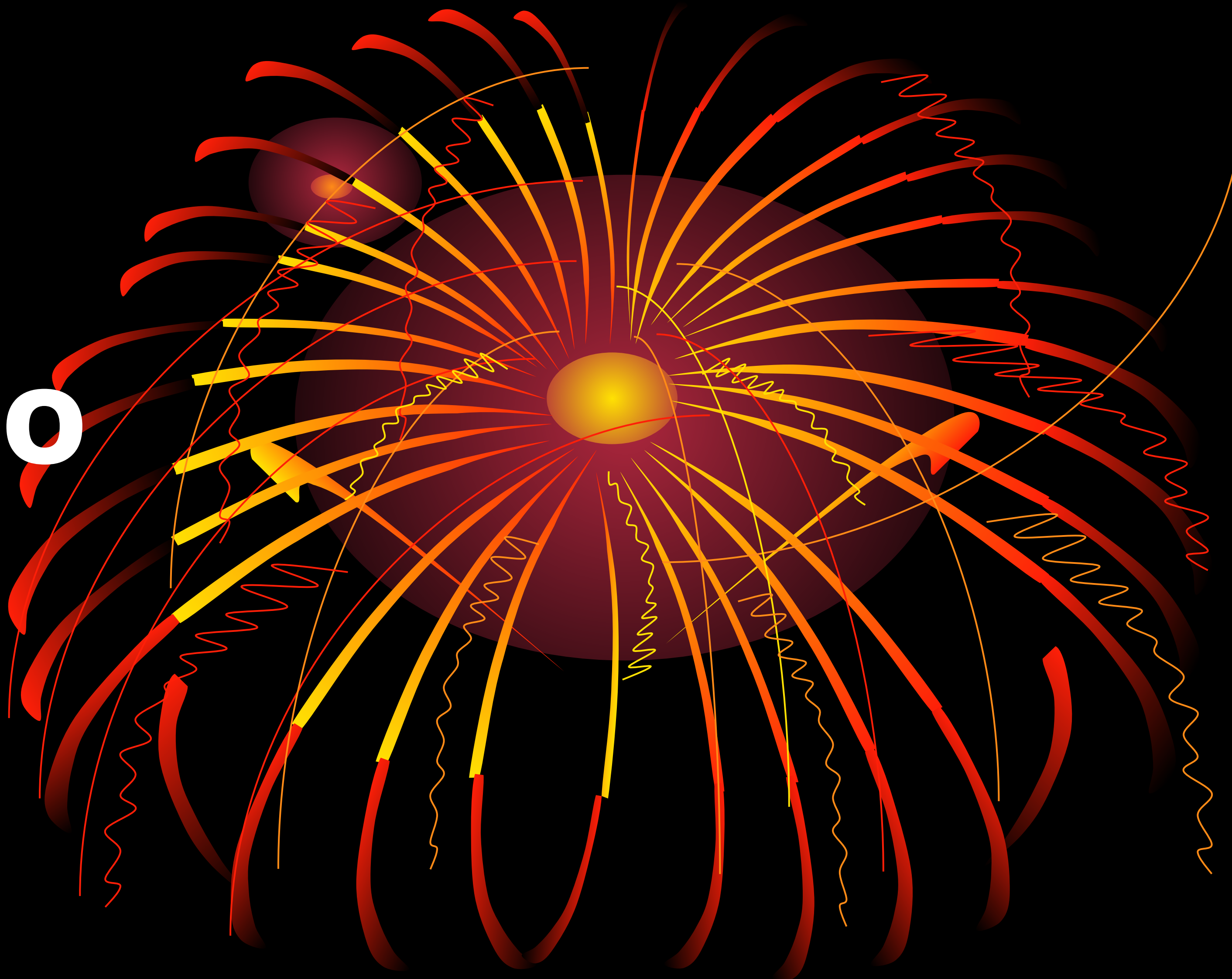
Prelude to an Exam
Allegro con brio

- Next Friday – EXAMINATION #2
- Watch those WebAssigns .. no more extensions.
- Monday will be
  - A Quiz on Circuits
  - A review of circuits and some other problems.
- Wednesday, more on Magnetism. Only day 1 on the exm. Watch for a new Webassign.

Magnetism

A Whole New Topic

DEMO

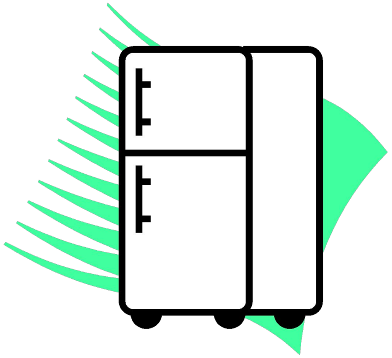


Lodestone (Mineral)

- Lodestones attracted iron filings.
- Lodestones seemed to attract each other.
- Used as a compass.
  - One end always pointed north.
- Lodestone is a natural magnet.

Magnetism

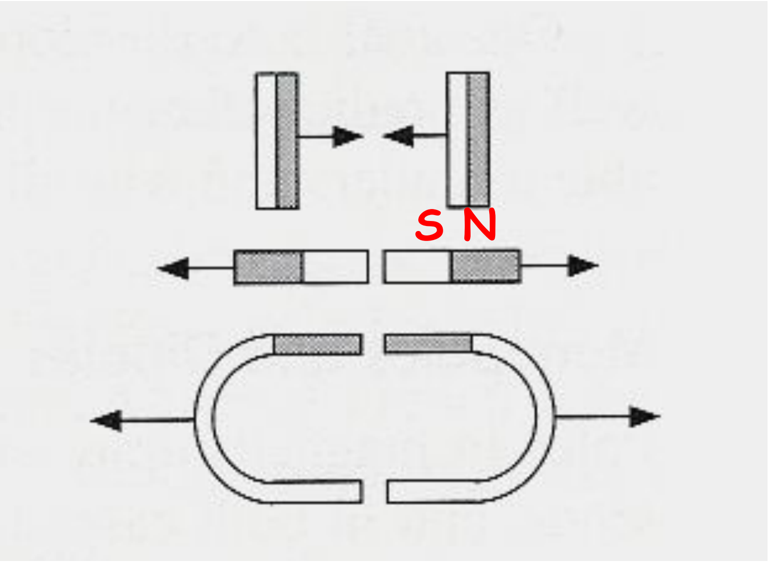
- Refrigerators are attracted to magnets!



Applications

- Motors
- Navigation – Compass
- Magnetic Tapes
  - Music, Data
- Television
  - Beam deflection Coil
- Magnetic Resonance Imaging
- High Energy Physics Research

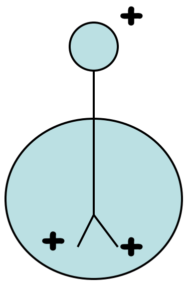
Magnets



Shaded End is NORTH Pole
Shaded End of a compass points to the NORTH.

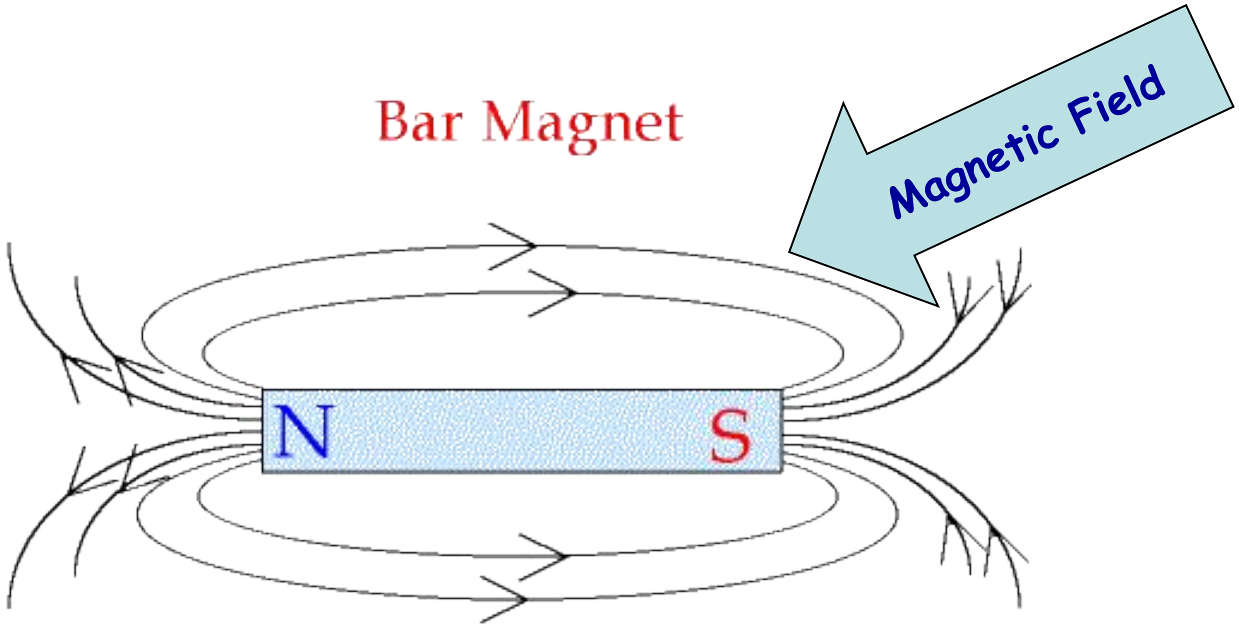
- Like Poles Repel
- Opposite Poles Attract
- Magnetic Poles are only found in pairs.
  - No magnetic monopoles have ever been observed.

Observations



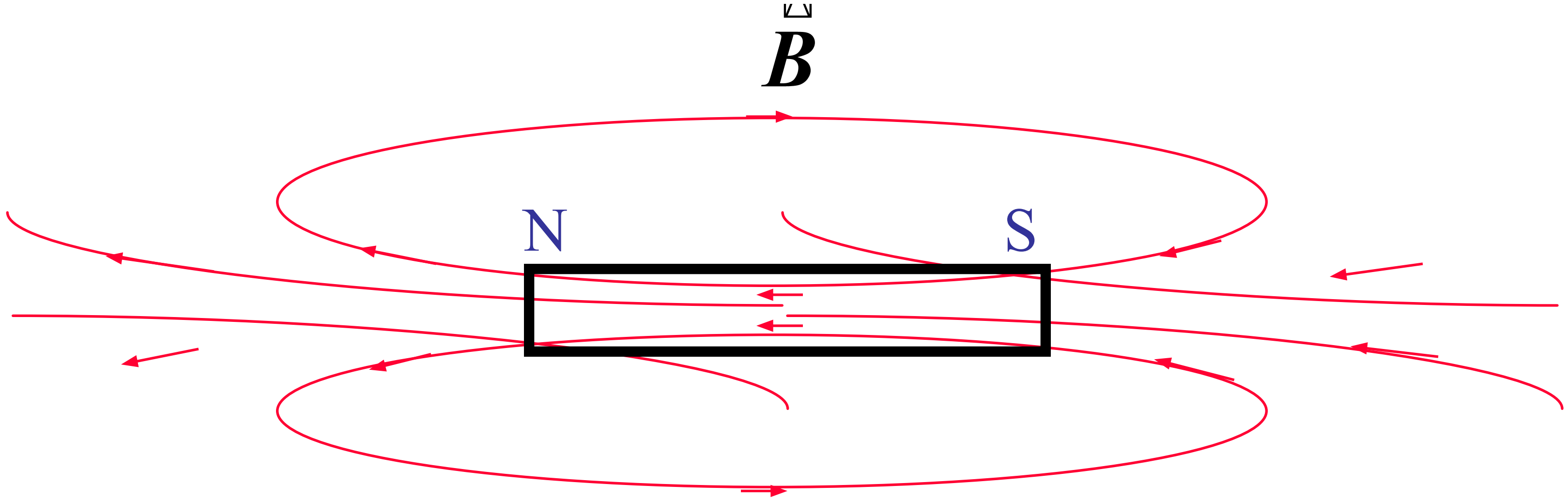
- Bring a magnet to a charged electroscope and nothing happens. No forces.
- Bring a magnet near some metals (Co, Fe, Ni …) and it will be attracted to the magnet.
  - The metal will be attracted to both the N and S poles independently.
  - Some metals are not attracted at all.
  - Wood is NOT attracted to a magnet.
  - Neither is water.
- A magnet will force a compass needle to align with it. (No big Surprise.)

Magnets

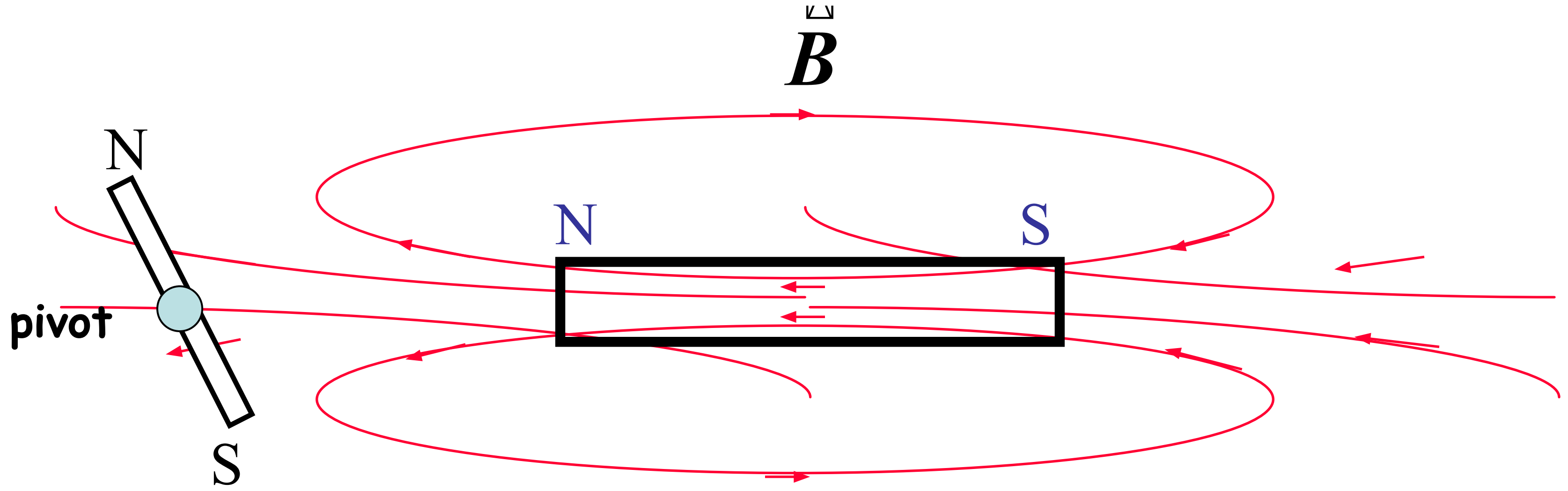


Cutting a bar magnet in half produces TWO bar magnets, each with N and S poles.

Consider a Permanent Magnet

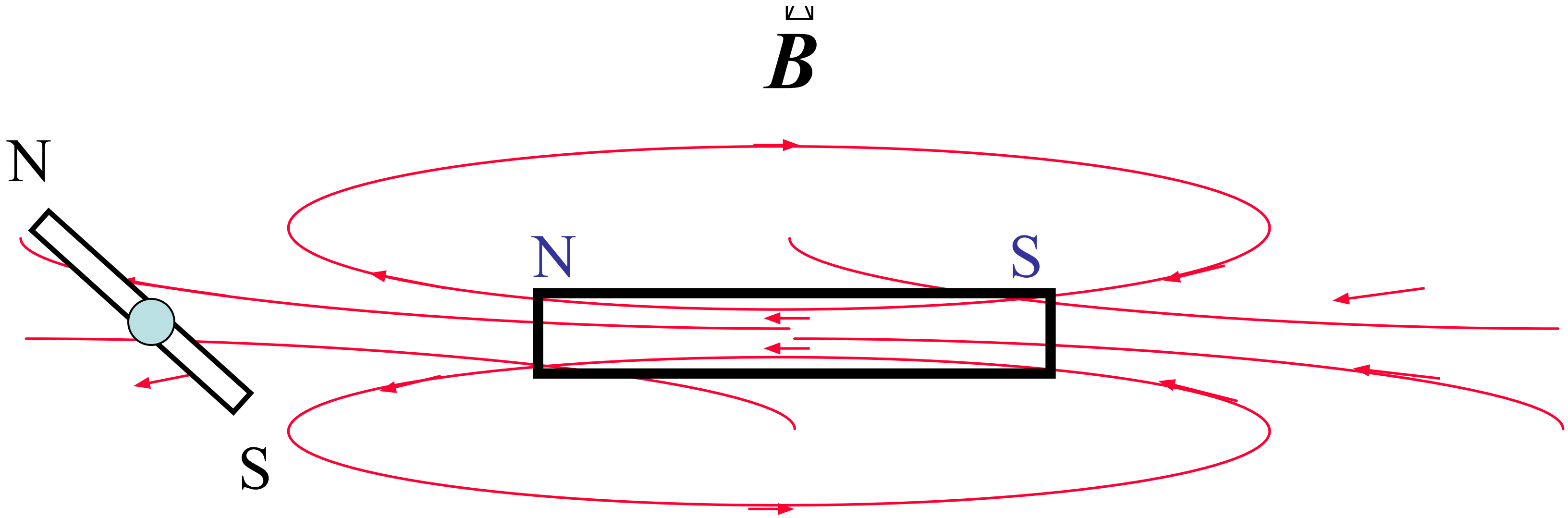


Introduce Another Permanent Magnet



The bar magnet (a magnetic dipole) wants to align with the B-field.

Field of a Permanent Magnet

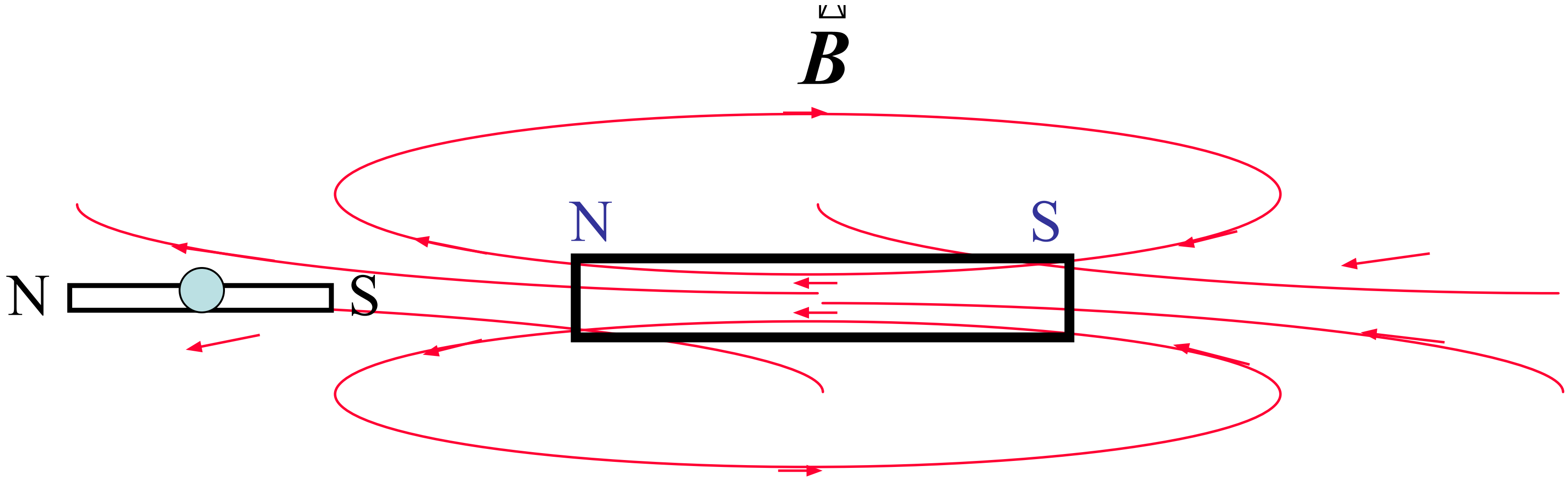


The south pole of the small bar magnet is attracted towards the north pole of the big magnet.

Also, the small bar magnet (a magnetic dipole) wants to align with the **B**-field.

The field **attracts** and **exerts a torque** on the small magnet.

Field of a Permanent Magnet



The bar magnet (a magnetic dipole) wants to align with the B-field.

The field exerts a <u>torque</u> on the dipole

The Magnetic Field

- Similar to Electric Field … exists in space.
  - Has Magnitude AND Direction.
- The "stronger" this field, the greater is the ability of the field to interact with a magnet.

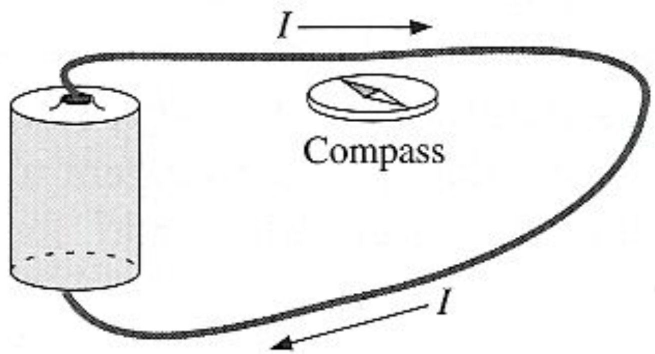
Convention For Magnetic Fields

$\vec{B}$

Field INTO Paper	Field OUT of Paper
X	□

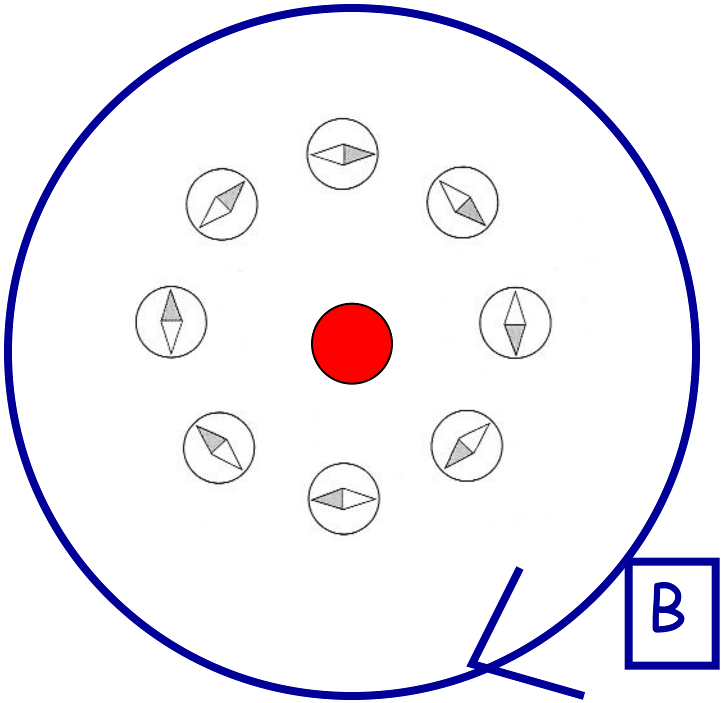
Experiments with Magnets Show

- Current carrying wire produces a circular magnetic field around it.



- Force on Compass Needle (or magnet) increases with current.

Current Carrying Wire



Current into the page.

Right hand Rule-

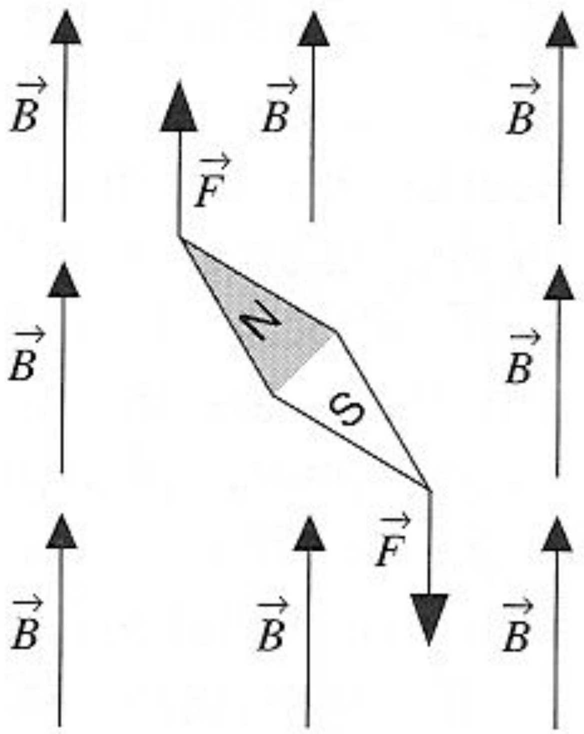
Thumb in direction of the current

Fingers curl in the direction of B

Current Carrying Wire

- B field is created at ALL POINTS in space surrounding the wire.
- The B field had magnitude and direction.
- Force on a magnet increases with the current.
- Force is found to vary as ~(1/d) from the wire.

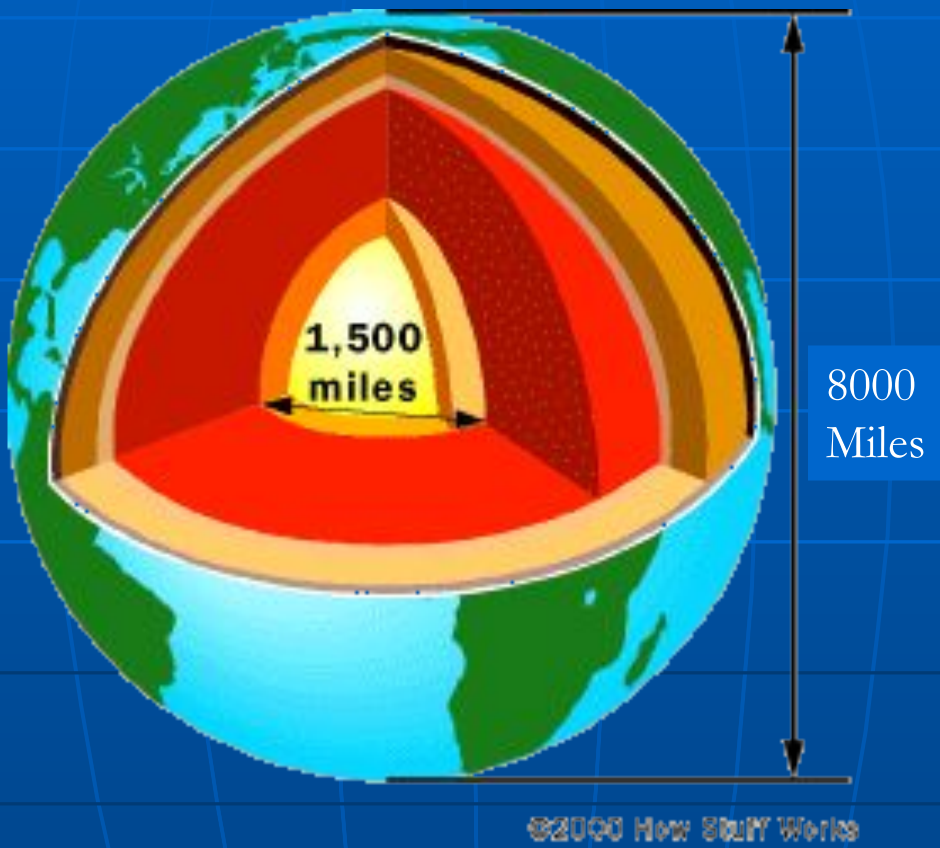
Compass and $\vec{B}$ Field



- Observations
  - **North** Pole of magnets tend to move toward the direction of B while S pole goes the other way.
  - Field exerts a **TORQUE** on a compass needle.
  - Compass needle is a **magnetic dipole**.
  - **North Pole of compass points toward the NORTH**.

Planet Earth

Inside it all.



On the surface it looks like this..

