

# *Chapter 4*

# **The Valuation of Long-Term Securities**



# ***After studying Chapter 4, you should be able to:***

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- 1. Distinguish among the various terms used to express value.**
- 2. Value bonds, preferred stocks, and common stocks.**
- 3. Calculate the rates of return (or yields) of different types of long-term securities.**
- 4. List and explain a number of observations regarding the behavior of bond prices.**



# ***The Valuation of Long-Term Securities***

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- **Distinctions Among Valuation Concepts**
- **Bond Valuation**
- **Preferred Stock Valuation**
- **Common Stock Valuation**
- **Rates of Return (or Yields)**



# *What is Value?*

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- **Liquidation value** represents the amount of money that could be realized if an asset or group of assets is sold separately from its operating organization.
- **Going-concern value** represents the amount a firm could be sold for as a continuing operating business.



# ***What is Value?***

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- **Book value** represents either
  - (1) **an asset**: the accounting value of an asset -- the asset's cost minus its accumulated depreciation;
  - (2) **a firm**: total assets minus liabilities and preferred stock as listed on the balance sheet.



# ***What is Value?***

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- **Market value** represents the market price at which an asset trades.
- **Intrinsic value** represents the price a security “ought to have” based on all factors bearing on valuation.



# ***Bond Valuation***

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- **Important Terms**
- **Types of Bonds**
- **Valuation of Bonds**
- **Handling Semiannual Compounding**



# ***Important Bond Terms***

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- A **bond** is a long-term debt instrument issued by a corporation or government.
- The **maturity value** (**MV**) [or face value] of a bond is the stated value. In the case of a U.S. bond, the face value is usually \$1,000.





# ***Important Bond Terms***

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- The bond's **coupon rate** is the stated rate of interest; the annual interest payment divided by the bond's face value.
- The **discount rate** (capitalization rate) is dependent on the risk of the bond and is composed of the risk-free rate plus a premium for risk.



# Different Types of Bonds

A perpetual bond is a bond that *never* matures. It has an infinite life.

$$V = \frac{I}{(1 + k_d)^1} + \frac{I}{(1 + k_d)^2} + \dots + \frac{I}{(1 + k_d)^\infty}$$

$$= \sum_{t=1}^{\infty} \frac{I}{(1 + k_d)^t} \quad \text{or} \quad I (\text{PVIFA}_{k_d, \infty})$$

$$V = I / k_d \quad [\text{Reduced Form}]$$



# ***Perpetual Bond Example***

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Bond P has a \$1,000 face value and provides an 8% annual coupon. The appropriate discount rate is 10%. What is the value of the perpetual bond?

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$$I = \$1,000 ( 8\% ) = \$80.$$

$$k_d = 10\%.$$

$$V = I / k_d \text{ [Reduced Form]}$$
$$= \$80 / 10\% = \$800.$$



# Different Types of Bonds

A non-zero coupon-paying bond is a coupon paying bond with a finite life.

$$V = \frac{I}{(1 + k_d)^1} + \frac{I}{(1 + k_d)^2} + \dots + \frac{I + MV}{(1 + k_d)^n}$$

$$= \sum_{t=1}^n \frac{I}{(1 + k_d)^t} + \frac{MV}{(1 + k_d)^n}$$

$$V = I (\text{PVIFA}_{k_d, n}) + MV (\text{PVIF}_{k_d, n})$$



# Coupon Bond Example

Bond C has a \$1,000 face value and provides an 8% annual coupon for 30 years. The appropriate discount rate is 10%. What is the value of the coupon bond?

$$\begin{aligned} V &= \$80 (\text{PVIFA}_{10\%, 30}) + \$1,000 (\text{PVIF}_{10\%, 30}) = \\ &= \$80 (9.427) + \$1,000 (.057) \\ &= \$754.16 + \$57.00 = \$811.16. \end{aligned}$$

[Table IV] [Table II]



# Different Types of Bonds

A **zero coupon bond** is a bond that pays no interest but sells at a deep discount from its face value; it provides compensation to investors in the form of price appreciation.

$$V = \frac{MV}{(1 + k_d)^n} = MV (PVIF_{kd, n})$$



# ***Zero-Coupon Bond Example***

**Bond Z has a \$1,000 face value and a 30 year life. The appropriate discount rate is 10%. What is the value of the zero-coupon bond?**

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$$\begin{aligned} V &= \$1,000 (\text{PVIF}_{10\%, 30}) &= \\ \$1,000 (.057) & &= \$57.00 \end{aligned}$$



# ***Semiannual Compounding***

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**Most bonds *in the U.S.* pay interest twice a year (1/2 of the annual coupon).**

## **Adjustments needed:**

- (1) Divide  $k_d$  by 2**
- (2) Multiply  $n$  by 2**
- (3) Divide  $I$  by 2**





# Semiannual Compounding

A non-zero coupon bond adjusted for semiannual compounding.

$$V = \frac{I/2}{(1 + k_d/2)^1} + \frac{I/2}{(1 + k_d/2)^2} + \dots + \frac{I/2 + MV}{(1 + k_d/2)^{2*n}}$$

$$= \sum_{t=1}^{2*n} \frac{I/2}{(1 + k_d/2)^t} + \frac{MV}{(1 + k_d/2)^{2*n}}$$

$$= I/2 (PVIFA_{k_d/2, 2*n}) + MV (PVIF_{k_d/2, 2*n})$$



# Semiannual Coupon Bond Example

Bond C has a \$1,000 face value and provides an 8% semiannual coupon for 15 years. The appropriate discount rate is 10% (annual rate).  
What is the value of the coupon bond?

$$V = \$40 (