Inside: Warmer than Floriduh

Much Warmer than Floriduh

Finally

In Between

- The molten iron core exists in a magnetic field that had been created from other sources (sun…).
- The fluid is rotating in this field.
- This motion causes a current in the molten metal.
- The current causes a magnetic field.
- The process is self-sustaining.
- The driving force is the heat (energy) that is generated in the core of the planet.

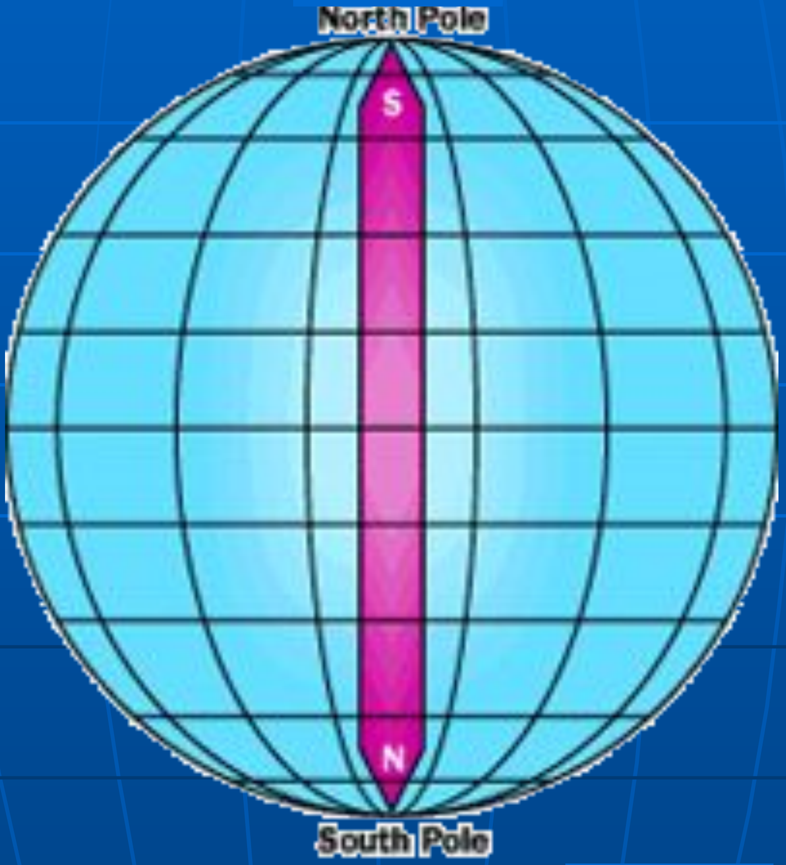
After molten lava emerges from a volcano, it solidifies to a rock. In most cases it is a black rock known as basalt, which is faintly magnetic, like iron emerging from a melt. Its magnetization is in the direction of the local magnetic force at the time when it cools down.

Instruments can measure the magnetization of basalt. Therefore, if a volcano has produced many lava flows over a past period, scientists can analyze the magnetizations of the various flows and from them get an idea on how the direction of the local Earth's field varied in the past. Surprisingly, this procedure suggested that times existed when the magnetization had the **opposite** direction from today's. All sorts of explanation were proposed, but in the end the only one which passed all tests was that in the distant past, indeed, the magnetic polarity of the Earth was sometimes reversed.

Ancient Navigation

This planet is really screwed up!

NORTH
POLE

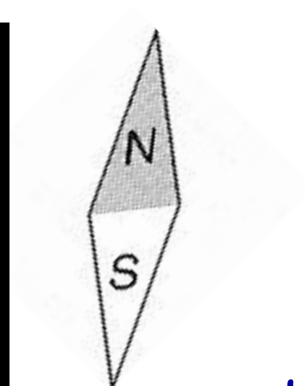


SOUTH POLE

Repeat

Navigation
DIRECTION
N

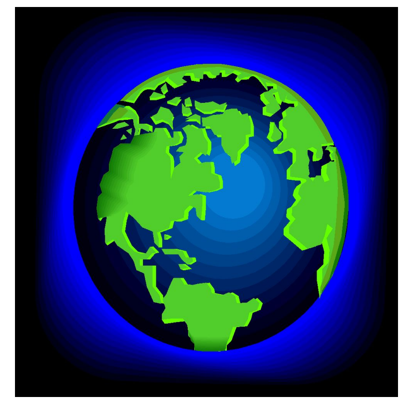
S



Compass
Direction

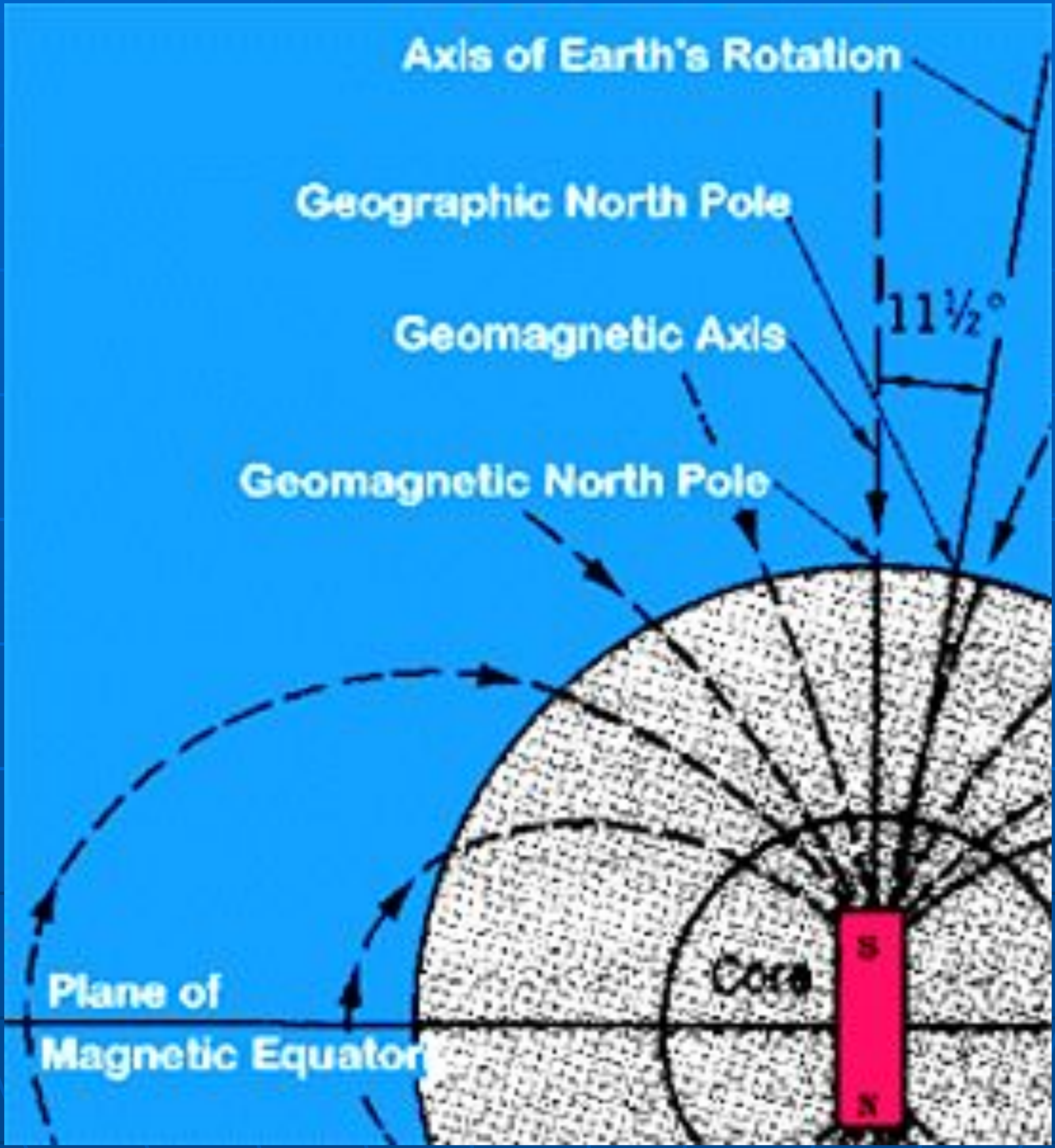
If N direction
is pointed to by
the NORTH pole
of the Compass
Needle, then the
pole at the NORTH
of our planet must
be a SOUTH MAGNETIC
POLE!

Navigation
DIRECTION
S

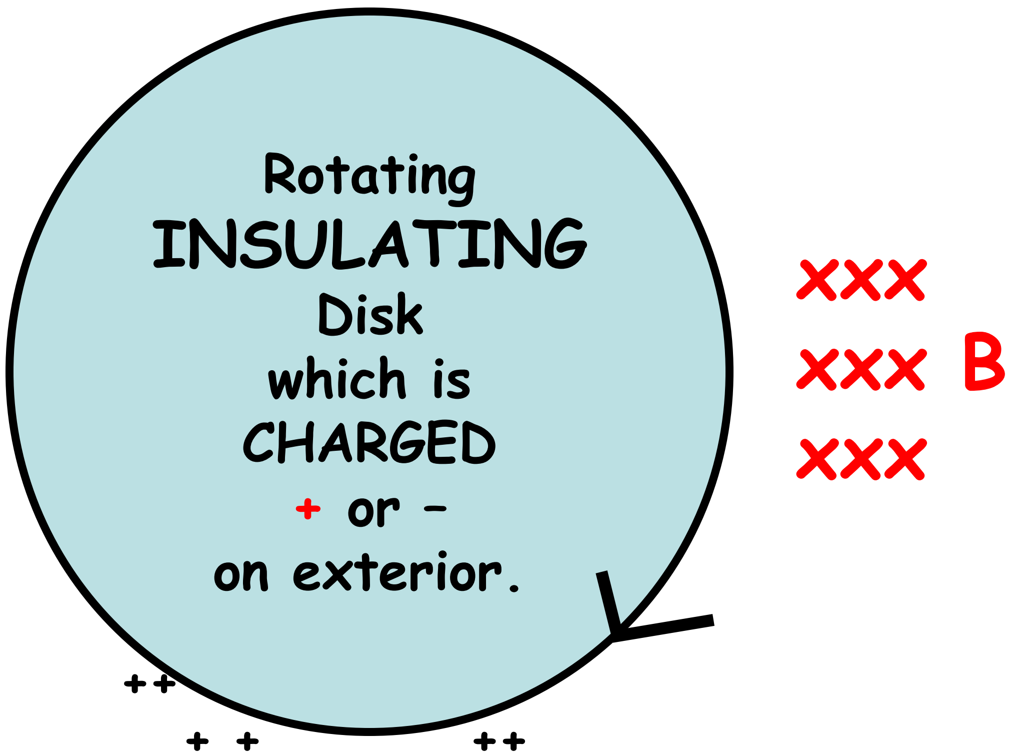


N

And it REVERSES from time to time.


Axis of Earth's Rotation
Geographic North Pole
11½°
Geomagnetic Axis
Geomagnetic North Pole
Core
S
N
Plane of
Magnetic Equator

Rowland’s Experiment



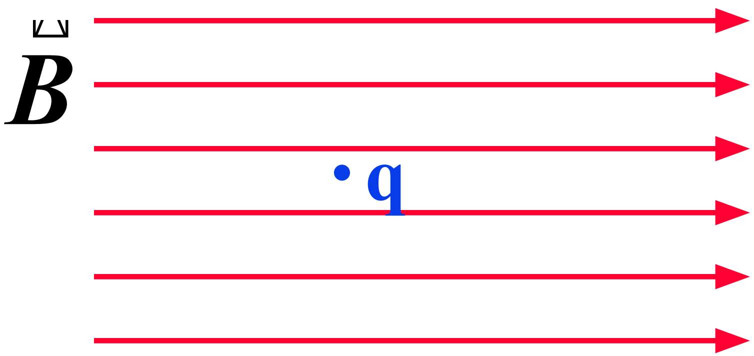
Field is created by any moving charge.

Increases with charge on the disk.

Increases with angular velocity of the disk.

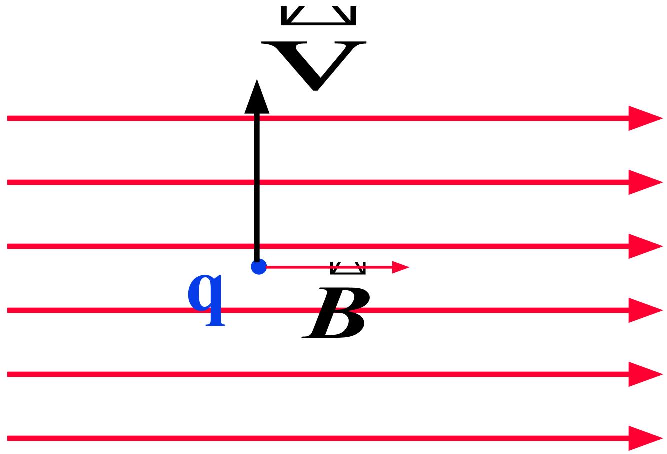
Electrical curent is a moving charge.

A Look at the Physics



There is NO force on a charge placed into a magnetic field if the charge is NOT moving.

There is no force if the charge moves parallel to the field.



- If the charge is moving, there is a force on the charge, ***perpendicular*** to both **v** and **B**.

$$\mathbf{F} = q\,\mathbf{v} \times \mathbf{B}$$

WHAT THE HECK IS THAT???

- A WHAT PRODUCT?
- A CROSS PRODUCT – Like an angry one??
- Alas, yes ….
  - $F = qv \times B$

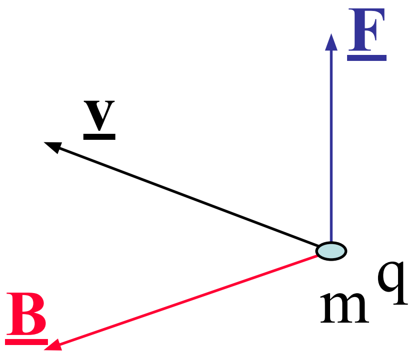
The Lorentz Force

This can be summarized as:

$$\vec{F} = q\vec{v} \times \vec{B}$$

or:

$$F = qvB\sin\theta$$



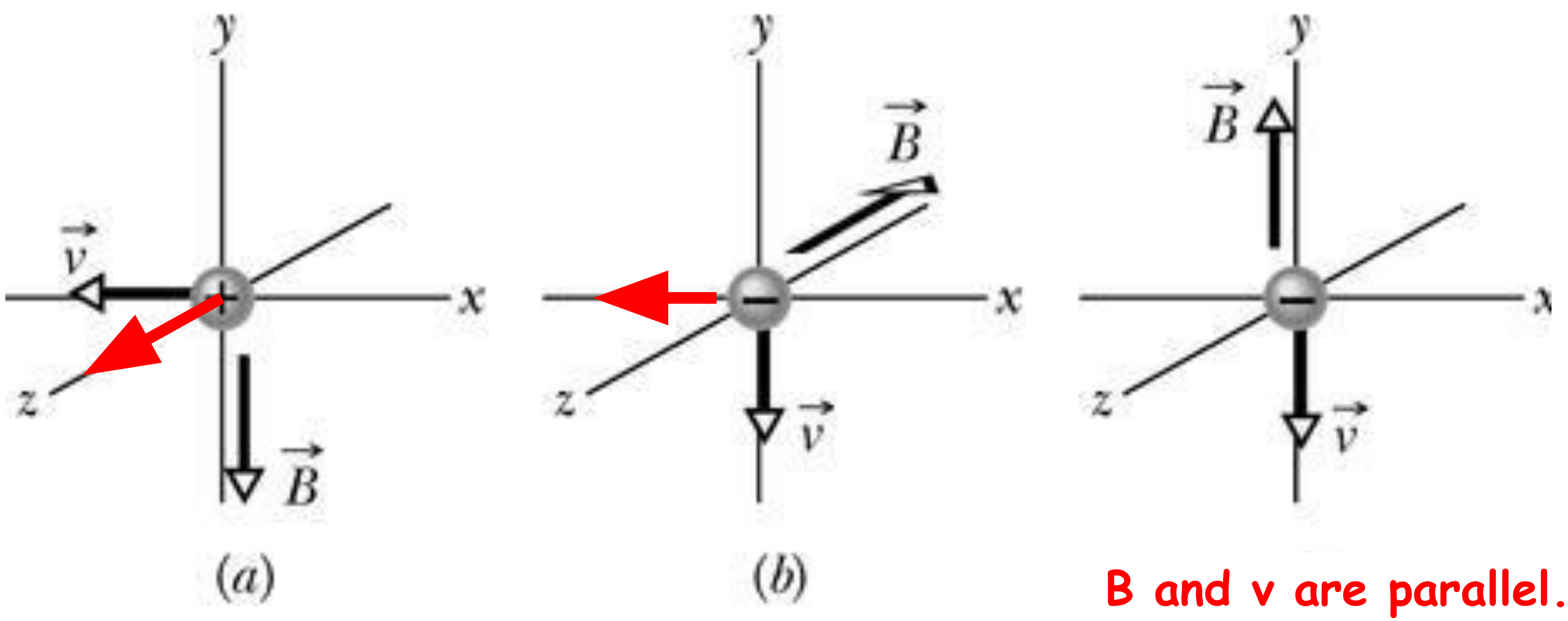
θ is the angle between B and V

Note

B is sort of the Force per unit (charge-velocity)

Whatever that is!!

Practice



B and v are parallel.
Crossproduct is zero.
So is the force.

Which way is the Force???

Units

$$F = Bqv\,Sin(\theta)$$

$$Units:$$

$$B = \frac{[F]}{[qv]} = \frac{N}{Cm/s} = \frac{\mathrm{N}}{\mathrm{Amp-m}}$$

$$1\,\mathrm{tesla} = 1\,\mathrm{T} = 1\,\mathrm{N/(A\text{-}m)}$$

teslas are HUGE!

At the Surface of the Earth	3×10^{-5} T
Typical Refrigerator Magnet	5×10^{-3} T
Laboratory Magnet	0.1 T
Large Superconducting Magnet	10 T

The Magnetic Force is *Different* From the Electric Force.

Whereas the electric force acts in the same direction as the field:

$$\vec{F} = q\vec{E}$$

The magnetic force acts in a direction orthogonal to the field:

$$\vec{F} = q\vec{v} \times \vec{B}$$

(Use "Right-Hand" Rule to determine direction of F)

And --- the charge must be moving !!

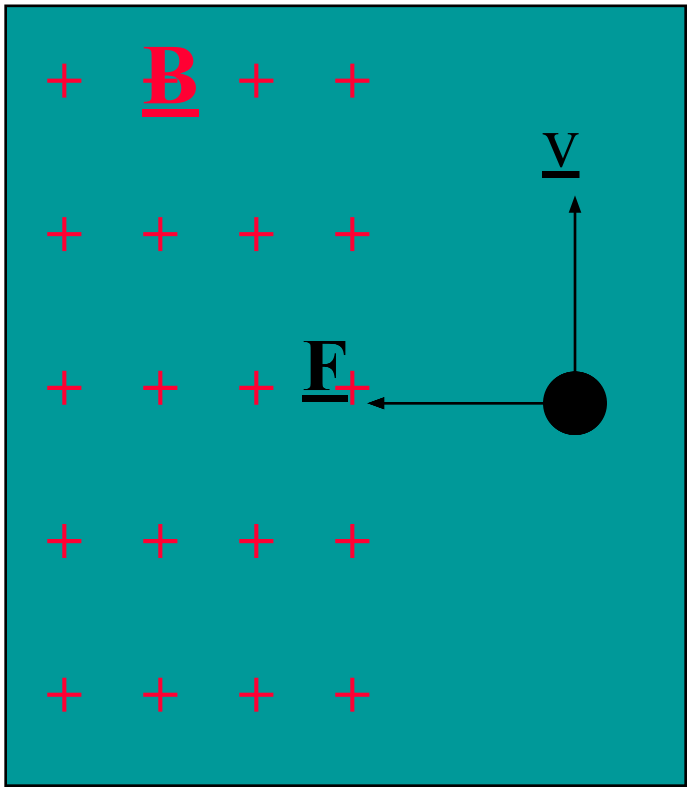
So…

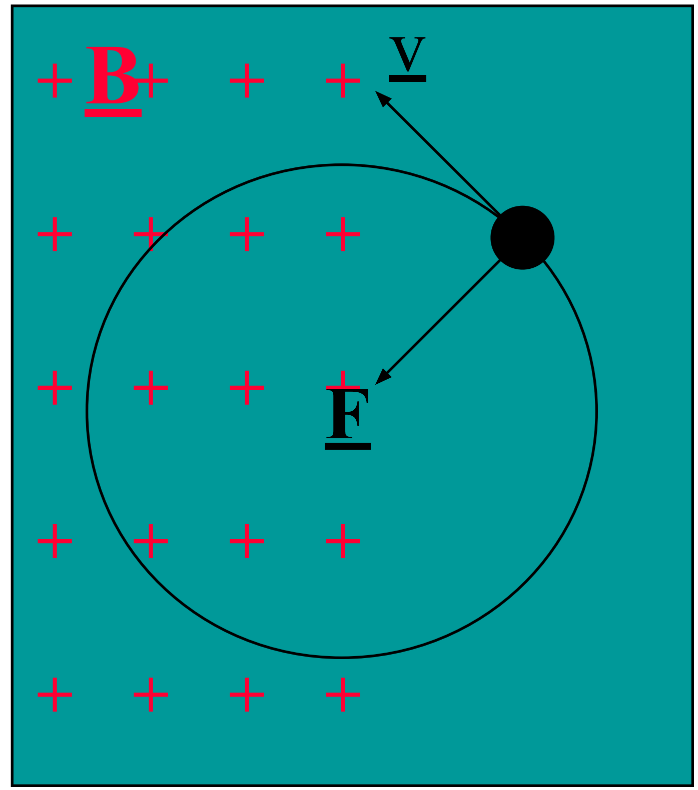
- A moving charge can create a magnetic field.
- A moving charge is acted upon by a magnetic field.

- In Magnetism, things move.
- In the Electric Field, forces and the field can be created by stationary charges.

Trajectory of Charged Particles in a Magnetic Field

(B field points *into* plane of paper.)





Trajectory of Charged Particles in a Magnetic Field

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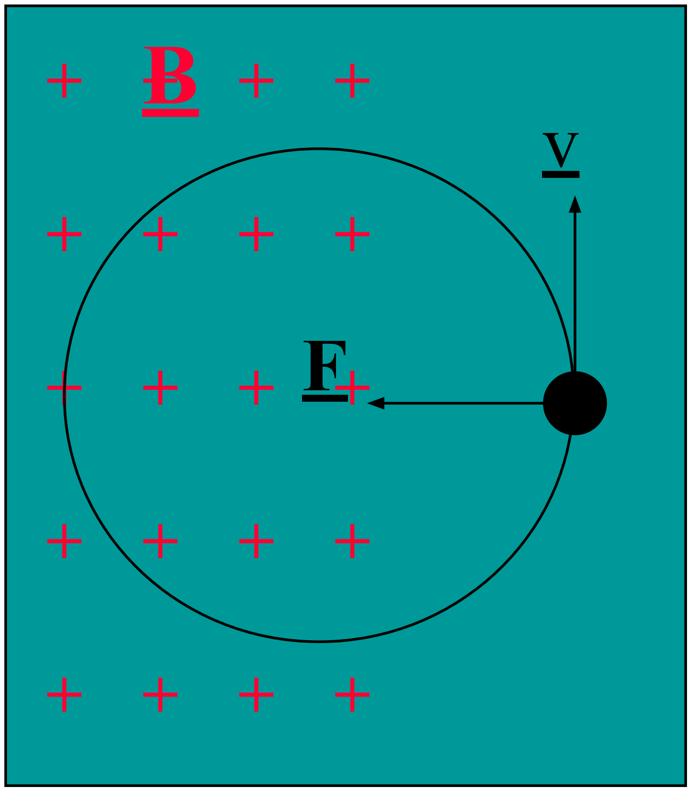
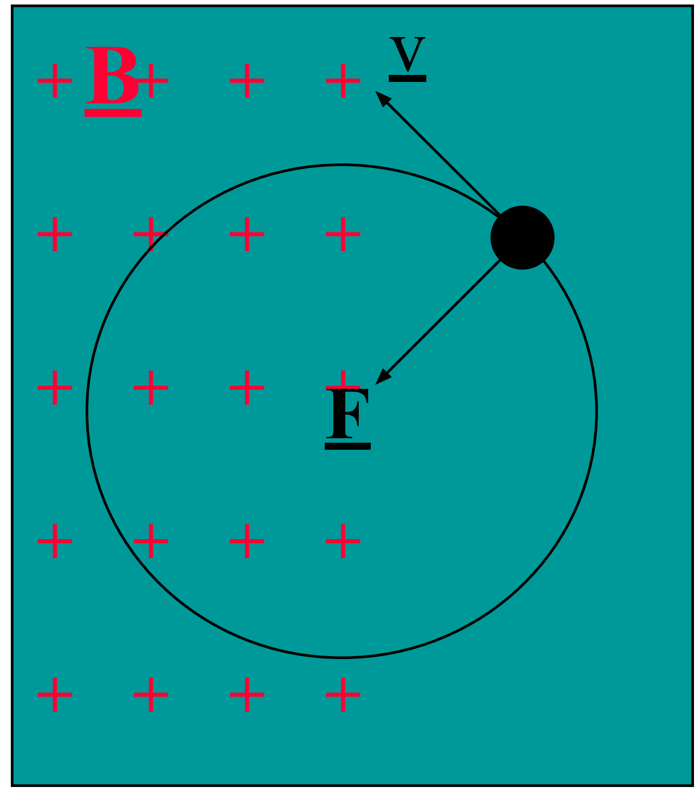
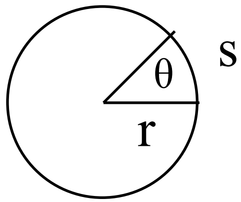




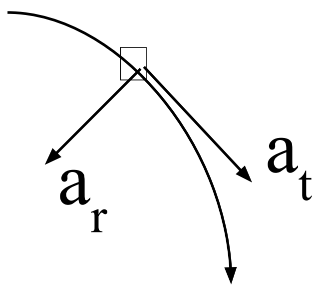
Magnetic Force is a ***centripetal*** force

Review of Rotational Motion



$\theta = s / r \Rightarrow s = \theta\, r \Rightarrow ds/dt = d\theta/dt\; r \Rightarrow v = \omega\, r$

θ = angle, ω = angular speed, α = angular acceleration

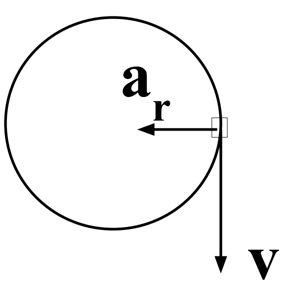


$a_t = r\,\alpha$ tangential acceleration

$a_r = v^2 / r$ radial acceleration

The radial acceleration changes the direction of motion, while the tangential acceleration changes the speed.

Uniform Circular Motion



ω = constant $\Rightarrow$ v and a_r constant but direction changes

$a_r = v^2/r = \omega^2 r$

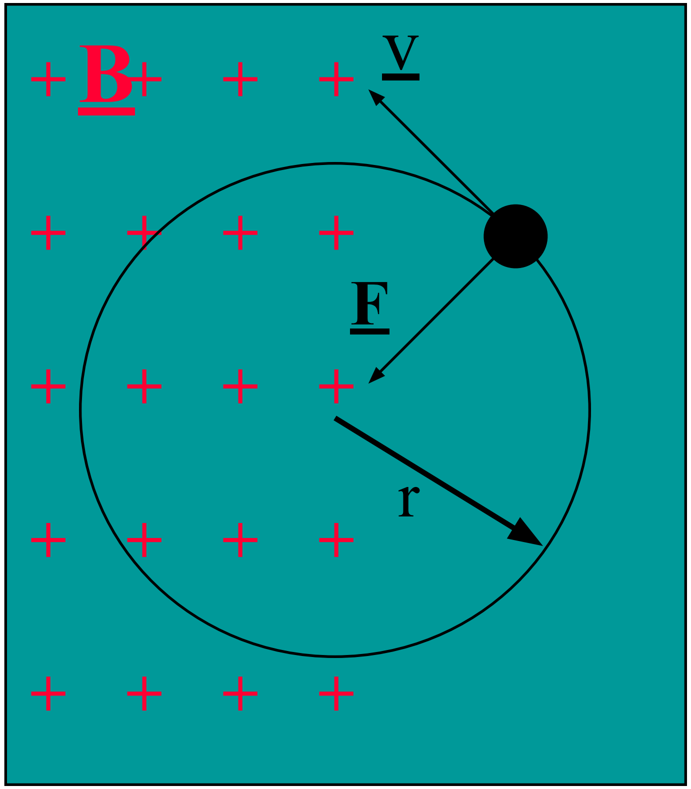
$KE = ½\, mv^2 = ½\, mw^2r^2$

$F = ma_r = mv^2/r = m\omega^2 r$

YES !

You have to remember this stuff.

Radius of a Charged Particle Orbit in a Magnetic Field



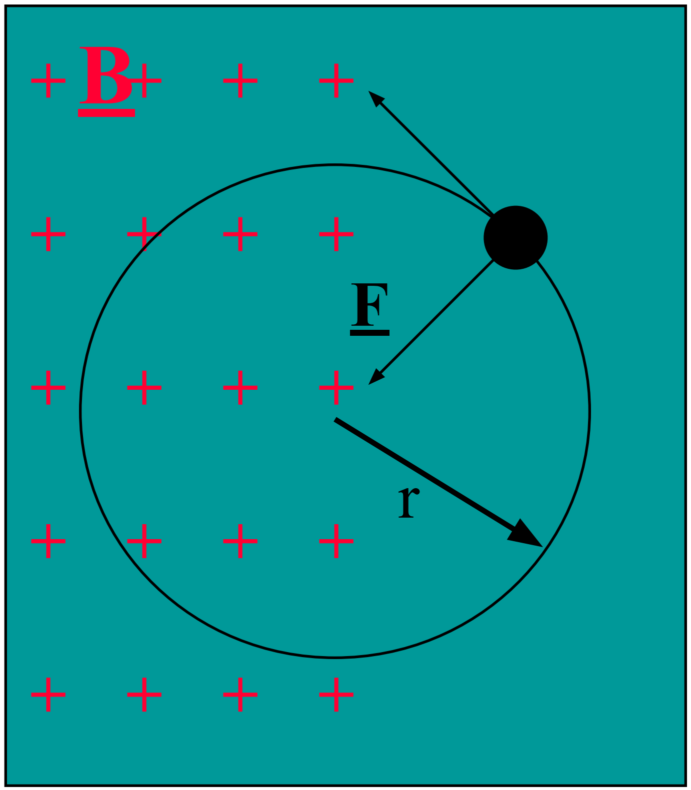
Centripetal Force = Magnetic Force

$$\therefore \frac{m\mathrm{v}^2}{r} = q\mathrm{v}B$$

$$\Rightarrow \qquad r = \frac{m\mathrm{v}}{qB}$$

Note: as $\vec{F} \perp \vec{\mathrm{v}}$, the magnetic force does no work!

Cyclotron Frequency



The time taken to complete one orbit is:

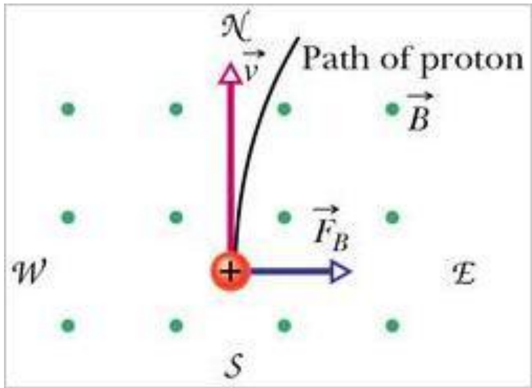
$$T = \frac{2\pi r}{v} = \frac{2\pi}{v}\frac{mv}{qB}$$

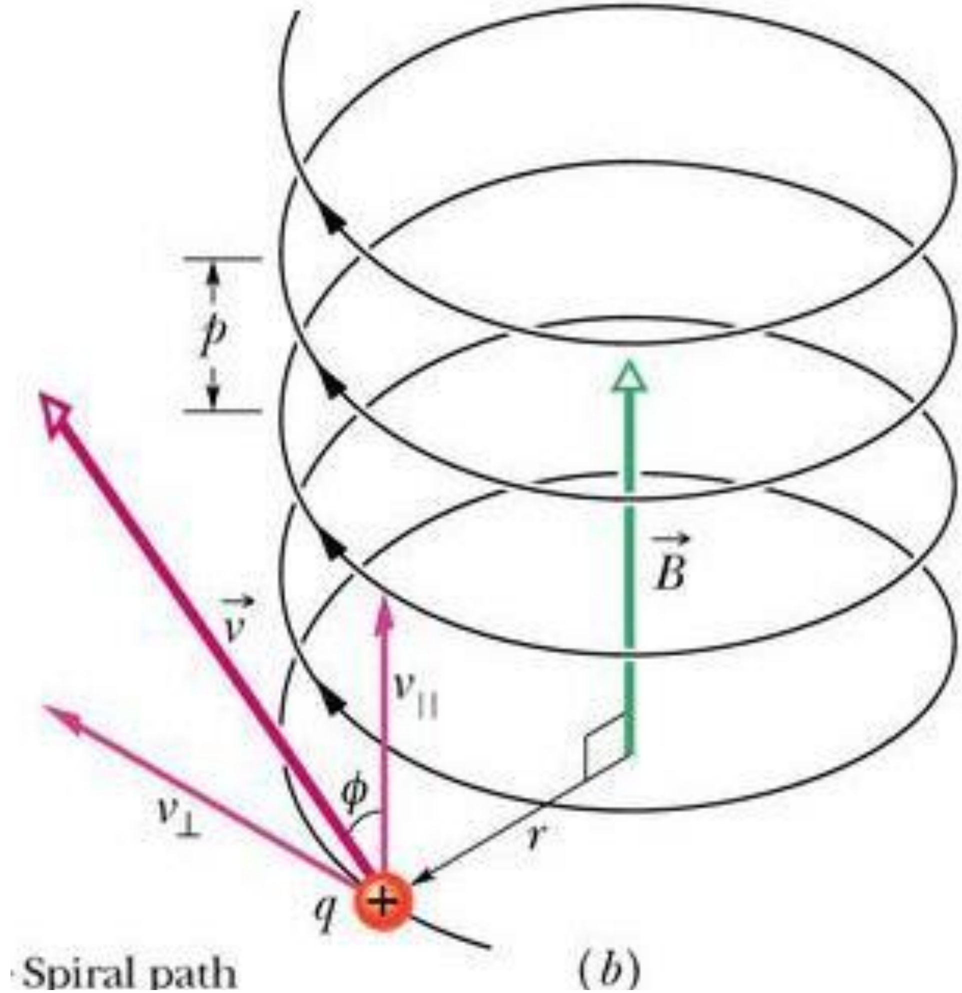
V cancels !

$$f = \frac{1}{T} = \frac{qB}{2\pi m}$$

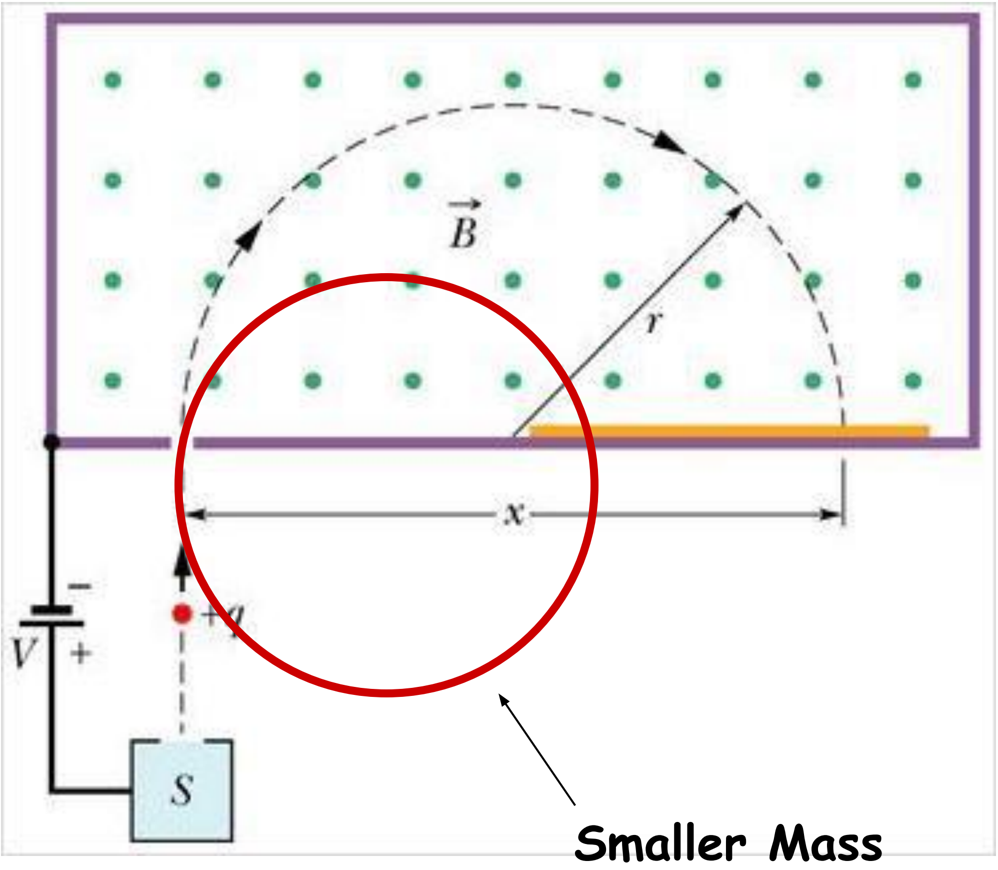
$$\omega_c = 2\pi f = \frac{qB}{m}$$

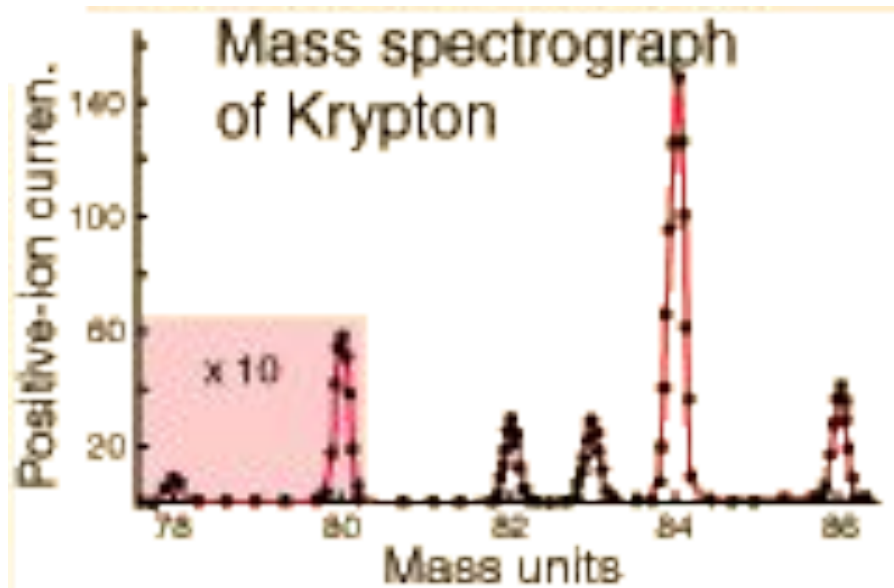
More Circular Type Motion in a Magnetic Field



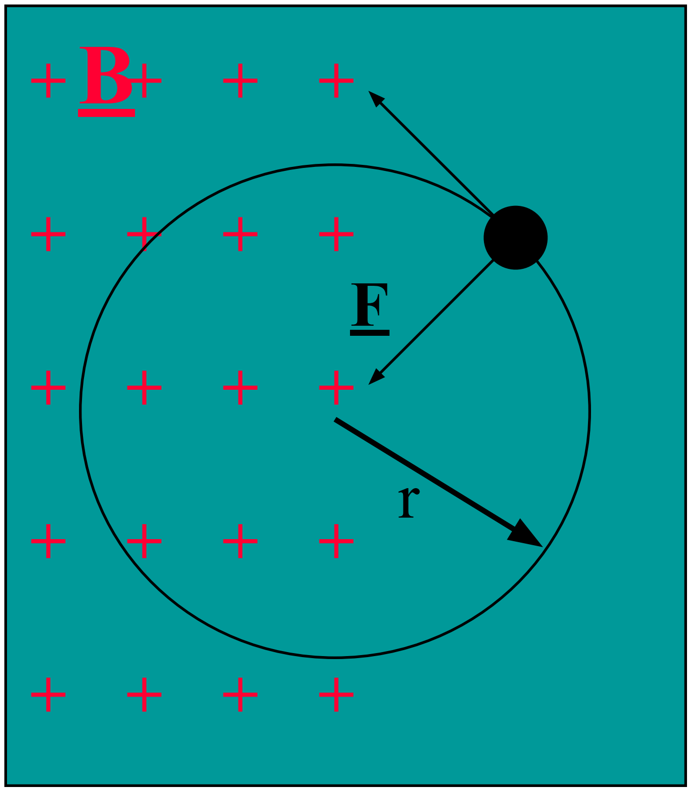


Mass Spectrometer




Mass spectrograph
of Krypton
Positive-ion curren.
140
100
60
20
x 10
78
80
82
84
86
Mass units

Cyclotron Frequency



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$$T = \frac{2\pi r}{v}$$

$$= \frac{2\pi}{v}\frac{mv}{qB}$$

V cancels !

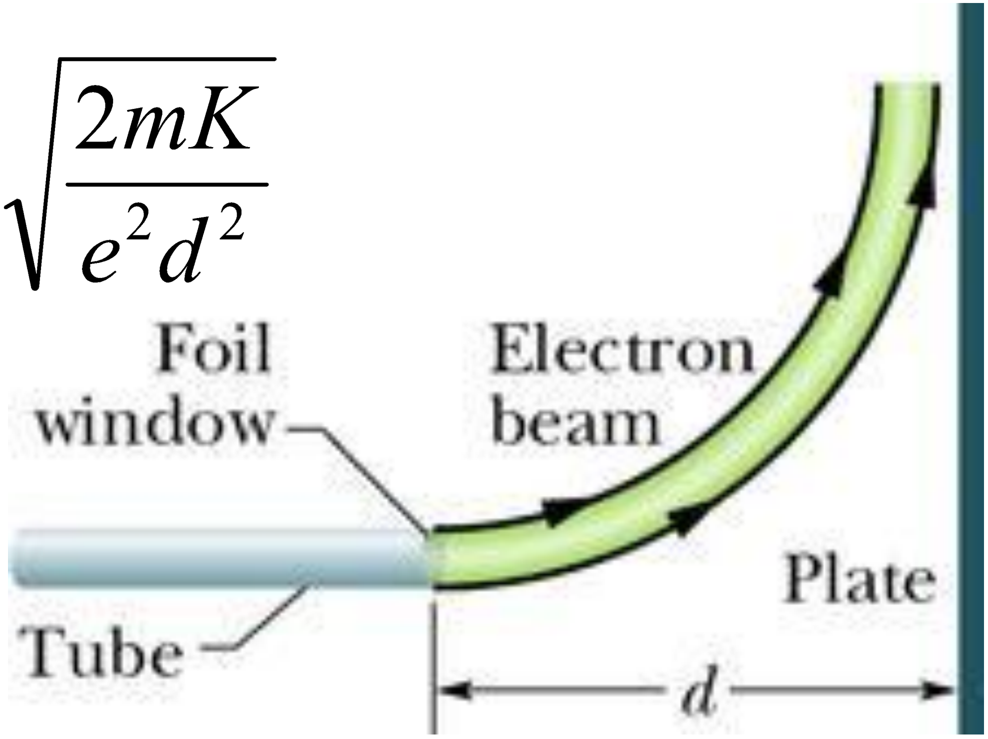
$$f = \frac{1}{T} = \frac{qB}{2\pi m}$$

$$\omega_c = 2\pi f = \frac{qB}{m}$$

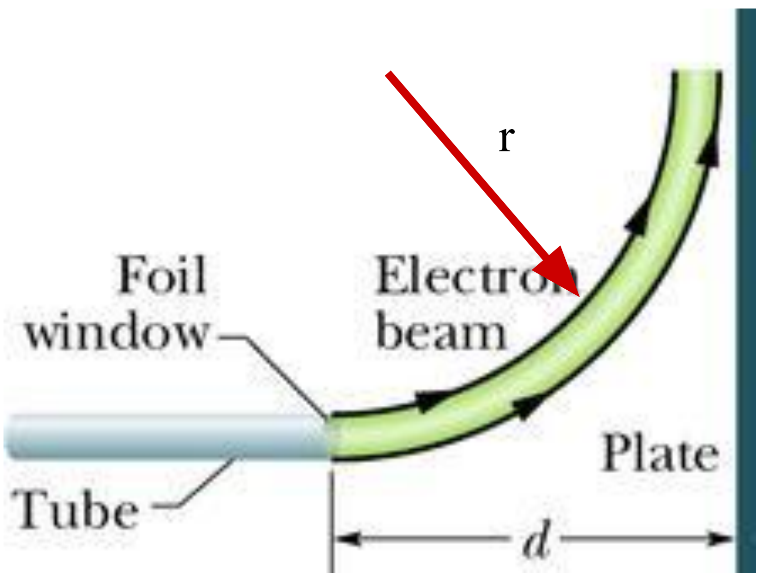
An Example

A beam of electrons whose kinetic energy is *K* emerges from a thin-foil "window" at the end of an accelerator tube. There is a metal plate a distance *d* from this window and perpendicular to the direction of the emerging beam. Show that we can prevent the beam from hitting the plate if we apply a uniform magnetic field B such that

$$B \geq \sqrt{\frac{2mK}{e^2 d^2}}$$



Problem Continued



From Before

$$r = \frac{mv}{qB}$$

$$K = \frac{1}{2}mv^2 \text{ so } v = \sqrt{\frac{2K}{m}}$$

$$r = \frac{m}{eB}\sqrt{\frac{2K}{m}} = \sqrt{\frac{2mK}{e^2 B^2}} \leq d$$

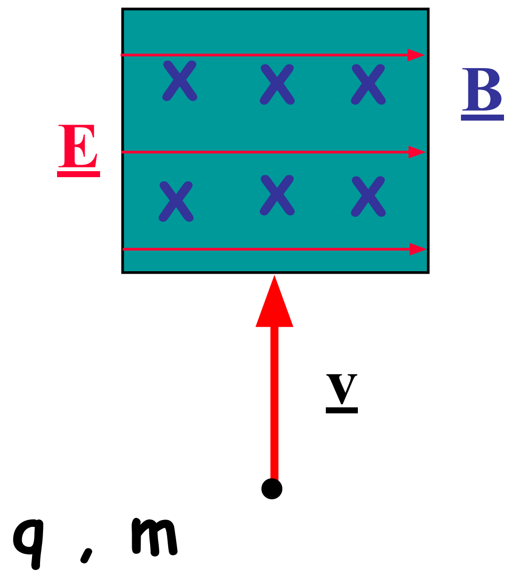
Solve for B:

$$B \geq \sqrt{\frac{2mK}{e^2 d^2}}$$

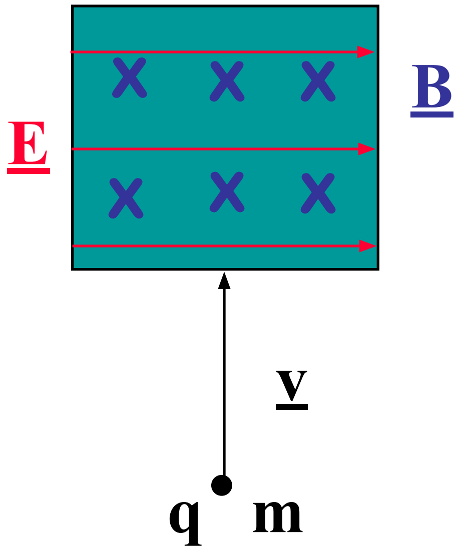
Some New Stuff

Magnetism and Forces

Let's Look at the effect of crossed **E** and **B** Fields:



What is the relation between the intensities of the electric and magnetic fields for the particle to move in a straight line ?.

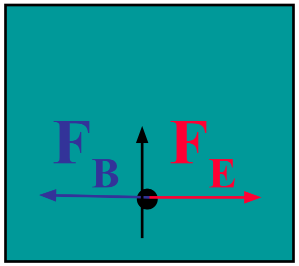


$F_E = q\,E$ and $F_B = q\,v\,B$

If $F_E = F_B$ the particle will move following a straight line trajectory

$$q\,E = q\,v\,B$$

$$\boxed{v = E / B}$$



What does this mean??

$v = E / B$

- This equation only contains the E and B fields in it.
- Mass is missing!
- Charge is missing!
- This configuration is a velocity filter!

"Real" Mass Spectrometer

- Create ions from injected species.
  - This will contain various masses, charges and velocities.
  - These are usually accelerated to a certain ENERGY (KeV) by an applied electric field.
  - The crossed field will only allow a selected velocity to go forward into the MS.
  - From before: $R=mv/Bq$

Components of MS:

Accelerate the ions through a known potential difference.

$$\frac{1}{2}mv^2 = qV_{applied}$$

So

$$\frac{q}{m} = \frac{1}{2}v^2\frac{1}{V_{applied}}$$

The velocity can be selected via an E x B field and the MS will separate by:

$$R = \frac{mv}{Bq}$$

Unknown is mass to charge ratio which can be sorted from the spectrum

Remember:

THESE "E and B" GUYS ARE VECTORS!

Let's Look at an example...

VECTOR CALCULATIONS

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

Problem: A Vector Example

A proton of charge +e and mass m is projected into a uniform magnetic field **B=Bi** with an initial velocity **v=v$_{0x}$i +v$_{0y}$j.** **Find the velocity at a later time.**

$$\mathbf{F} = e\mathbf{v} \times \mathbf{B}$$

$$\mathbf{F} = \begin{vmatrix} i & j & k \\ v_x & v_y & v_z \\ B & 0 & 0 \end{vmatrix} = eB(v_z\mathbf{j} - v_y\mathbf{k}) = m\mathbf{a}$$

$$ma_x = m\frac{dv_x}{dt} \text{ etc. for y and z.}$$

Equating components :

$$\frac{\mathrm{dv_x}}{\mathrm{dt}} = 0 \qquad \frac{\mathrm{dv_y}}{\mathrm{dt}} = \frac{eB}{m}v_z \quad \text{and} \quad \frac{\mathrm{dv_z}}{\mathrm{dt}} = -\frac{eB}{m}v_y$$

v_x is constant

More

$$\text{Let } \omega = \frac{\text{eB}}{\text{m}}$$

$$\frac{d^2 v_y}{dt^2} = \omega \frac{dv_z}{dt} = -\omega^2 v_y$$

$$\frac{d^2 v_y}{dt^2} + \omega^2 v_y = 0$$

Simple circular motion!

$$\text{v}_\text{y} = v_{0y} \cos(\omega t)$$

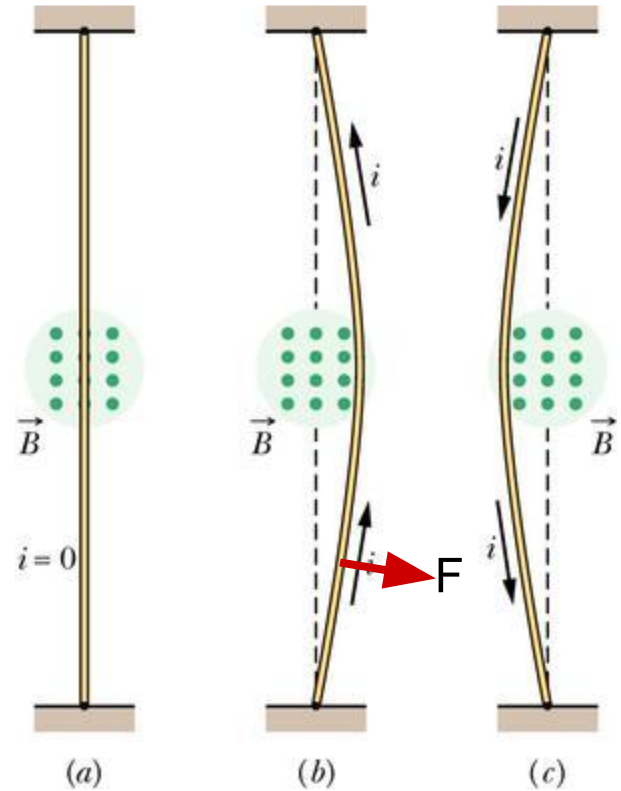
Same thing for z.

New Topic

Forces on Wires

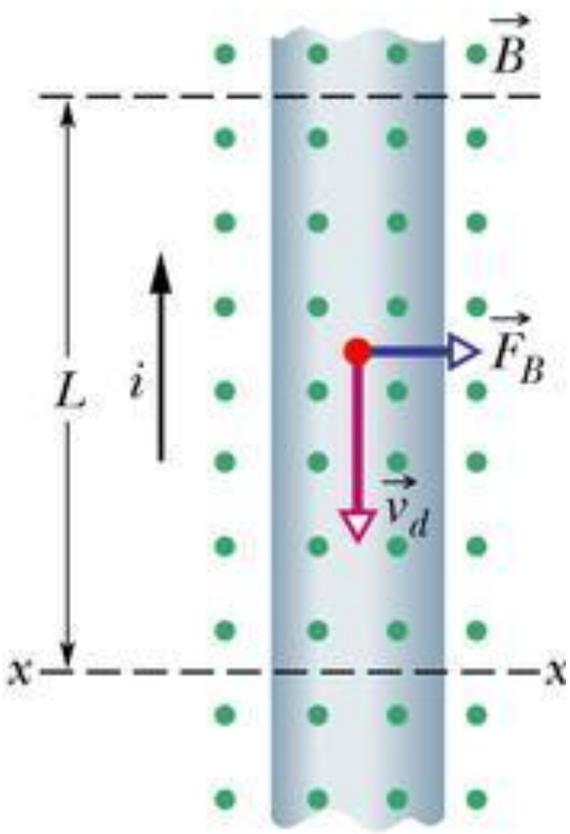
Wires

- A wire with a current contains moving charges.
- A magnetic field will apply a force to those moving charges.
- This results in a force on the wire itself.
  - The electron's sort of PUSH on the side of the wire.



Remember: Electrons go the "other way".

The Wire in More Detail



Assume all electrons are moving with the same velocity v_d.

$$q = i \times t = i\frac{L}{v_d}$$

$$F = qv_dB = i\frac{L}{v_d}v_dB = iLB$$

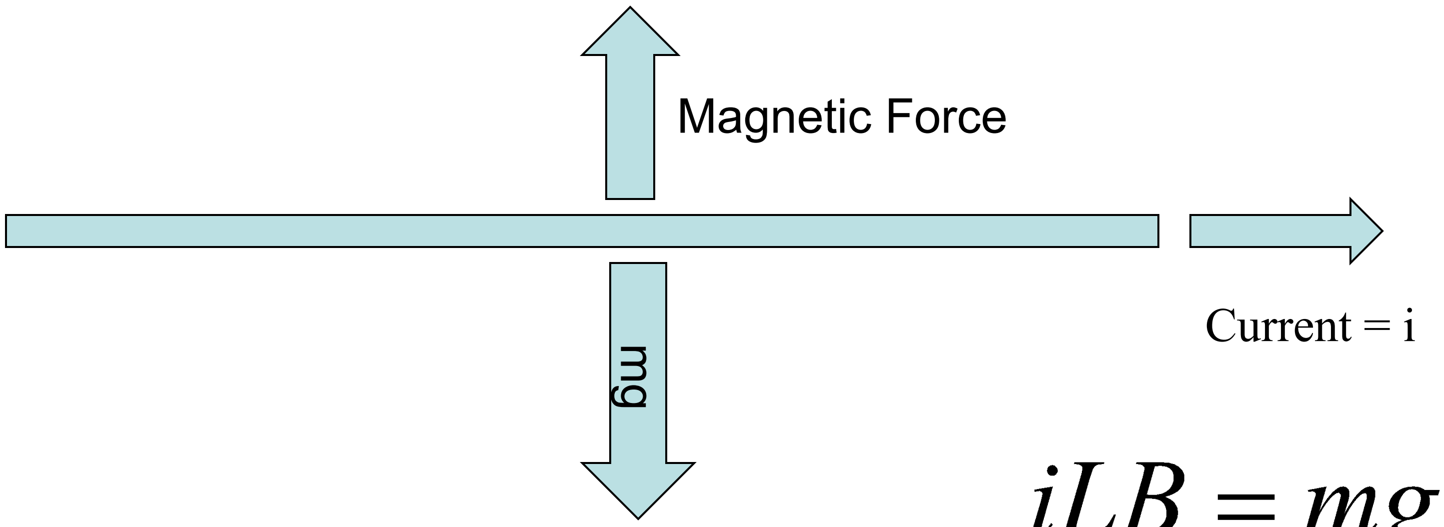
vector :

$$\mathbf{F} = i\mathbf{L} \times \mathbf{B}$$

L in the direction of the motion of POSITIVE charge (i).

B out of plane of the paper

Magnetic Levitation



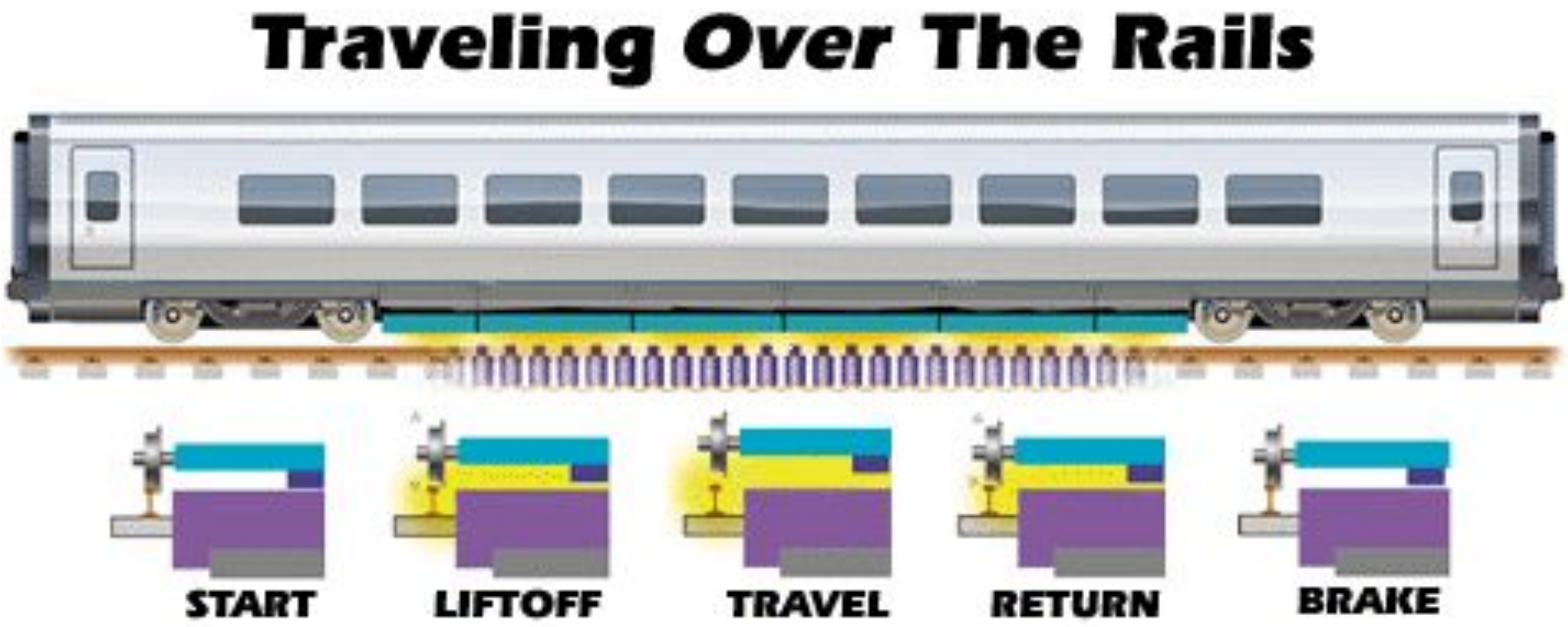
Where does B point????

Into the paper.

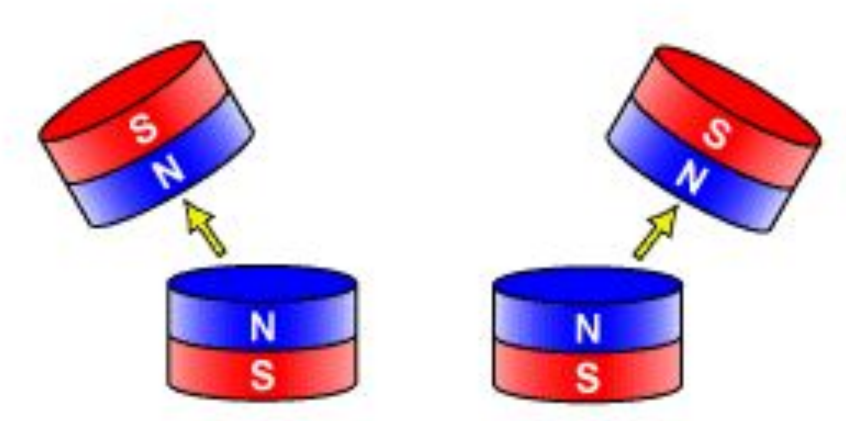
$$iLB = mg$$

$$B = \frac{mg}{iL}$$

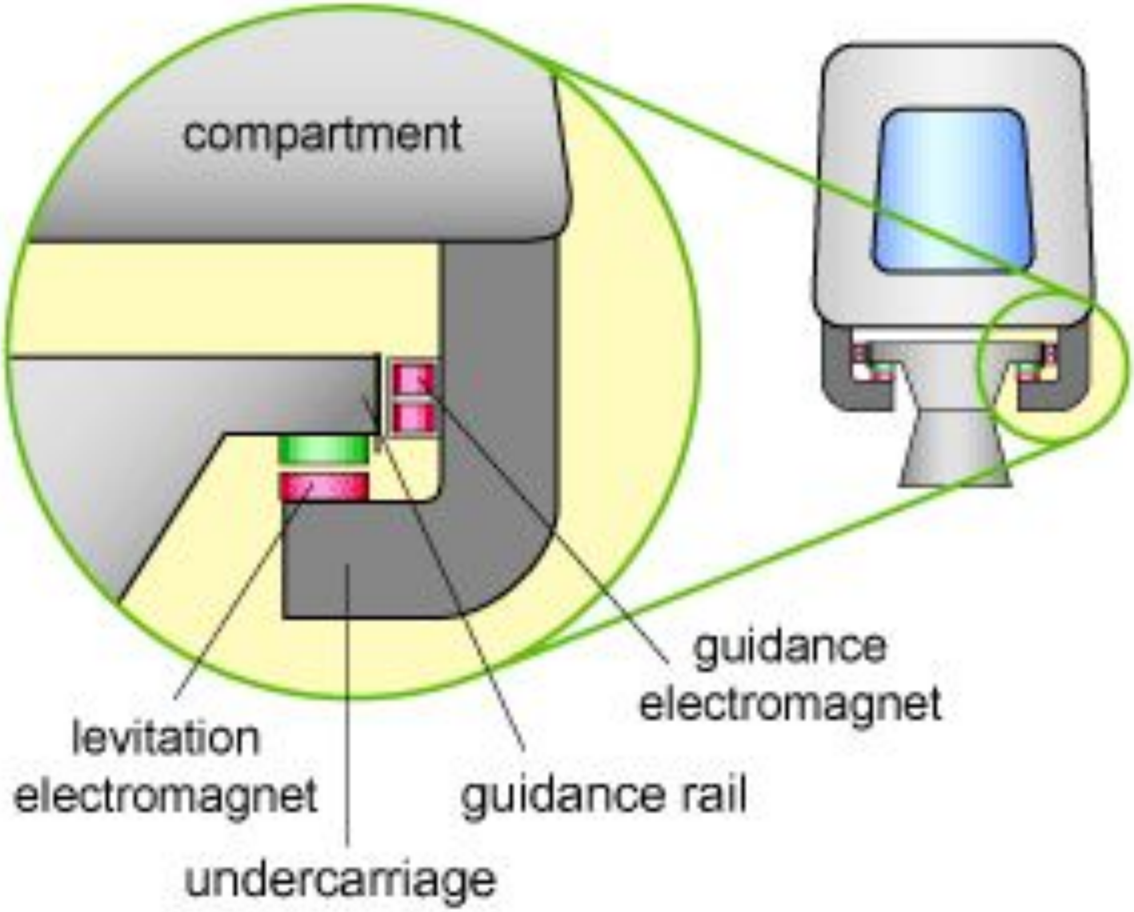
MagLev


Traveling Over The Rails
START
LIFTOFF
TRAVEL
RETURN
BRAKE

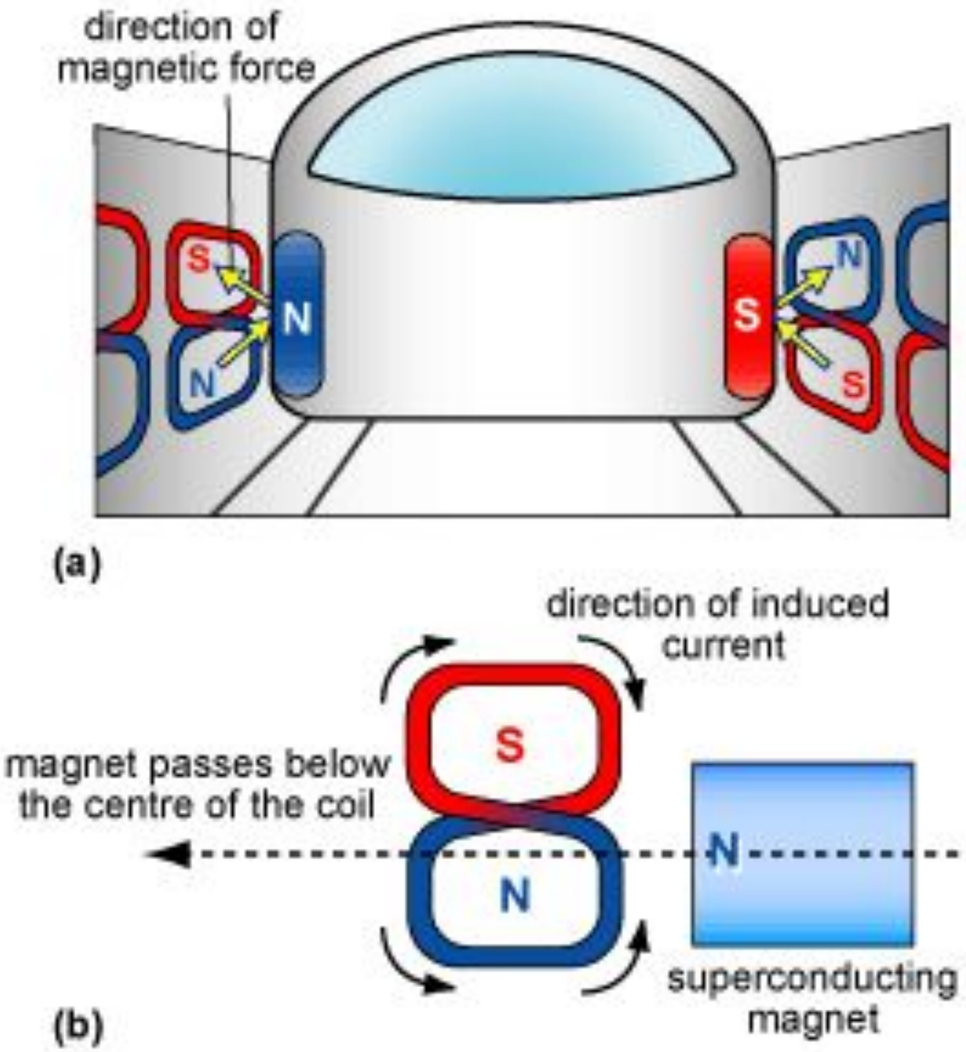
Magnetic Repulsion



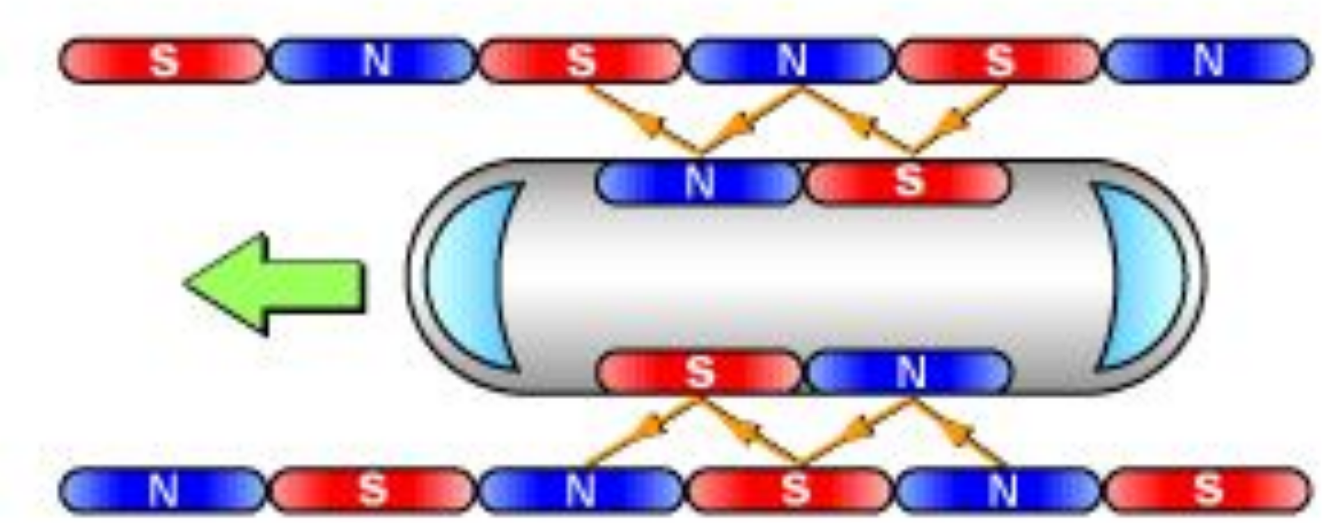
Detail


compartment
guidance
electromagnet
levitation
electromagnet
guidance rail
undercarriage

Moving Right Along ….



Acceleration

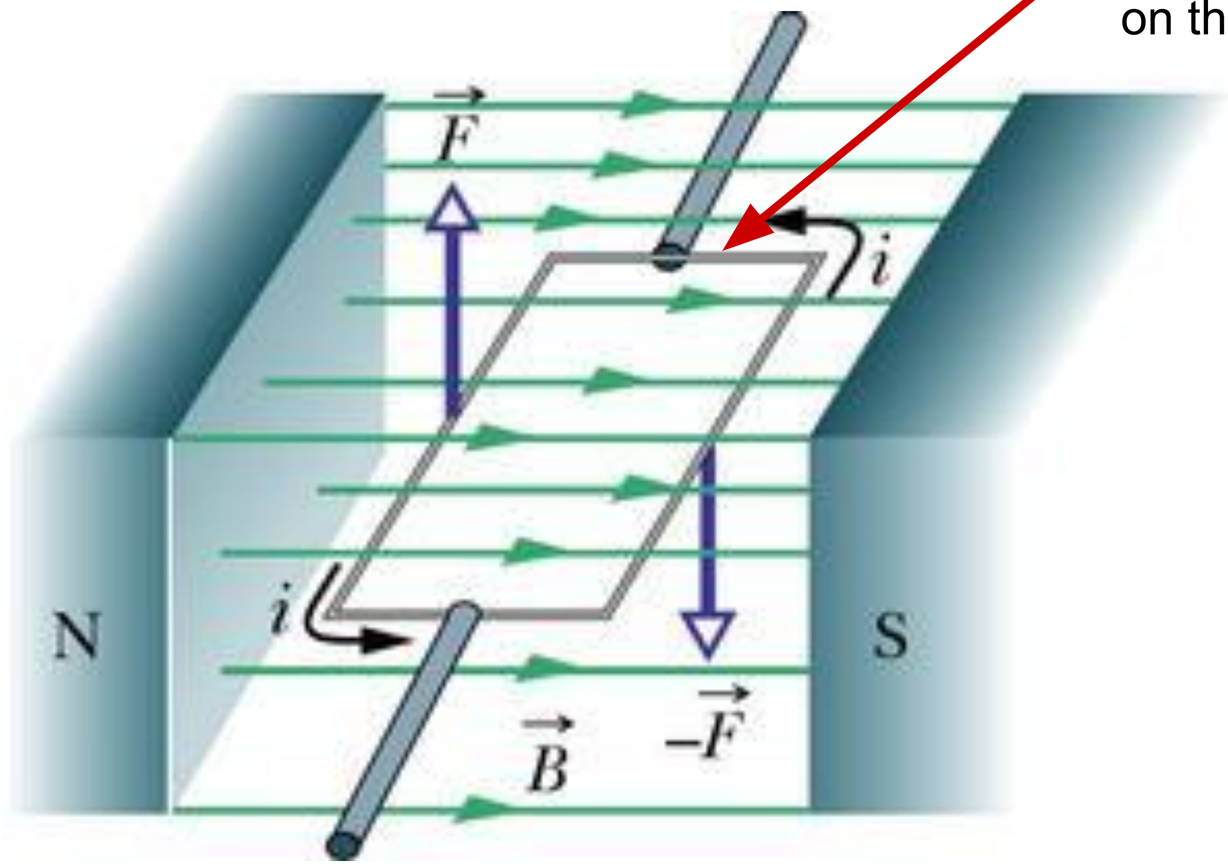


Don’t Buy A Ticket Quite Yet..

- This is still experimental.
- Much development still required.
- Some of these attempts have been abandoned because of the high cost of building a MagLev train.
- Probably 10-20 years out.
- Or More.

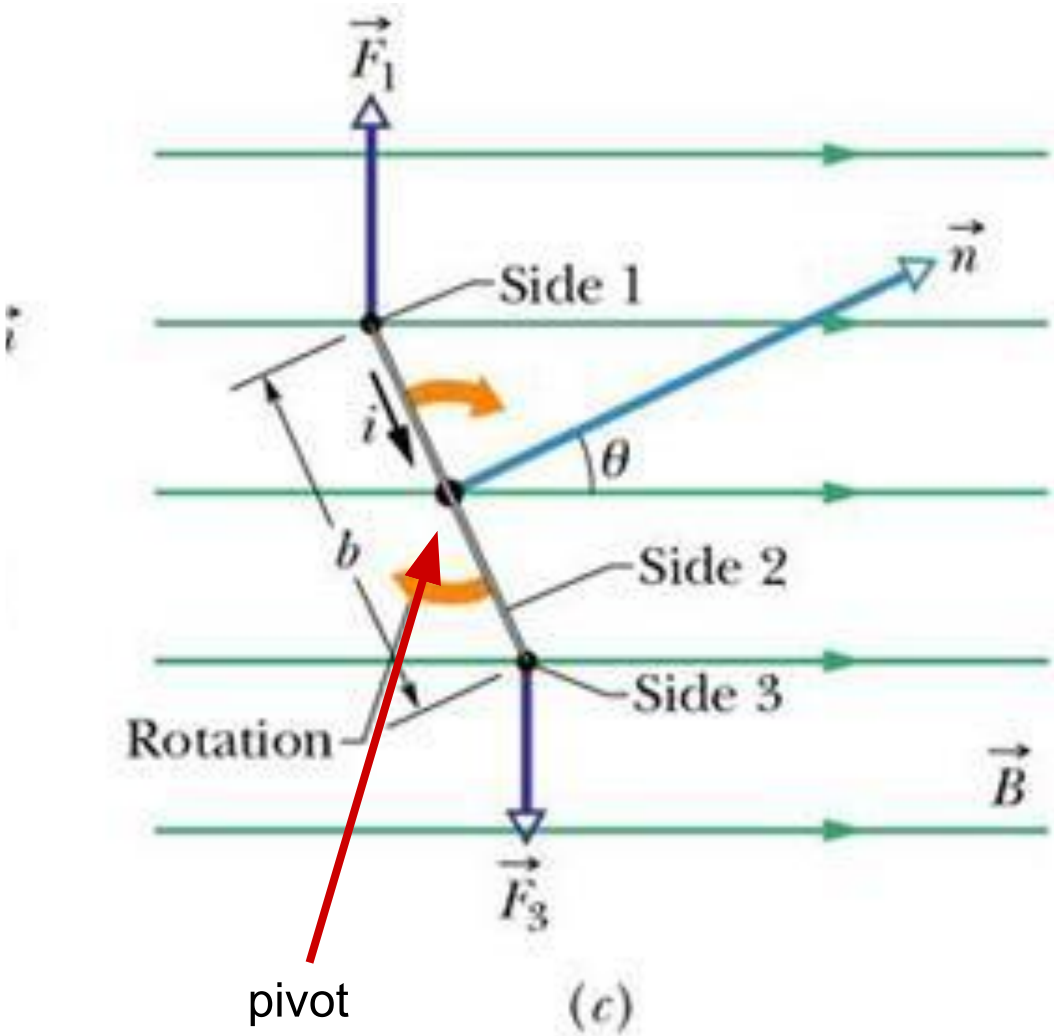
Current Loop

What is force
on the ends??



Loop will tend to rotate due to the <u>torque</u> the field applies to the loop.

The Loop

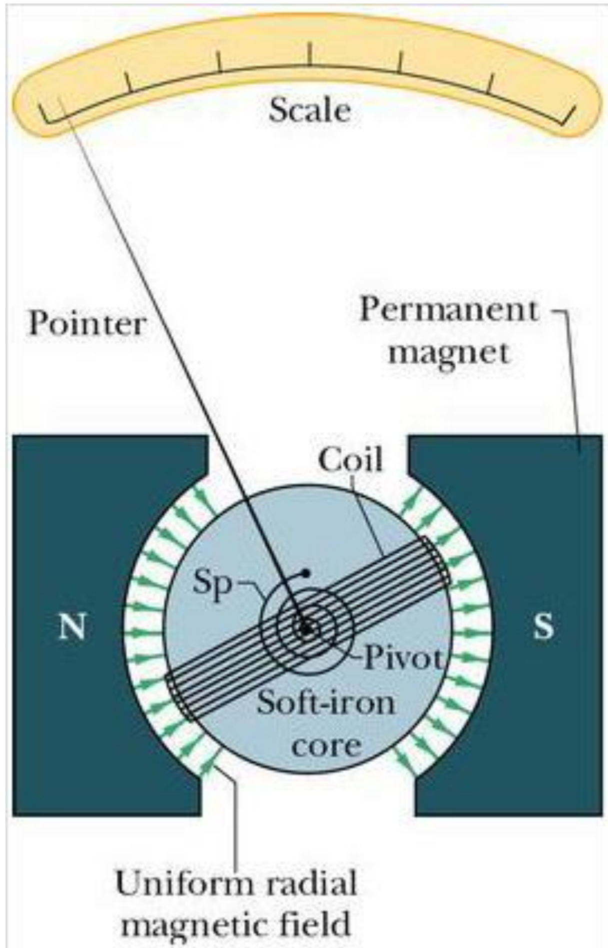


OBSERVATION

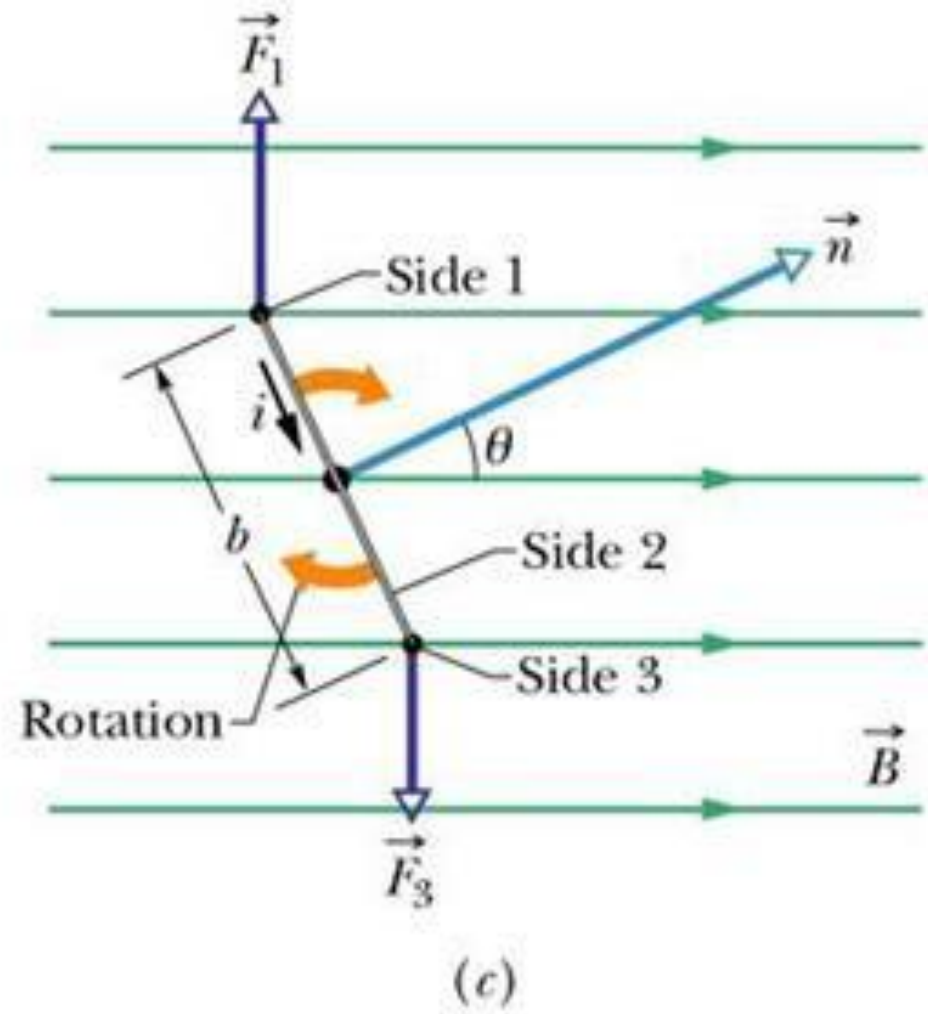
Force on Side 2 is out of the paper and that on the opposite side is into the paper. No net force tending to rotate the loop due to either of these forces.

The net force on the loop is also zero,

An Application
The Galvanometer



The other sides



$\tau_1 = F_1\,(b/2)\mathrm{Sin}(\theta)$
$= (B\ i\ a) \times (b/2)\mathrm{Sin}(\theta)$

total torque on the loop is: $2\tau_1$

Total torque:

$\tau = (iaB)\ b\mathrm{Sin}(\theta)$
$= iAB\mathrm{Sin}(\theta)$

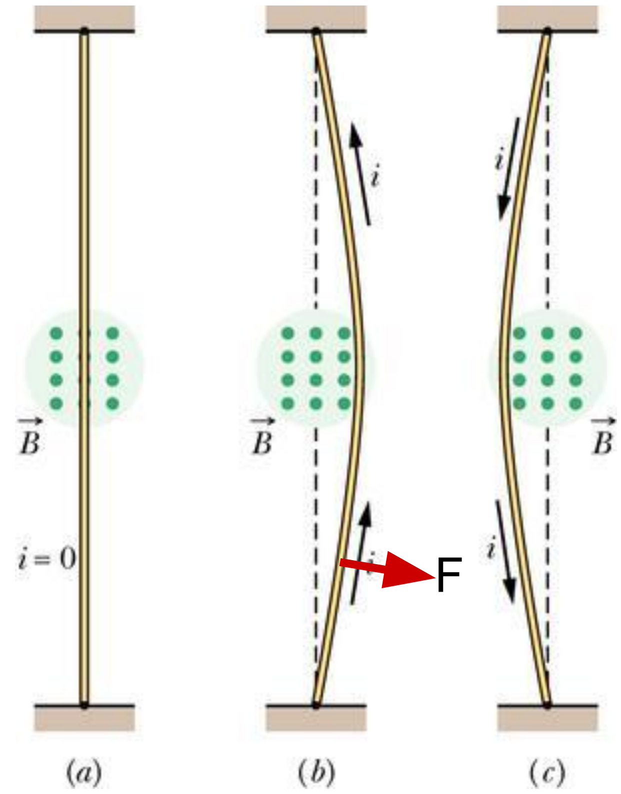
(A=Area)

Watcha Gonna Do

- Quiz Today
- Return to Magnetic Material
- Exams not yet returned. Sorry.

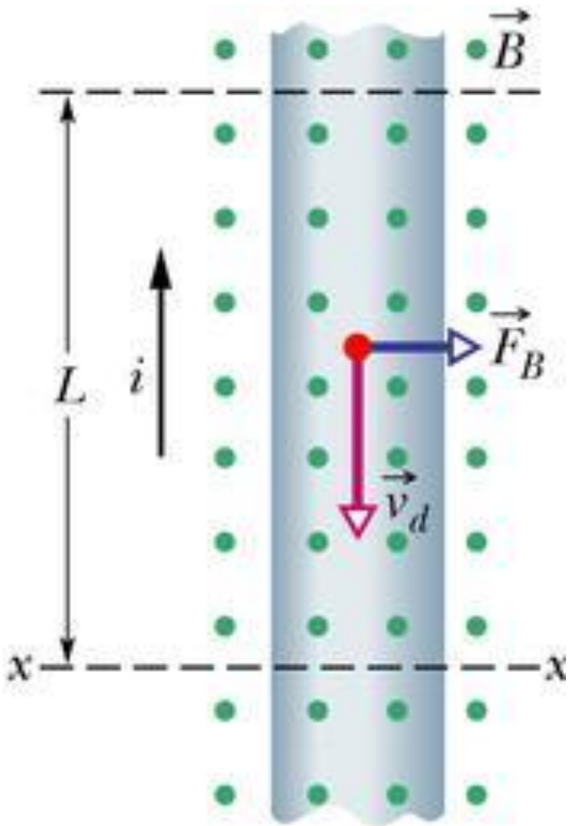
Wires

- A wire with a current contains moving charges.
- A magnetic field will apply a force to those moving charges.
- This results in a force on the wire itself.
  - The electron's sort of PUSH on the side of the wire.



Remember: Electrons go the "other way".

The Wire in More Detail



B out of plane of the paper

Assume all electrons are moving with the same velocity v_d.

$$q = i \times t = i\frac{L}{v_d}$$

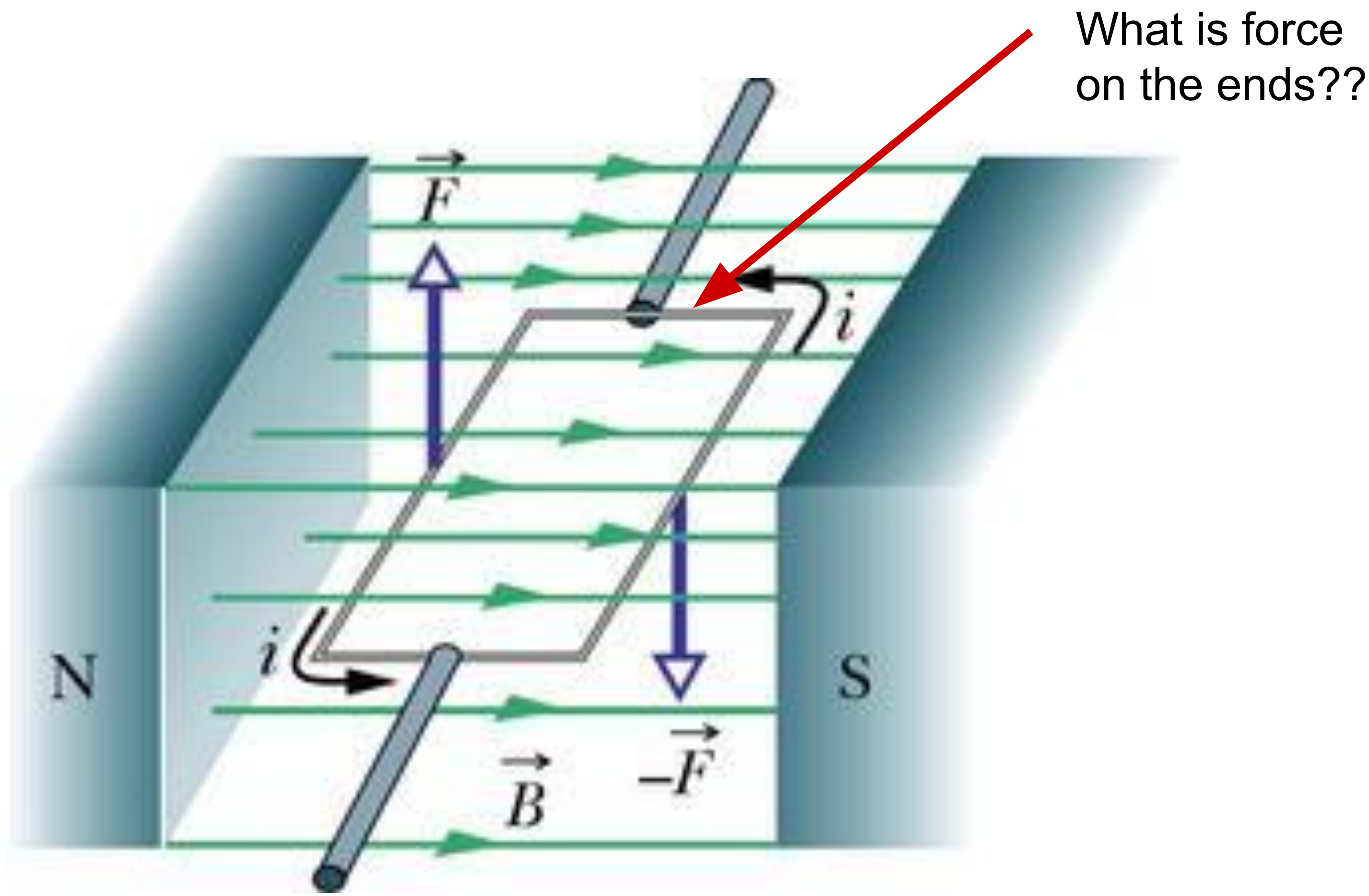
$$F = qv_d B = i\frac{L}{v_d}v_d B = iLB$$

vector :

$$\mathbf{F} = i\mathbf{L} \times \mathbf{B}$$

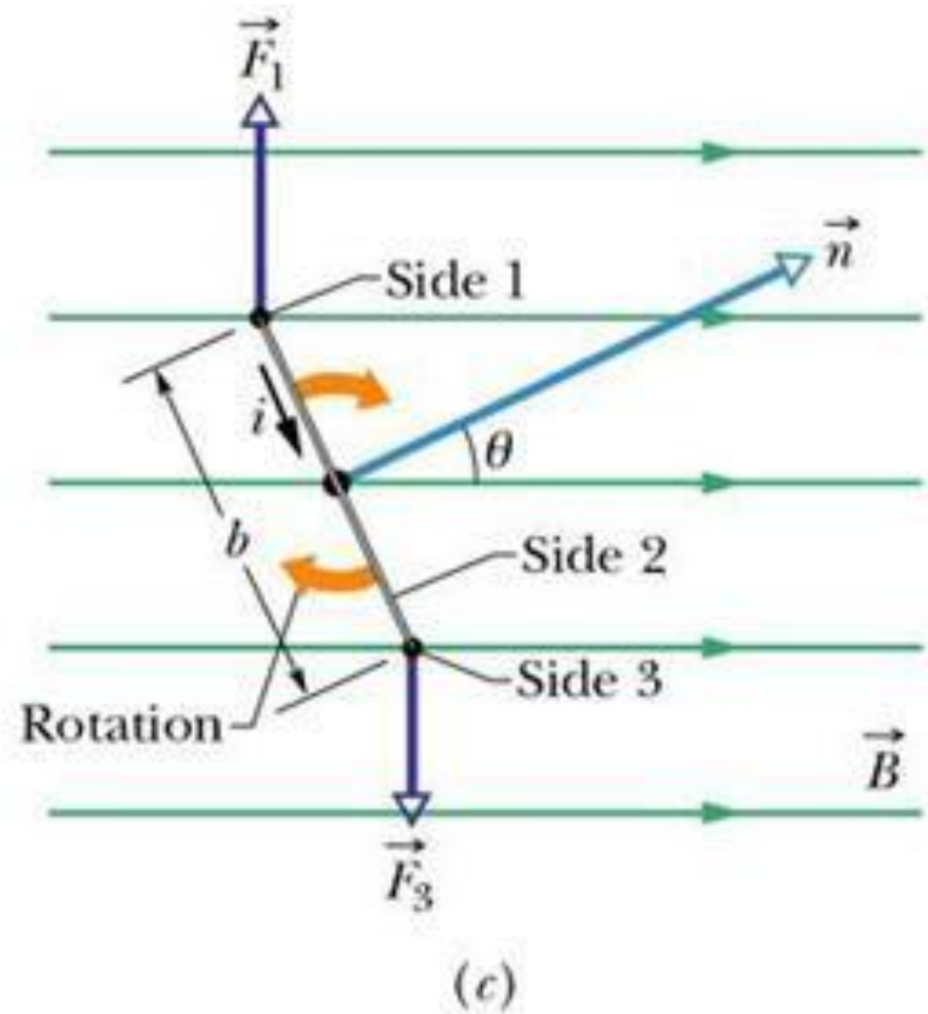
L in the direction of the motion of POSITIVE charge (i).

Current Loop



Loop will tend to rotate due to the <u>torque</u> the field applies to the loop.

Last Time



$\tau_1 = F_1\,(b/2)\mathrm{Sin}(\theta)$
$= (B\ i\ a) \times (b/2)\mathrm{Sin}(\theta)$

total torque on the loop is: $2\tau_1$

Total torque:

$\tau = (iaB)\ b\mathrm{Sin}(\theta)$
$= iAB\mathrm{Sin}(\theta)$

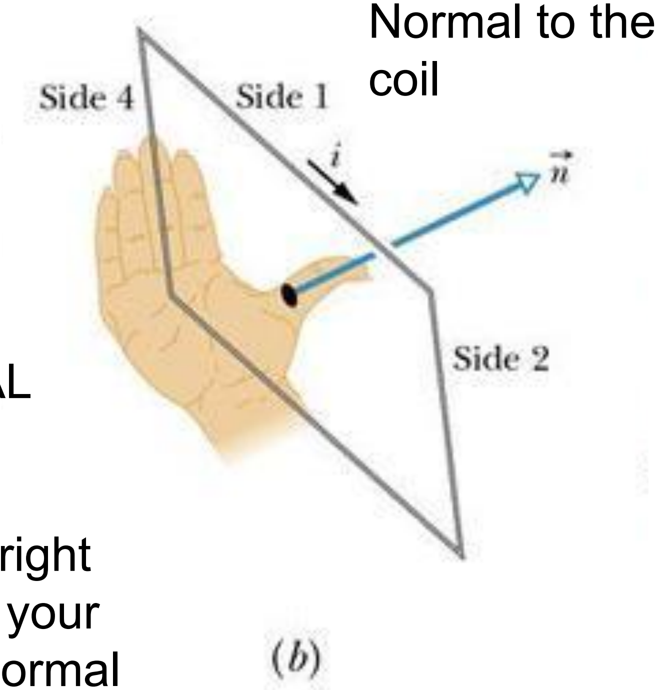
(A=Area)

A Coil

For a COIL of N turns, the net torque on the coil is therefore:

$$\tau = NiABSin(\theta)$$

Normal to the coil

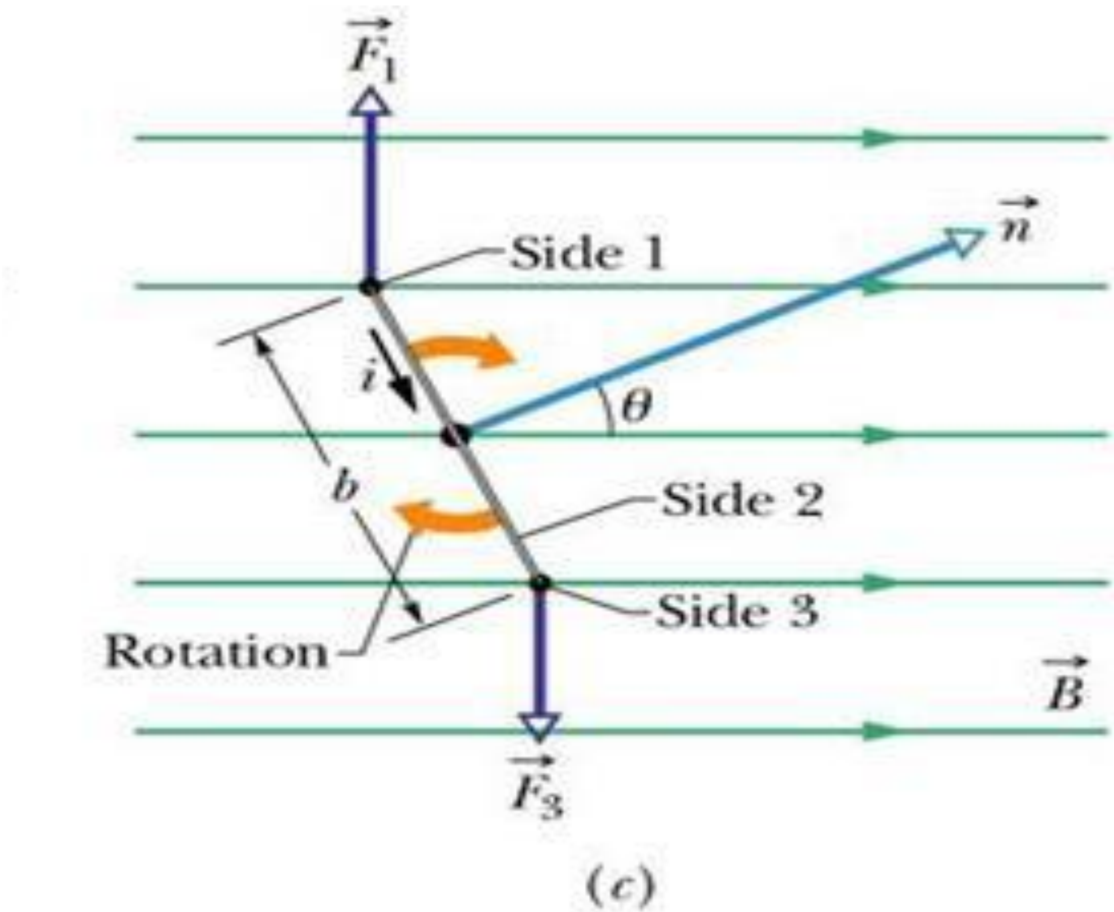


RIGHT HAND RULE TO FIND NORMAL TO THE COIL:

"Point or curl you're the fingers of your right hand in the direction of the current and your thumb will point in the direction of the normal to the coil.

Don't hurt yourself doing this!

Dipole Moment Definition



Define the magnetic dipole moment of the coil μ as:

$$\mu = NiA$$

We can convert this to a vector with A as defined as being normal to the area as in the previous slide.

Current Loop

$$\tau = iAB\sin\theta$$

Consider a coil with N turns of wire.

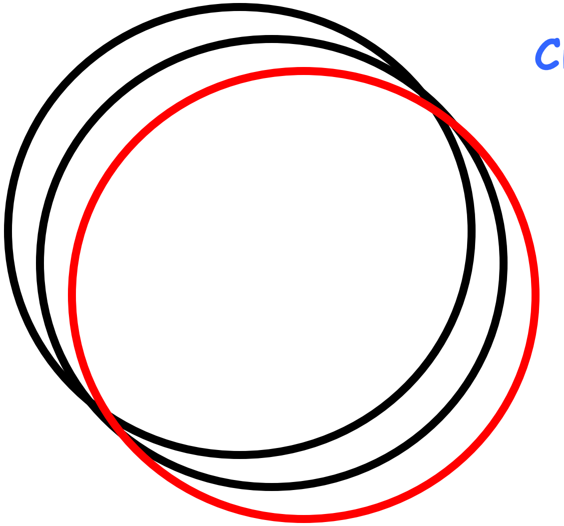
Define Magnetic Moment

$$\boldsymbol{\mu} = Ni\mathbf{A}$$

and

$$\tau = \boldsymbol{\mu} \times \mathbf{B}$$

A length *L* of wire carries a current *i*. Show that if the wire is formed into a circular coil, then the maximum torque in a given magnetic field is developed when the coil has one turn only, and that maximum torque has the magnitude … *well, let's see.*



Circumference = L/N

$$2\pi r = \frac{L}{N}$$

$$r = \frac{L}{2\pi N}$$

Problem continued…

$$\tau = NiAB \text{ since } \sin(\mu, \text{B}) \text{ is maximum}$$

$$\text{when the angle is } 90^{\circ}$$

$$A = \pi r^2$$

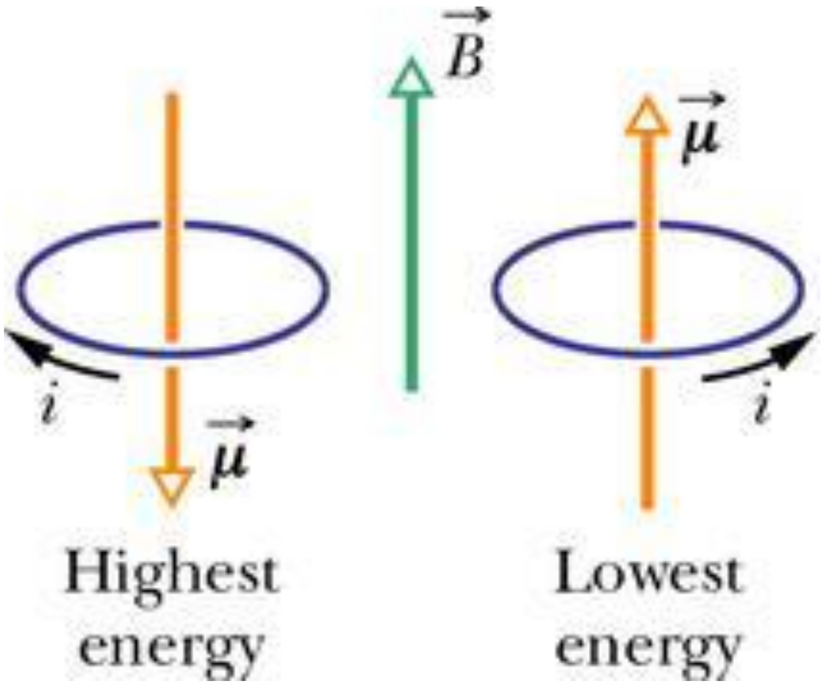
$$\tau = NiB\pi\left(\frac{L}{2\pi N}\right)^2$$

$$\tau = iB\pi\left(\frac{L\sqrt{N}}{2\pi N}\right)^2 = iB\pi\,\frac{L^2}{4\pi^2 N} \quad \text{(BiA)}$$

$$\text{Maximum when N} = 1 \text{ and}$$

$$\tau = \frac{iBL^2}{4\pi}$$

Energy



Like the electric dipole

$U(\theta) = -\mu \bullet \mathbf{B}$

μ and B want to be aligned!

Prove this on your own!

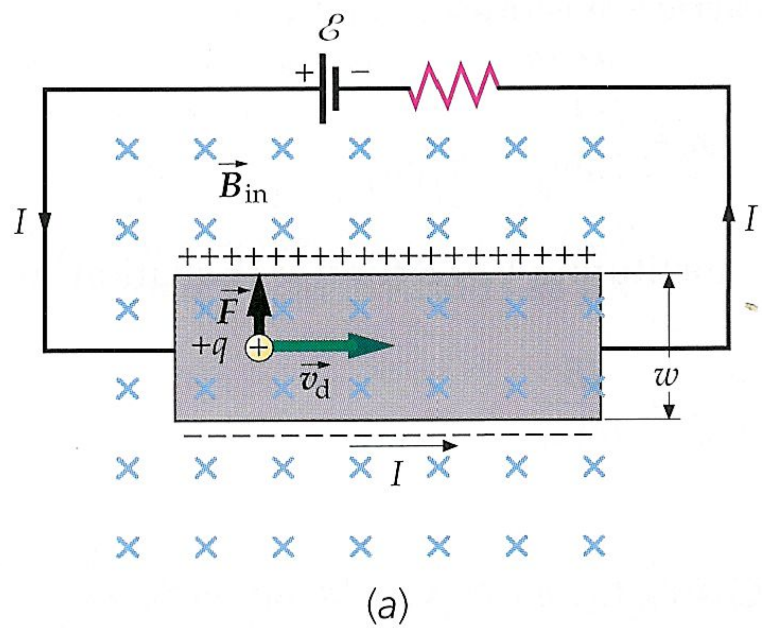
Similar to Electric Dipole Moment

The Hall Effect

What Does it Do?

- **Allows the measurement of Magnetic Field if a material is known.**
- **Allows the determination of the “type” of current carrier in semiconductors if the magnetic field is known.**
  - **Electrons**
  - **Holes**

Hall Geometry (+ Charge)



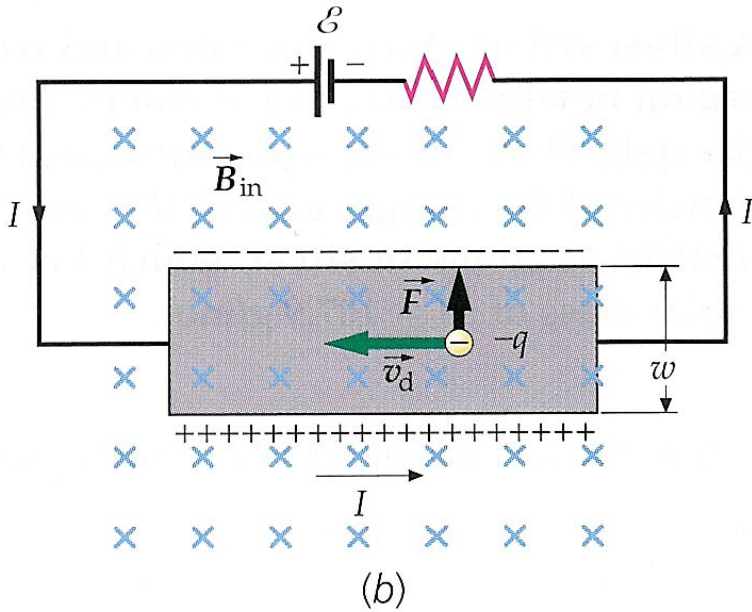
- Current is moving to the right. (v_d)

  Magnetic field will force the charge to the top.

  This leaves a deficit (-) charge on the bottom.

  This creates an electric field and a potential difference.

Negative Carriers



- Carrier is negative.
- Current still to the right.
- Force pushes negative charges to the top.
- Positive charge builds up on the bottom.
- Sign of the potential difference is reversed.

Hall Math

- Eventually, the field due to the Hall effect will allow the current to travel un-deflected through the conductor.

$$balance:$$

$$qv_d B = qE_{Hall} = q\frac{V_{Hall}}{w}$$

$$or$$

$$V_{Hall} = wv_d B$$

$$J = nev_d = i / A$$

$$v_d = \frac{i}{neA}$$

$$V_{Hall} = wv_d B = wB\frac{i}{neA}$$

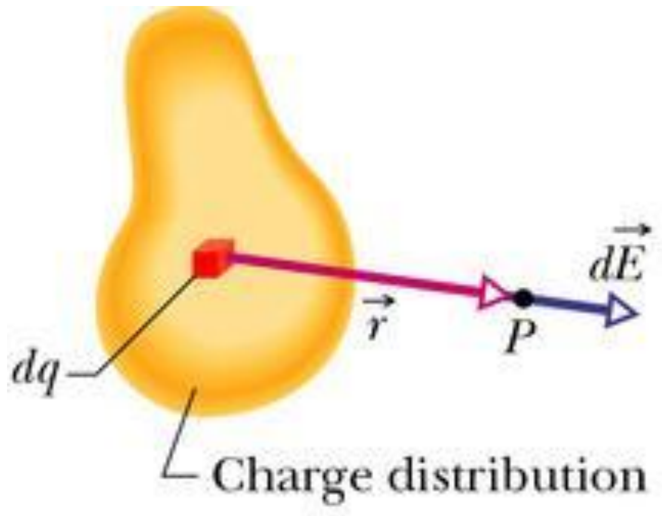
$$A = wt$$

$$V_{Hall} = wB\frac{i}{newt} = \frac{iB}{net}$$

Magnetic Fields Due to Currents

Chapter 30

Try to remember…

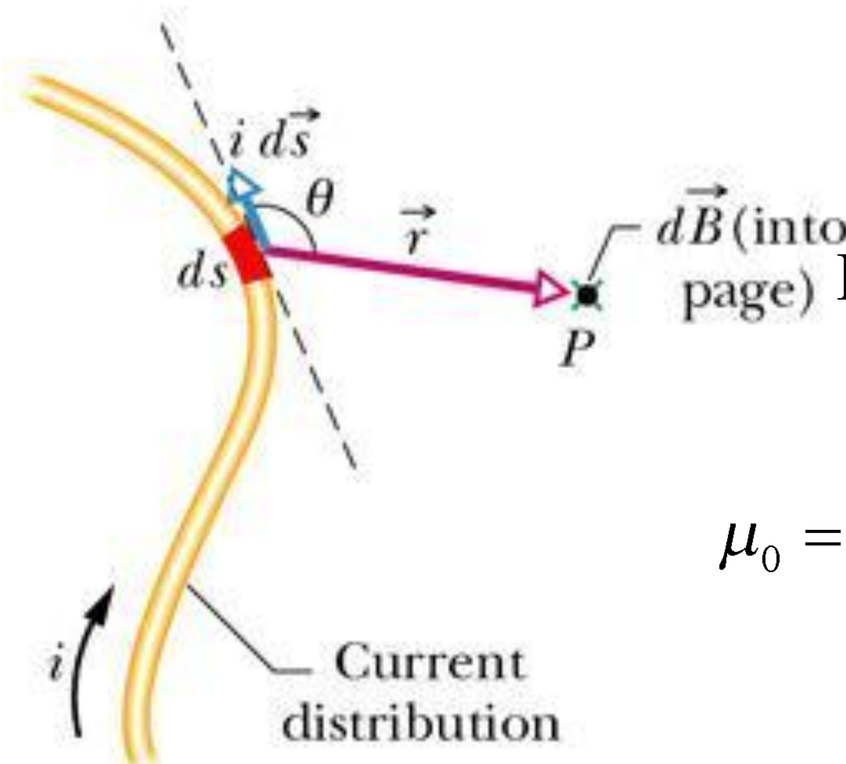


$$d\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\int\left(\frac{\mathbf{r}}{r}\right)\frac{dq}{r^2} = \frac{1}{4\pi\varepsilon_0}\int\frac{\mathbf{r}dq}{r^3}$$

$$\frac{r}{|\mathbf{r}|} = \text{UNIT VECTOR}$$

For the Magnetic Field, current "elements" create the field.

This is the Law of Biot-Savart



In a similar fashion to **E** field :

$$\mathbf{B} = \frac{\mu_0}{4\pi}\int \frac{id\mathbf{s}\times\mathbf{r}_{unit}}{r^2} = \frac{\mu_0}{4\pi}\int \frac{id\mathbf{s}\times\mathbf{r}}{r^3}$$

permeability

$$\mu_0 = 4\pi\times 10^{-7}\,Tm/A = 1.26\times 10^{-7}\,Tm$$

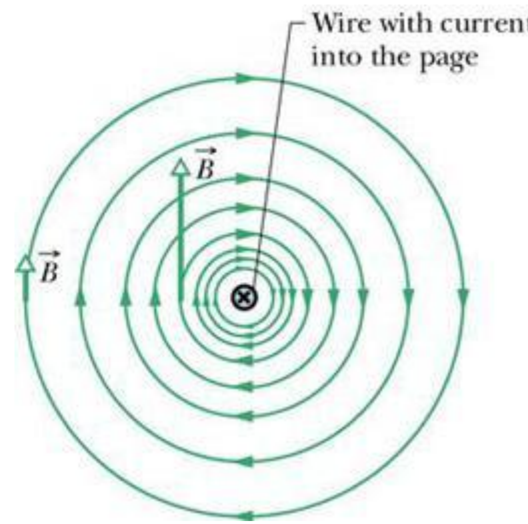
BY DEFINITION

This, defines B!

Magnetic Field of a Straight Wire

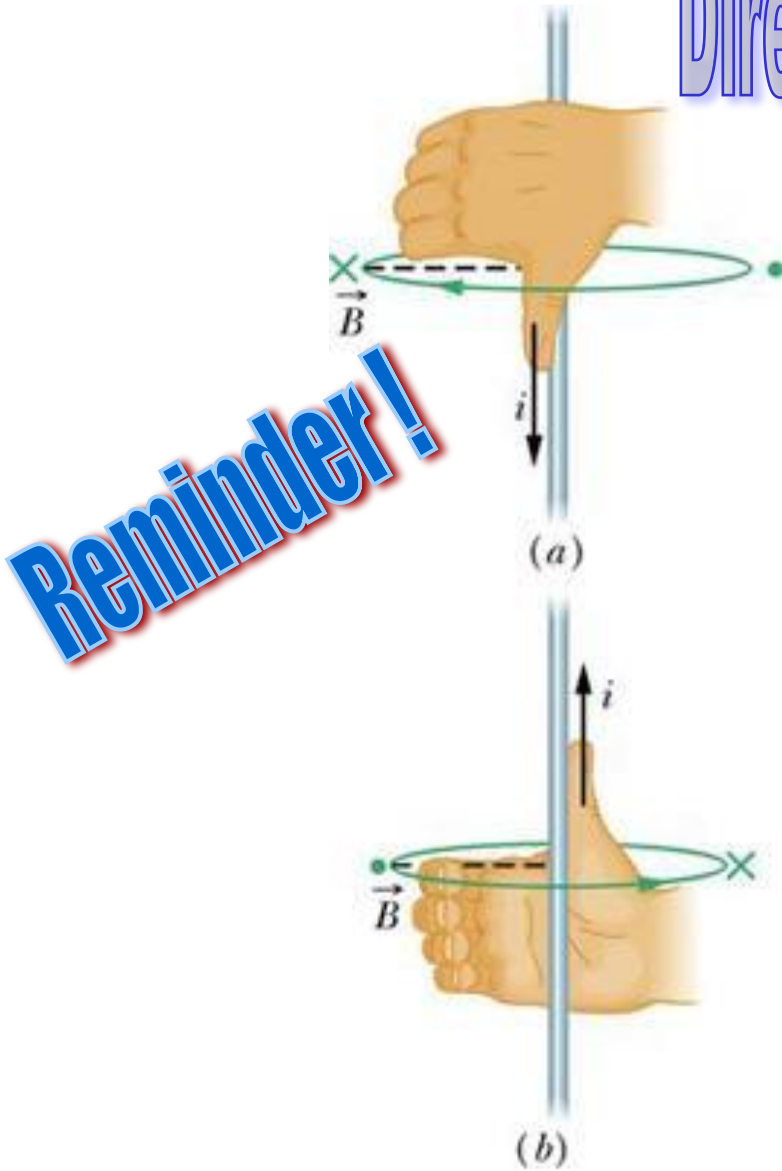
- We intimated via magnets that the Magnetic field associated with a straight wire seemed to vary with 1/d.
- We can now PROVE this!

From the Past



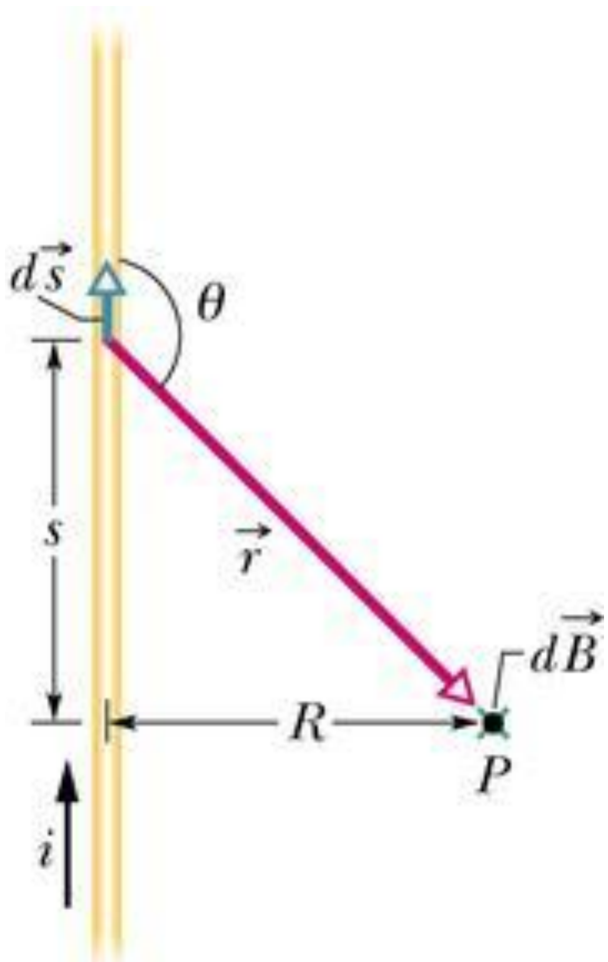
Using Magnets

Directions: The Right Hand Rule



Right-hand rule: Grasp the element in your right hand with your extended thumb pointing in the direction of the current. Your fingers will then naturally curl around in the direction of the magnetic field lines due to that element.

Let's Calculate the FIELD



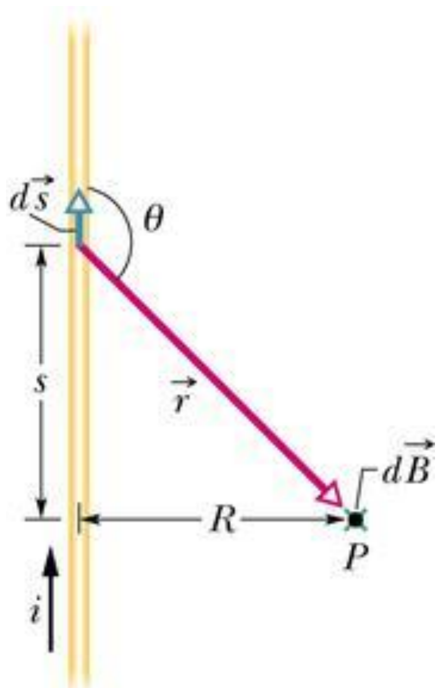
Note:

For ALL current elements

d**s** X **r**

is into the page

The Details

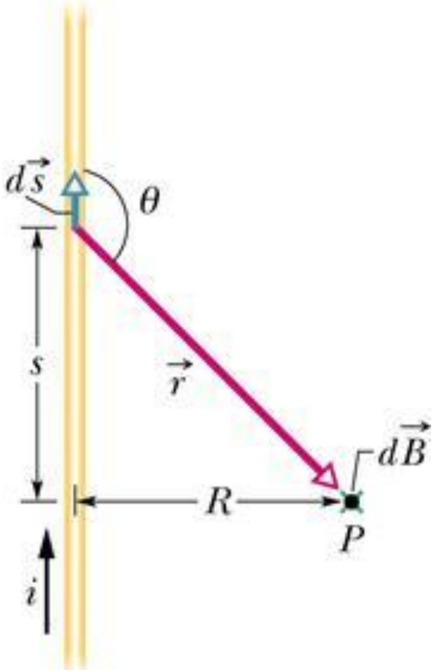


$$dB = \frac{\mu_0}{4\pi}\frac{ids\sin(\theta)}{r^2}$$

Negative portion of the wire contributes an equal amount so we integrate from 0 to ∞ and DOUBLE it.

$$\mathrm{B} = \frac{\mu_0 i}{2\pi}\int_0^\infty \frac{\sin(\theta)ds}{r^2}$$

Moving right along



$$r = \sqrt{s^2 + R^2}$$

$$\sin\theta = \sin(\pi - \theta) = \frac{R}{\sqrt{s^2 + R^2}}$$

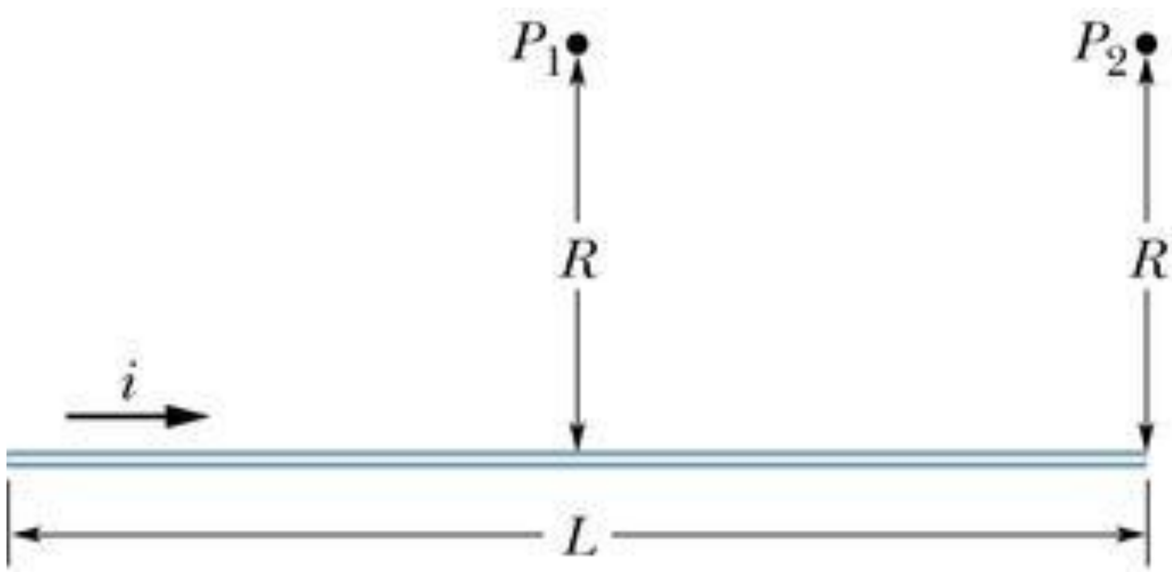
So

$$B = \frac{\mu_0 i}{2\pi}\int_0^{\infty}\frac{r\,ds}{\left(s^2 + R^2\right)^{3/2}} = \frac{\mu_0 i}{2\pi R}$$

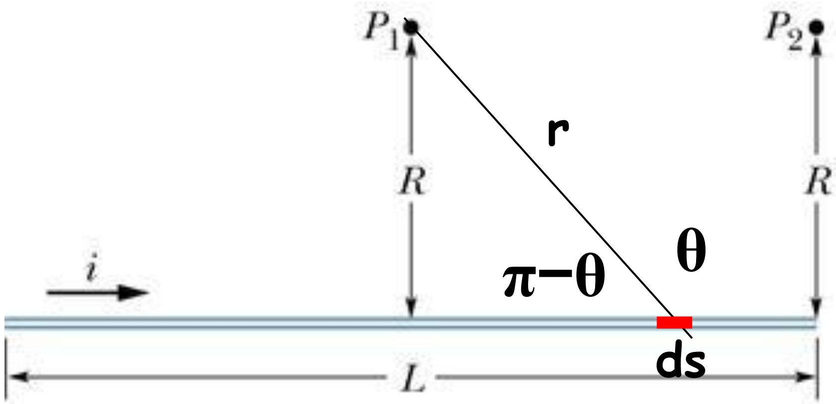
1/d

Verify this.

A bit more complicated
A **finite** wire



P_1



$$NOTE : \sin(\theta) = \sin(\pi - \theta)$$

$$ds \times r = ds\, r \sin(\theta)$$

$$\sin(\theta) = \frac{R}{r}$$

$$dB = \frac{\mu_0 i}{4\pi} \frac{ds \sin(\theta)}{r^2}$$

$$r = \left(s^2 + R^2\right)^{1/2}$$

More P_1

$$B = \frac{\mu_0 i}{4\pi} \int_{-L/2}^{+L/2} \frac{ds}{\left(s^2 + R^2\right)^{3/2}}$$

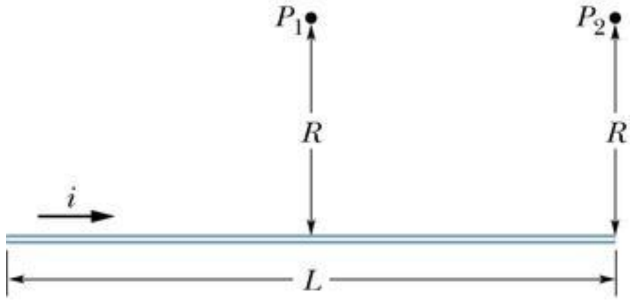
and

$$B = \frac{\mu_0 i}{2\pi R} \frac{L}{\sqrt{L^2 + 4R^2}}$$

when $\mathrm{L} \Rightarrow \infty$,

$$\mathrm{B} \Rightarrow \frac{\mu_0 i}{2\pi R}$$

P2

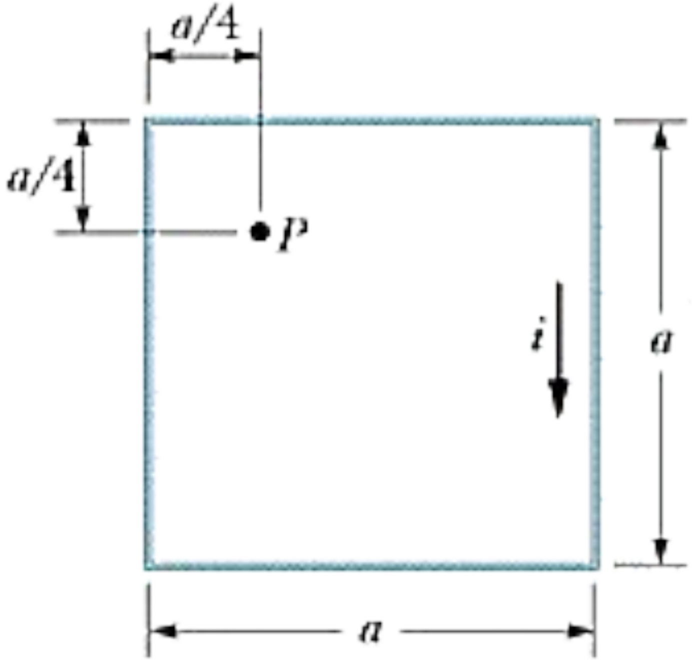


$$B = \frac{\mu_0 i R}{4\pi} \int_{-L}^{0} \frac{ds}{\left(s^2 + R^2\right)^{3/2}}$$

or

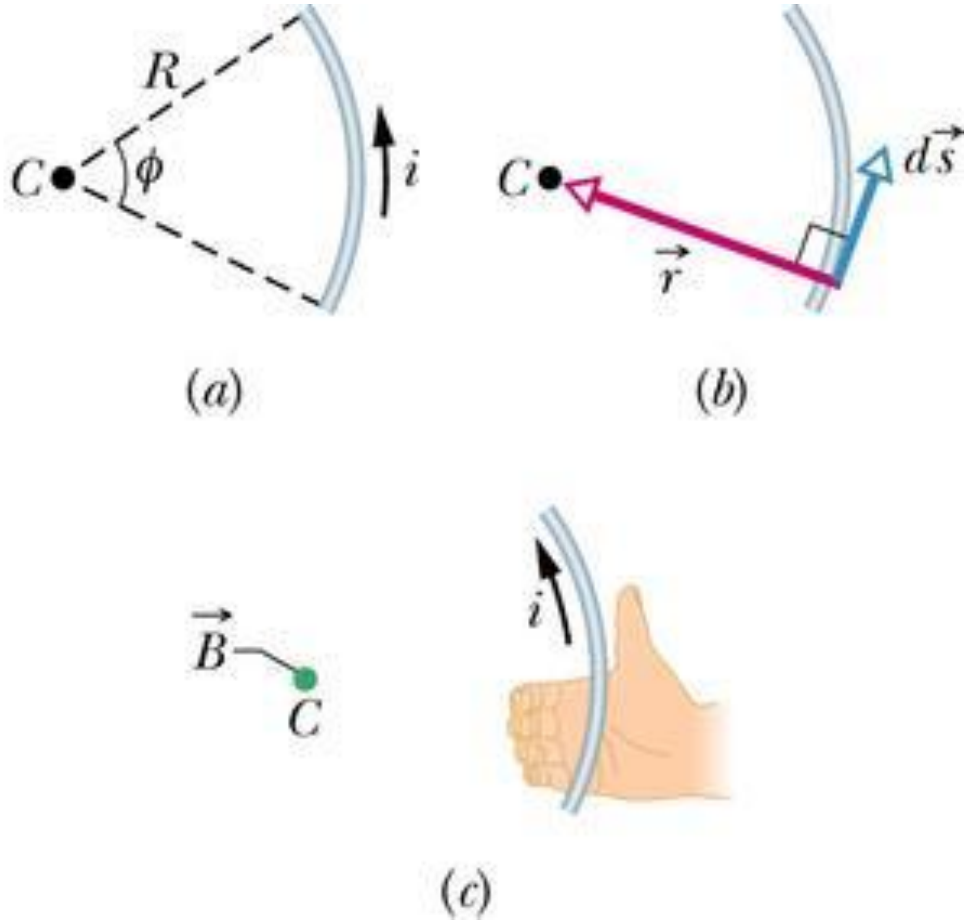
$$B = \frac{\mu_0 i}{4\pi R} \frac{L}{\sqrt{s^2 + R^2}}$$

APPLICATION:
Find the magnetic field B at point *P* in for *i* = 10 *A* and *a* = 8.0 cm.

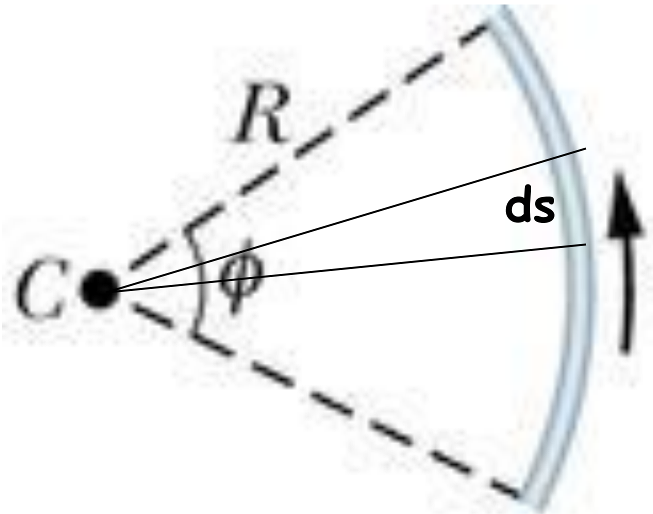


A Combination of P2 geometries.

Circular Arc of Wire



More arc…



$$ds = Rd\phi$$

$$dB = \frac{\mu_0}{4\pi}\frac{ids}{R^2} = \frac{\mu_0}{4\pi}\frac{iRd\phi}{R^2}$$

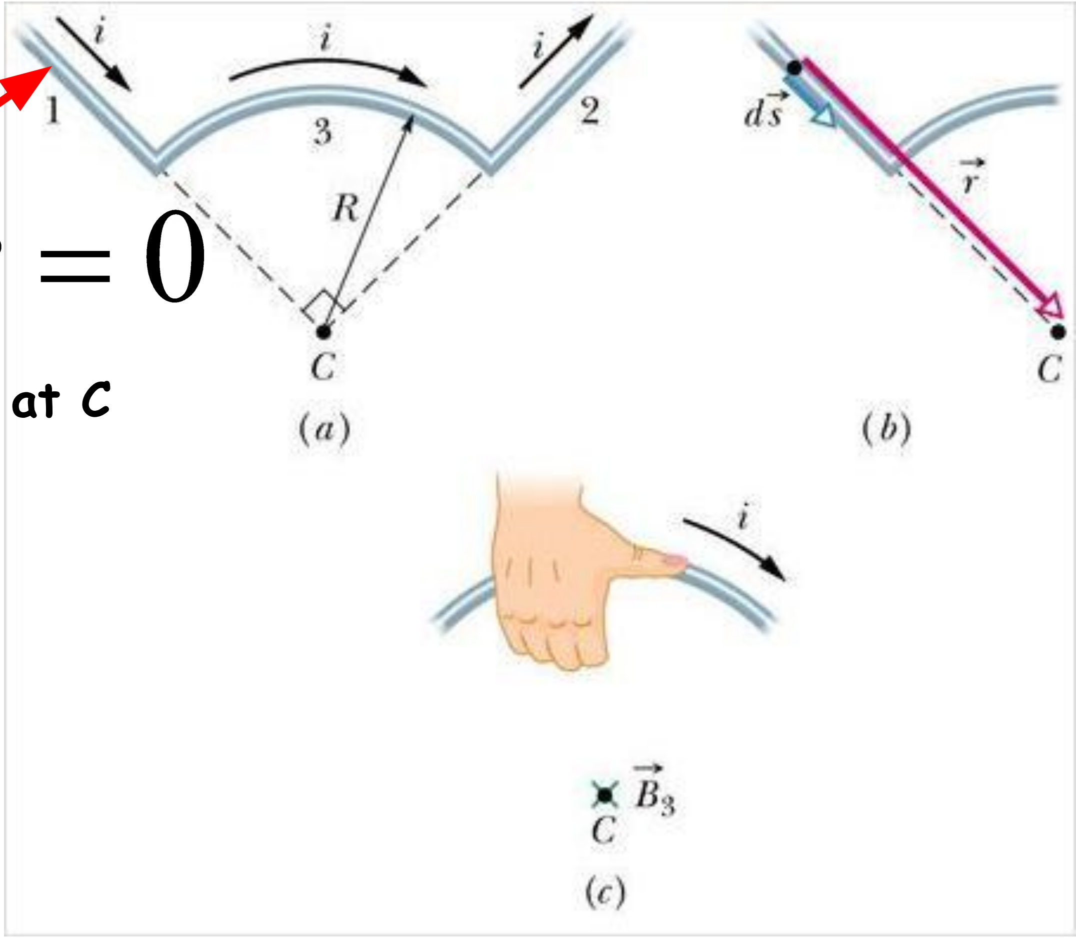
$$B = \int dB = \int_0^{\phi} \frac{\mu_0}{4\pi}\frac{iRd\phi}{R^2} = \frac{\mu_0 i}{4\pi R}\int_0^{\phi} d\phi$$

$$B = \frac{\mu_0 i\phi}{4\pi R} \text{ at point C}$$

Howya Do Dat??

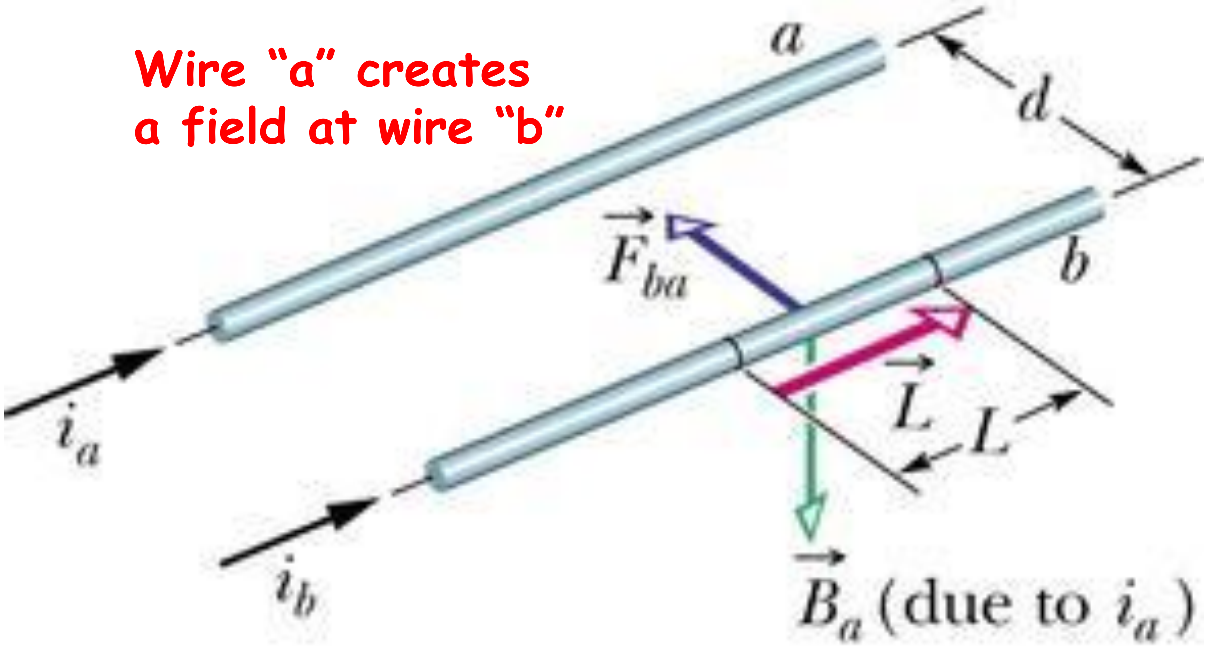
$$d\mathbf{s} \times \mathbf{r} = 0$$

No Field at C



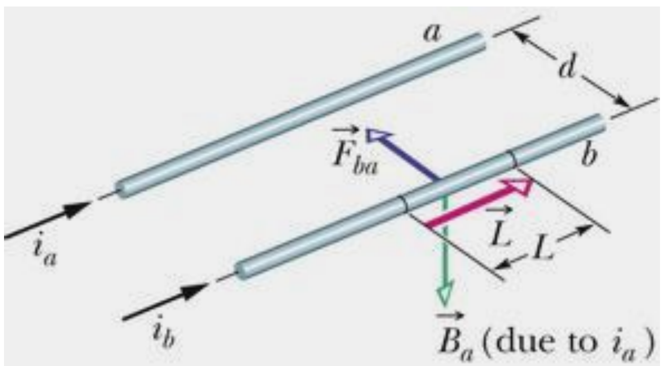
Force Between Two Current Carrying Straight Parallel Conductors

Wire "a" creates a field at wire "b"



Current in wire "b" sees a force because it is moving in the magnetic field of "a".

The Calculation



The FIELD at wire "b" due to wire "a" is what we just calculated :

$$B_{\text{at "b"}} = \frac{\mu_0 i_a}{2\pi d}$$

$$F_{\text{on "b"}} = i_b \mathbf{L} \times \mathbf{B}$$

Since **L** and **B** are at right angles...

$$\mathrm{F} = \frac{\mu_0 L i_a i_b}{2\pi d}$$

Definition of the Ampere

The force acting between currents in parallel wires is the basis for the definition of the ampere, which is one of the seven SI base units. The definition, adopted in 1946, is this: The ampere is that constant current which, if maintained in two straight, parallel conductors of infinite length, of negligible circular cross section, and placed 1 m apart in vacuum, would produce on each of these conductors a force of magnitude 2 x 10-7 newton per meter of length.

TRANSITION

AMPERE

Welcome to Andre’ Marie Ampere’s Law

Normally written as a “circulation” vector equation.

We will look at another form, but first…

Remember GAUSS'S LAW??

$$\oint \mathbf{E} \bullet d\mathbf{A} = \frac{q_{enclosed}}{\varepsilon_0}$$

Surface
Integral

Gauss's Law

- Made calculations easier than integration over a charge distribution.
- Applied to situations of HIGH SYMMETRY.
- Gaussian SURFACE had to be defined which was consistent with the geometry.

- AMPERE'S Law is the Gauss' Law of Magnetism! (Sorry)

The next few slides have been lifted from **Seb Oliver** on the internet

Whoever he is!

Biot-Savart

- The “Coulombs Law of Magnetism”

$$d\mathbf{B} = \left[\frac{\mu_0}{4\pi} \right] \frac{i\mathbf{ds} \times \hat{\mathbf{r}}}{\left| \mathbf{r} \right|^2}$$

Invisible Summary

- Biot-Savart Law

$$d\mathbf{B} = \left[\frac{\mu_0}{4\pi}\right]\frac{i\mathbf{ds}\times\hat{\mathbf{r}}}{|\mathbf{r}|^2}$$

  - (Field produced by wires)
  - Centre of a wire loop radius *R*

$$B = \frac{\mu_0 I}{2R}$$

  - Centre of a tight Wire Coil with *N* turns

$$B = \frac{\mu_0 NI}{2R}$$

  - Distance *a* from long straight wire

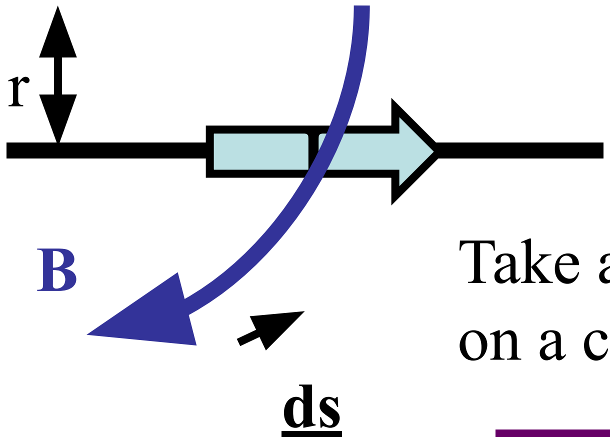
$$B = \frac{\mu_0 I}{2\pi a}$$

- Force between two wires

$$\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi a}$$

- Definition of Ampere

Magnetic Field from a long wire



Using Biot-Savart Law

$$\left|B\right| = \frac{\mu_0 I}{2\pi r}$$

Take a short vector on a circle, **ds**

$$\mathbf{B} \bullet \mathbf{ds} = \left|\mathbf{B}\right| \bullet \left|\mathbf{ds}\right| \cos\theta$$

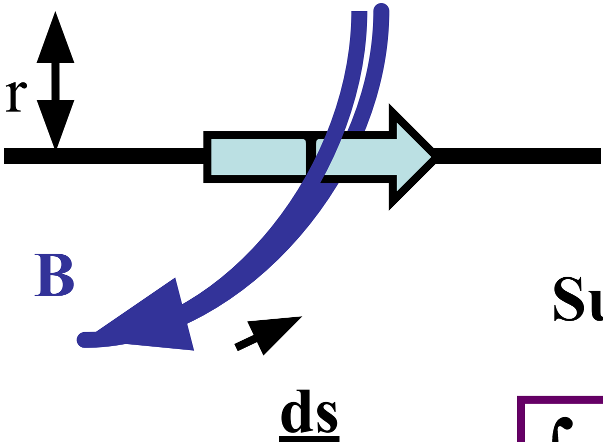
$$\theta = 0 \Rightarrow \cos\theta = 1$$

$$\mathbf{B} \bullet \mathbf{ds} = \left|\mathbf{B}\right|\left|\mathbf{ds}\right|$$

Thus the dot product of **B** & the short vector **ds** is:

$$\mathbf{B} \bullet \mathbf{ds} = \frac{\mu_0 I}{2\pi r} ds$$

Sum B·ds around a circular path



$$\mathbf{B}\bullet\mathbf{ds} = \frac{\mu_0 I}{2\pi r} ds$$

Sum this around the whole ring

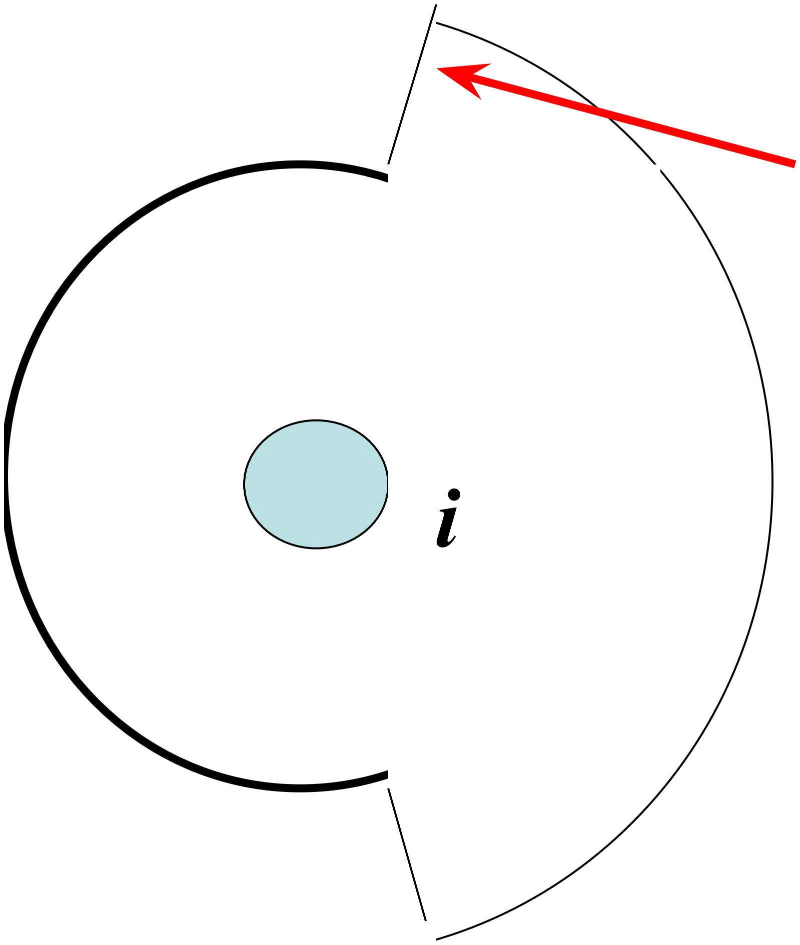
$$\int \mathbf{B}\bullet\mathbf{ds}$$ $$= \int \frac{\mu_0 I}{2\pi r} ds$$ $$= \int \frac{\mu_0 I}{2\pi r} ds$$

Circumference of circle

$$\int ds = 2\pi r$$

$$\Rightarrow \int \mathbf{B}\bullet\mathbf{ds} = \frac{\mu_0 I}{2\pi r} 2\pi r = \mu_0 I$$

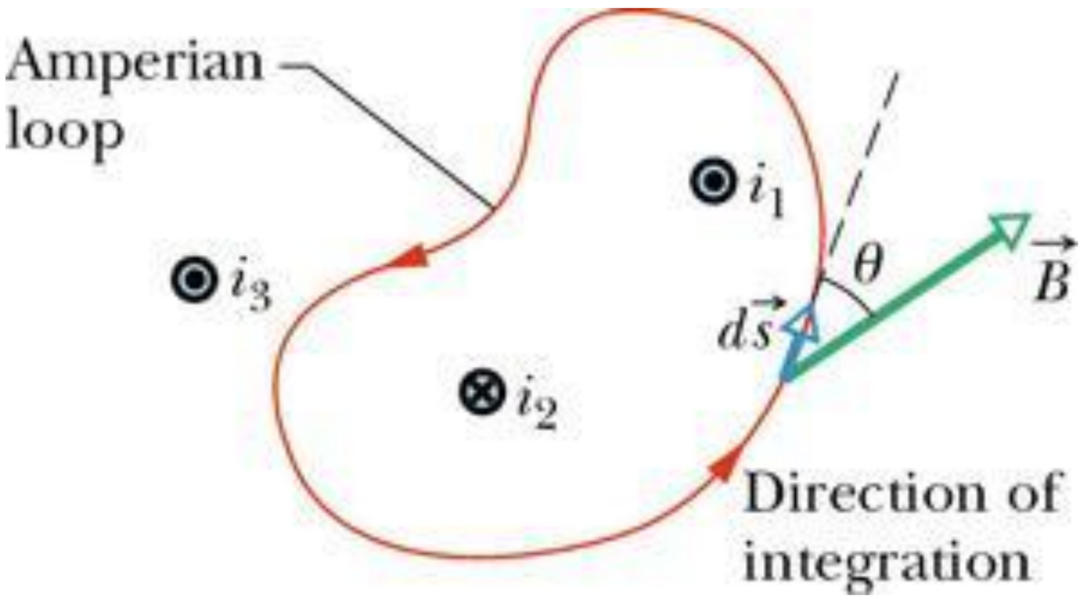
Consider a different path



$$\mathbf{B} \bullet \mathbf{ds} = 0$$

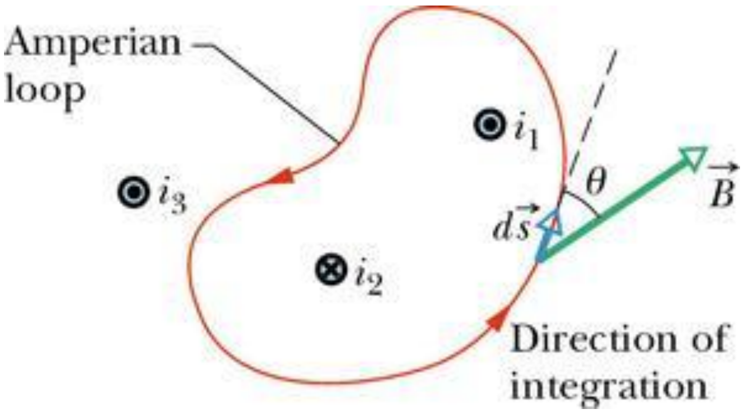
- Field goes as 1/r
- Path goes as r.
- Integral independent of r

SO, AMPERE'S LAW by SUPERPOSITION:



We will do a LINE INTEGRATION Around a closed path or LOOP.

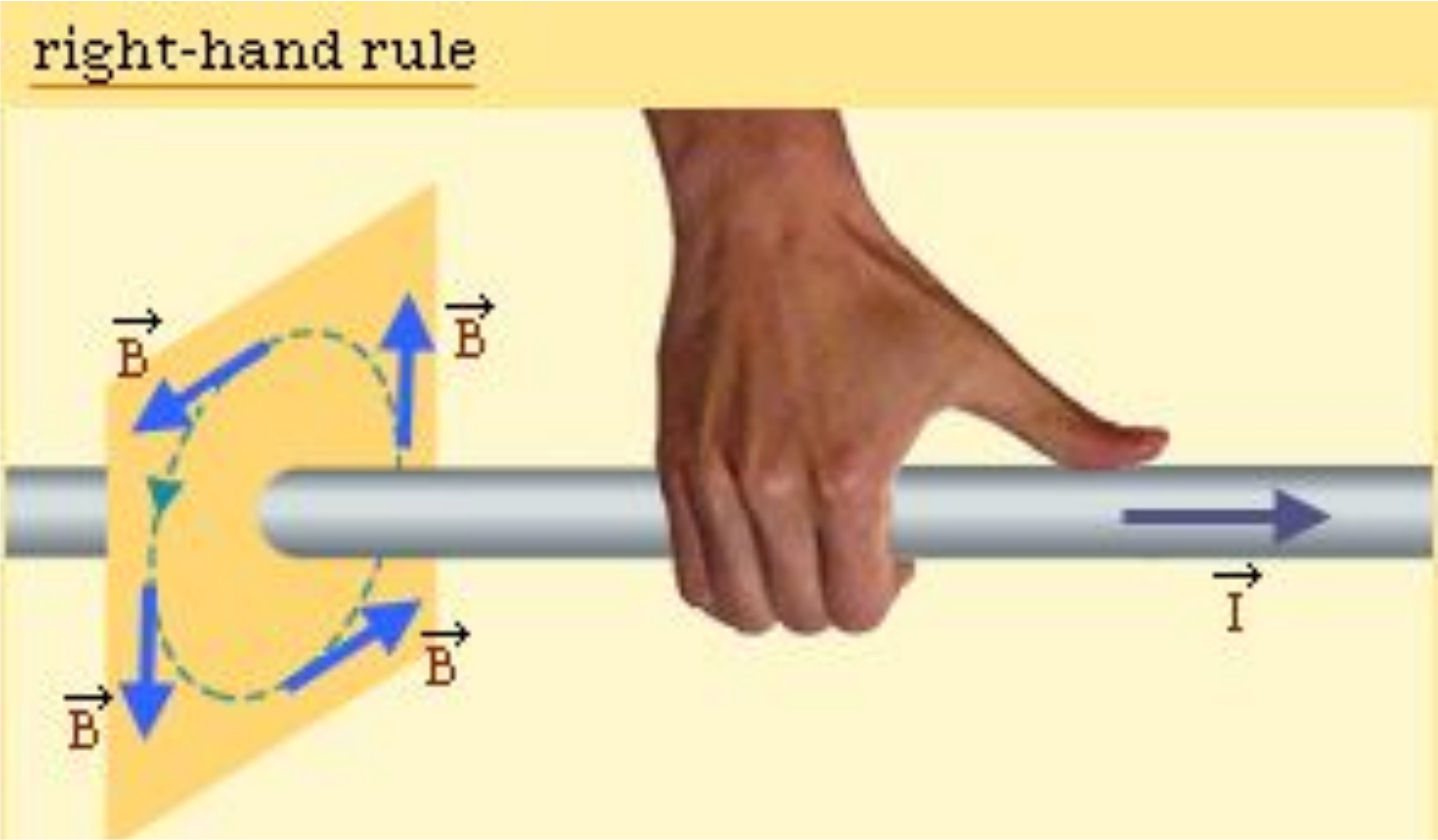
Ampere's Law



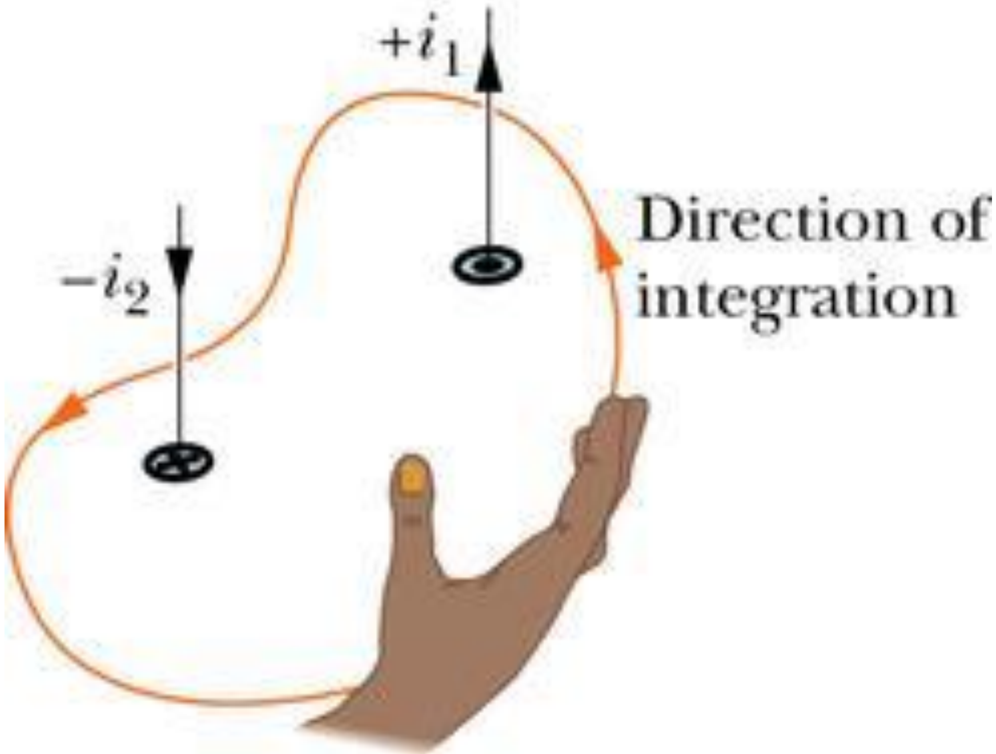
$$\oint \mathbf{B} \bullet d\mathbf{s} = \mu_0 i_{enclosed}$$

USE THE <u>RIGHT HAND</u> RULE IN THESE CALCULATIONS

The Right Hand Rule



Another Right Hand Rule



COMPARE

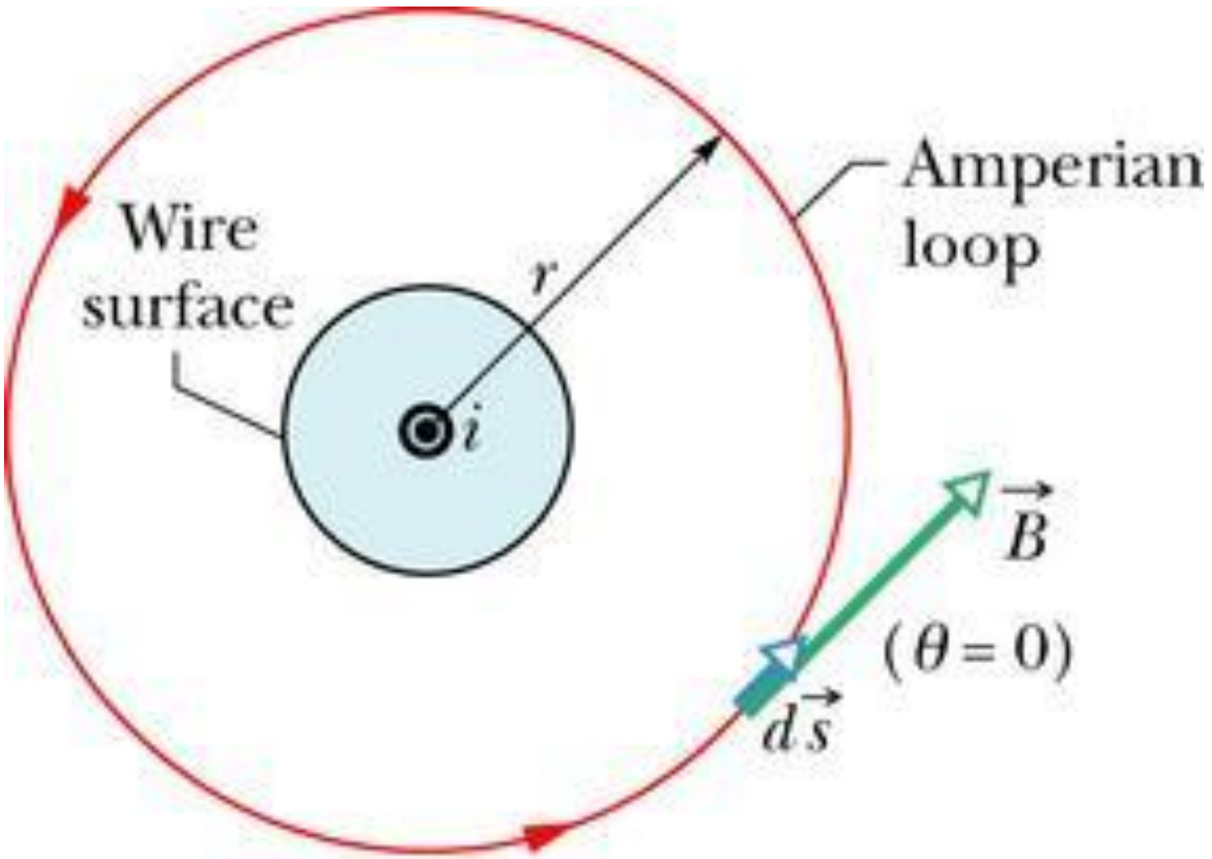
$$\oint \mathbf{B} \bullet d\mathbf{s} = \mu_0 i_{enclosed}$$

Line Integral

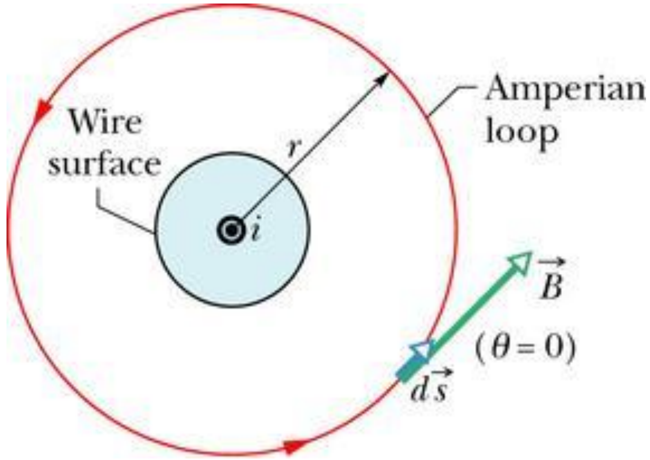
$$\oint \mathbf{E} \bullet d\mathbf{A} = \frac{q_{enclosed}}{\varepsilon_0}$$

Surface Integral

Simple Example



Field Around a Long Straight Wire

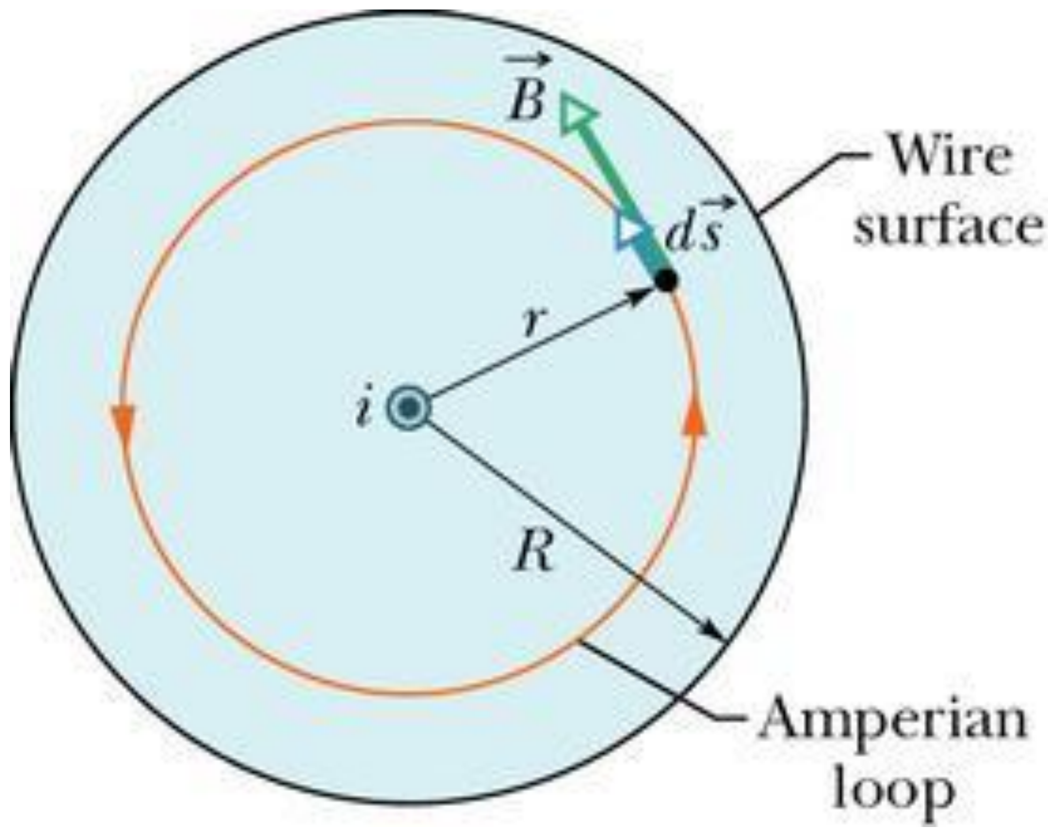


$$\oint \mathbf{B} \bullet d\mathbf{s} = \mu_0 i_{enclosed}$$

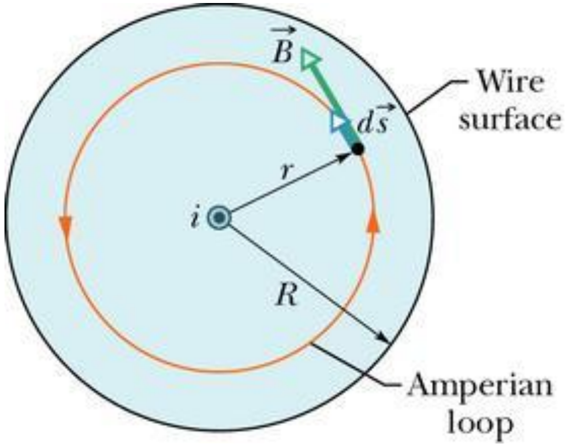
$$B \times 2\pi r = \mu_0 i$$

$$B = \frac{\mu_0 i}{2\pi r}$$

Field INSIDE a Wire Carrying UNIFORM Current



The Calculation



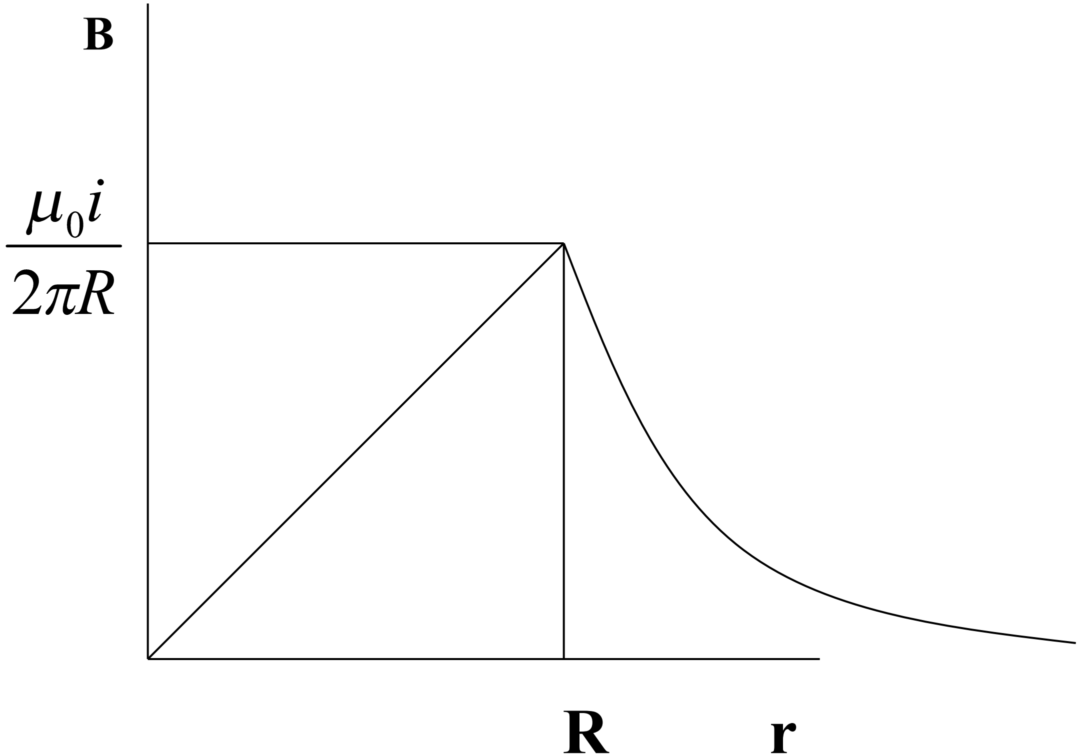
$$\oint \mathbf{B} \bullet d\mathbf{s} = B\oint ds = 2\pi r B = \mu_0 i_{enclosed}$$

$$i_{enclosed} = i\frac{\pi r^2}{\pi R^2}$$

and

$$B = \left(\frac{\mu_0 i}{2\pi R^2}\right) r$$

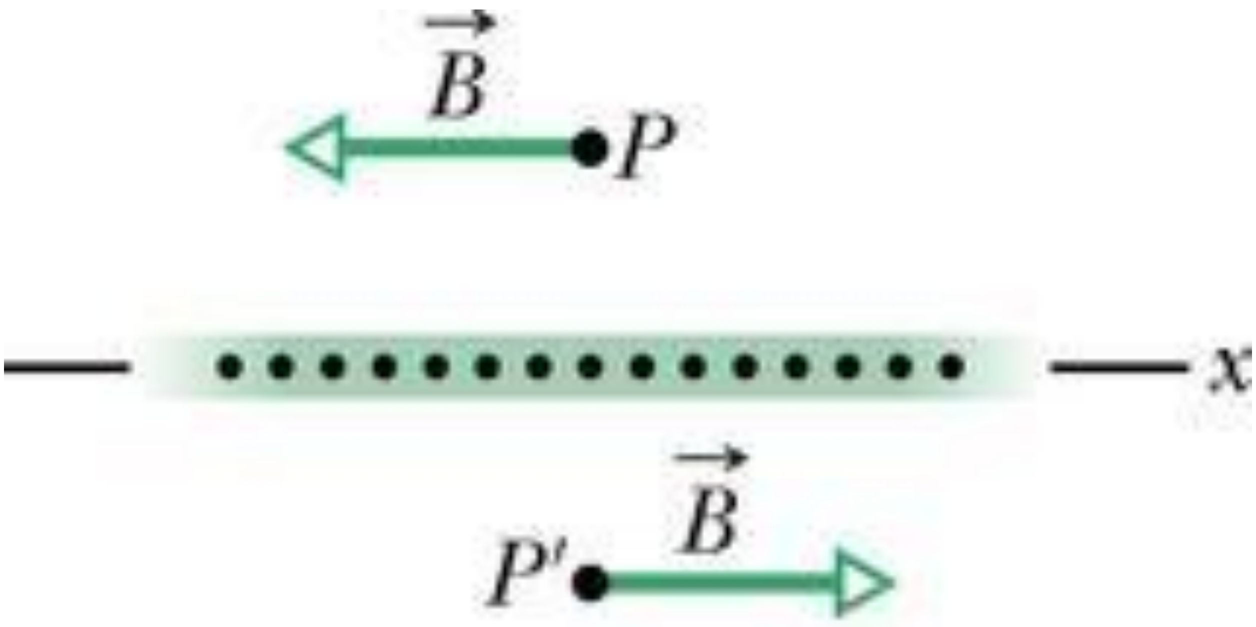
Graph ????


B
$\frac{\mu_0 i}{2\pi R}$
R
r

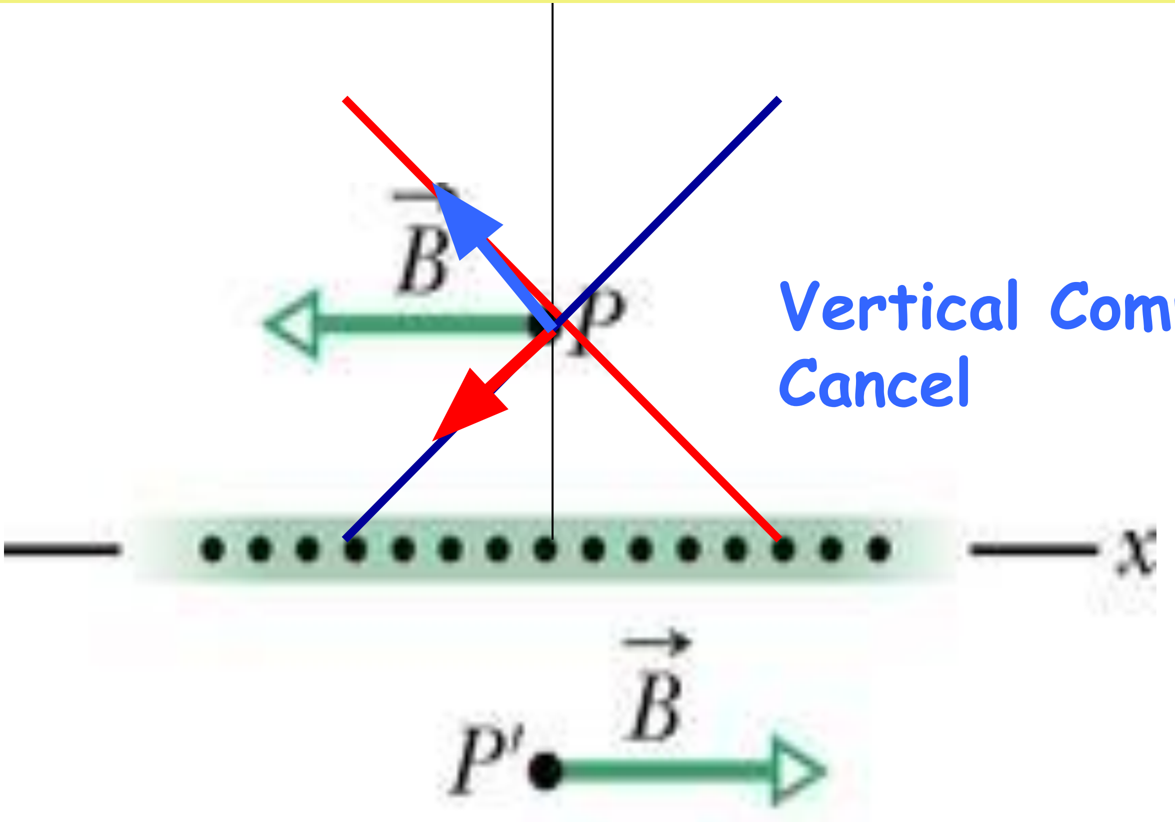
Procedure

- Apply Ampere's law only to highly symmetrical situations.
- Superposition works.
  - Two wires can be treated separately and the results added (VECTORIALLY!)
- The individual parts of the calculation can be handled (usually) without the use of vector calculations because of the symmetry.
- THIS IS SORT OF LIKE GAUSS's LAW **WITH AN ATTITUDE**!

The figure below shows a cross section of an infinite conducting sheet carrying a current per unit *x*-length of *l*; the current emerges perpendicularly out of the page. (a) Use the Biot–Savart law and symmetry to show that for all points *P* above the sheet, and all points *P´* below it, the magnetic field B is parallel to the sheet and directed as shown. (b) Use *Ampere's* law to find B at all points *P* and *P´* .

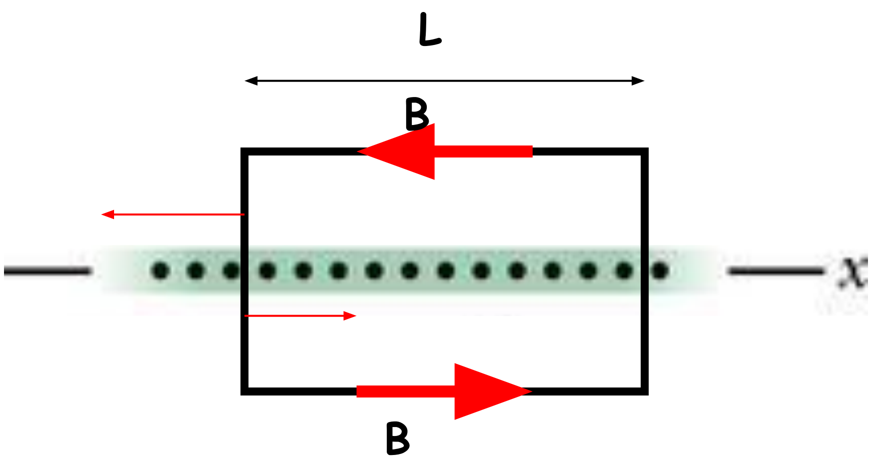


FIRST PART



Vertical Components Cancel

Apply Ampere to Circuit



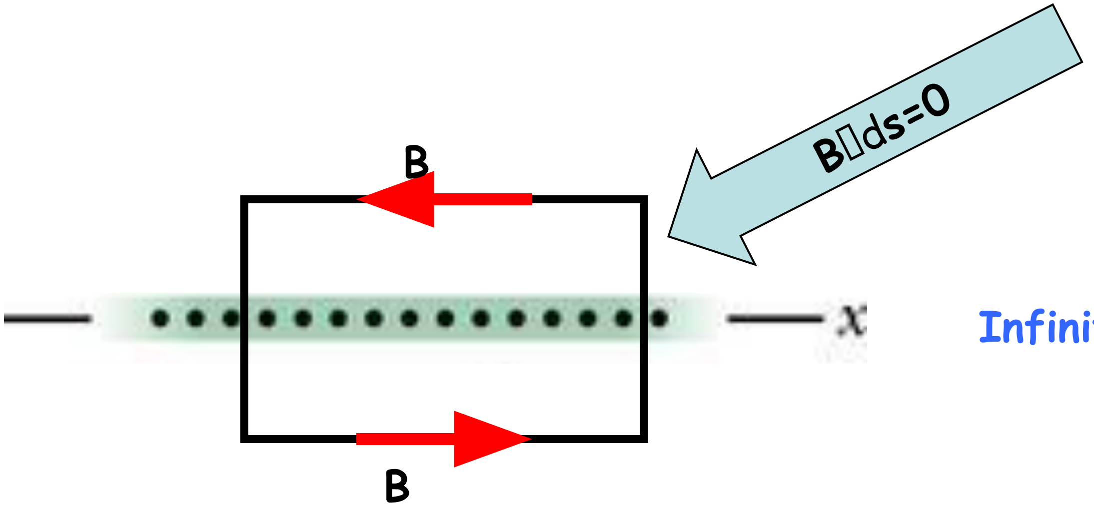
Infinite Extent

$\lambda =$ current per unit length

Current inside the loop is therefore :

$i = \lambda L$

The "Math"



Infinite Extent

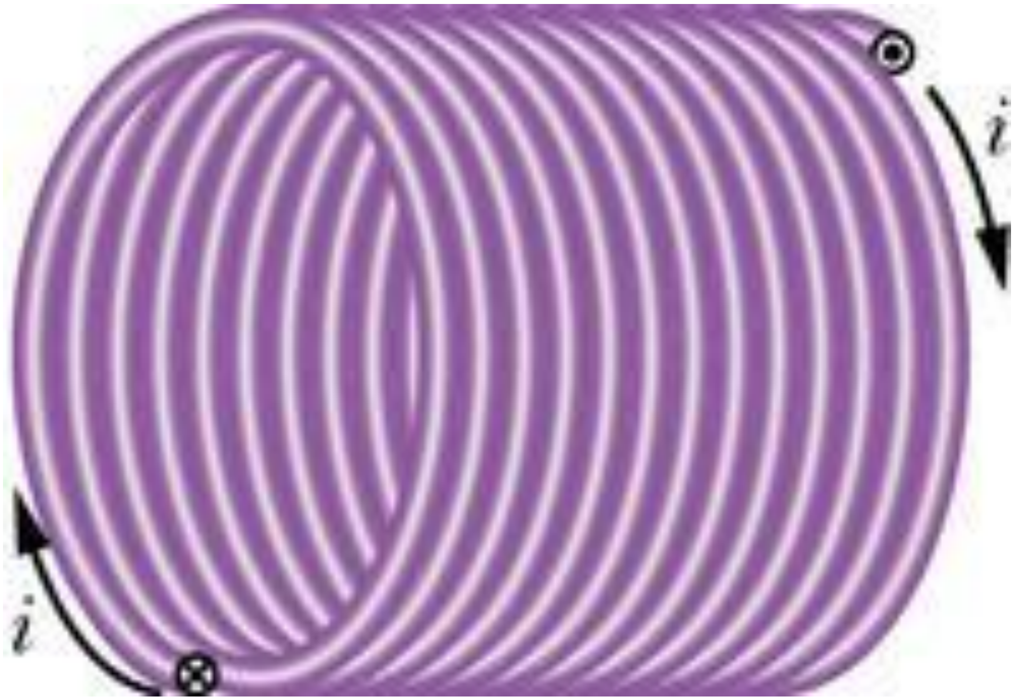
$$\oint \mathbf{B} \bullet d\mathbf{s} = \mu_0 i_{enclosed}$$

$$BL + BL = \mu_0 \lambda L$$

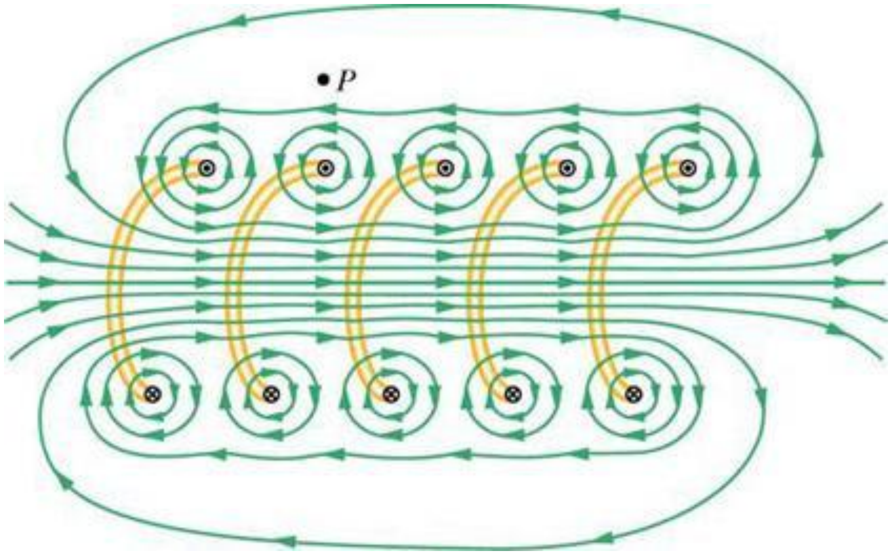
$$B = \frac{\mu_0 \lambda}{2}$$

Distance not a factor!

A Physical Solenoid



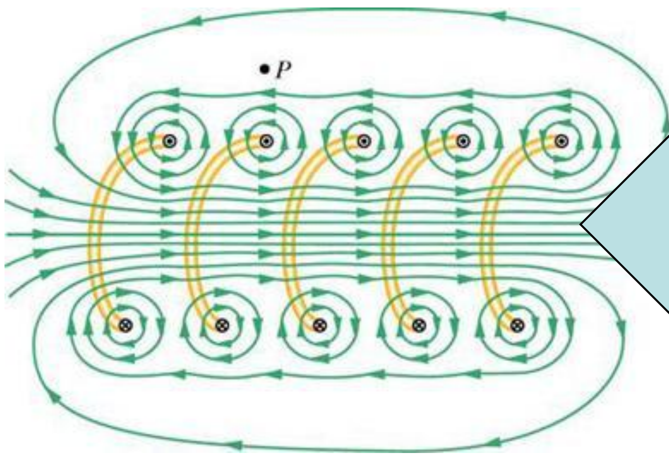
Inside the Solenoid



For an "**INFINITE**" (long) solenoid the previous problem and SUPERPOSITION suggests that the field OUTSIDE this solenoid is ZERO!

More on Long Solenoid

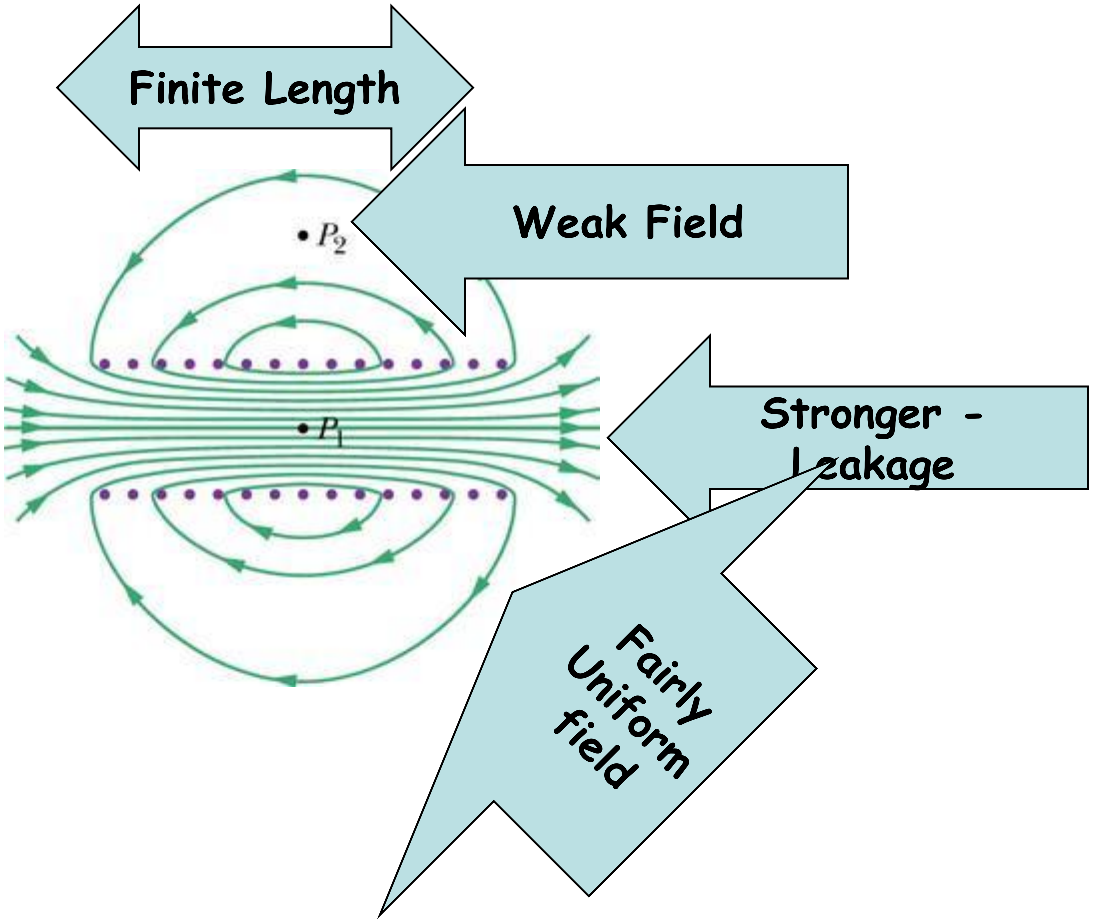
Field is ZERO!



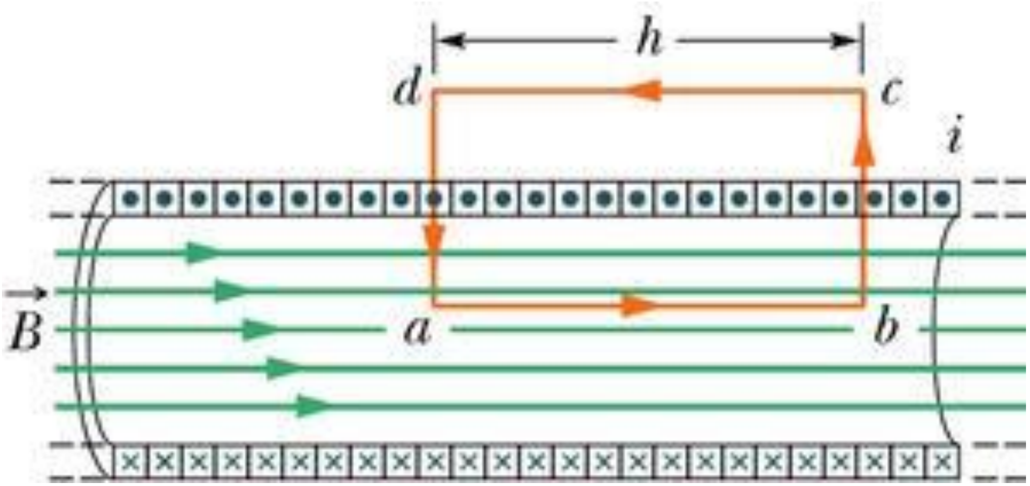
Field looks UNIFORM

Field is ZERO

The real thing.....



Another Way



$$Ampere:$$

$$\oint \mathbf{B} \bullet d\mathbf{s} = \mu_0 i_{enclosed}$$

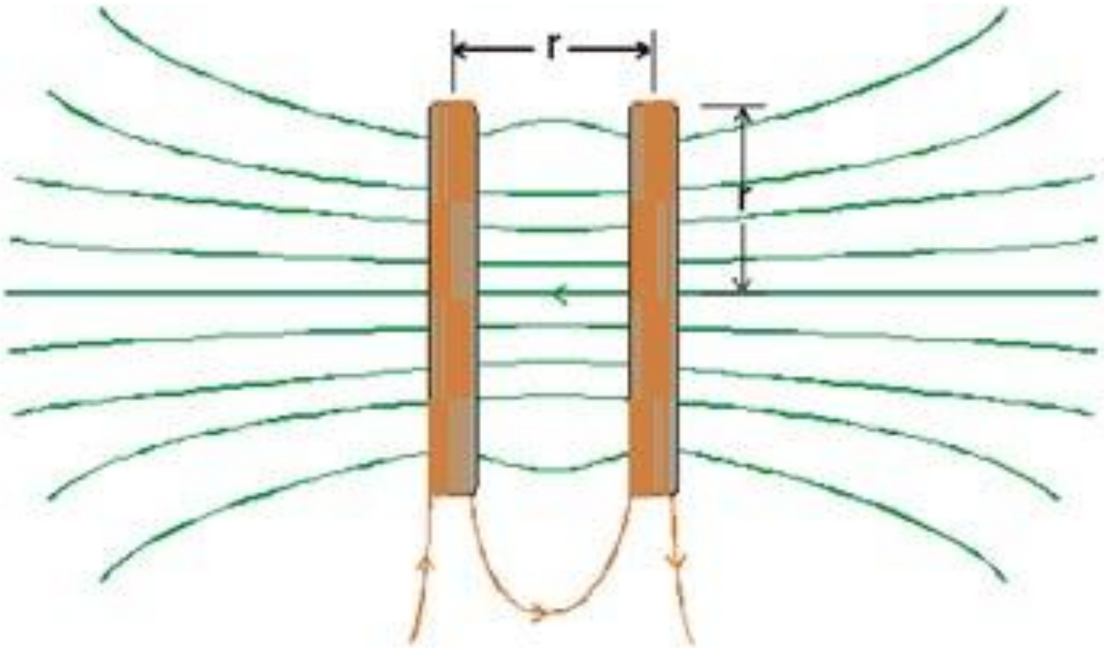
$$0h + Bh = \mu_0 nih$$

$$B = \mu_0 ni$$

Application

- Creation of Uniform Magnetic Field Region
- Minimal field outside
  - except at the ends!

Two Coils

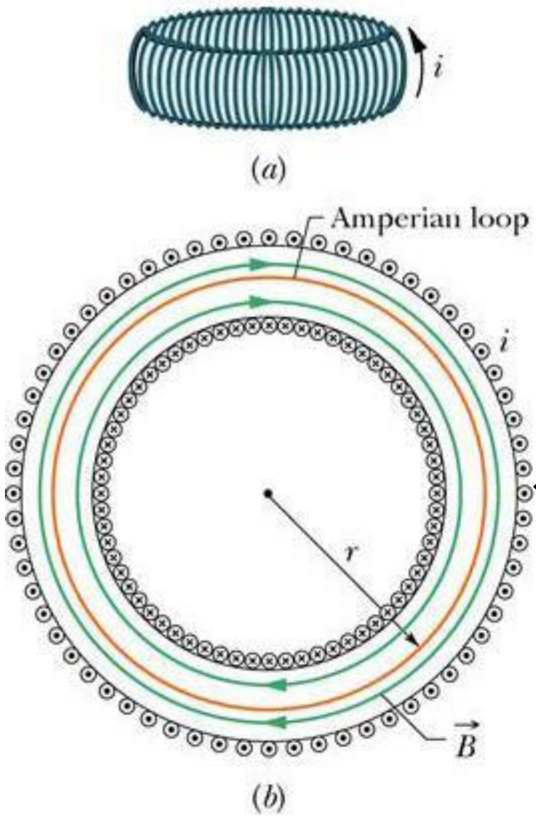


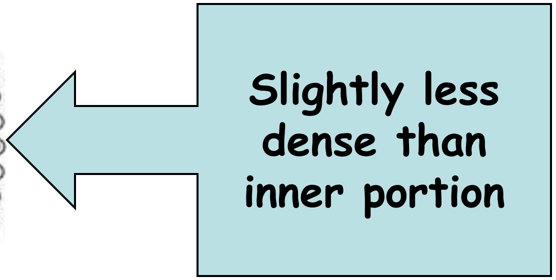
"Real" Helmholtz Coils

Used for experiments.

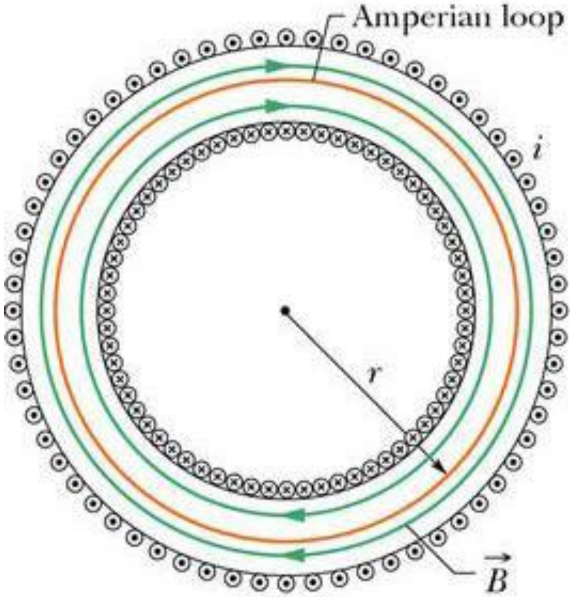
Can be aligned to cancel out the Earth's magnetic field for critical measurements.

The Toroid


i
(a)
Amperian loop
i
r
B
(b)


Slightly less dense than inner portion

The Toroid



Ampere again. We need only worry about the INNER coil contained in the path of integration:

$$\oint \mathbf{B} \bullet d\mathbf{s} = B \times 2\pi r = \mu_0 N i \text{ (N = } \textit{total} \text{ \# turns)}$$

so

$$B = \frac{\mu_0 N i}{2\pi r}$$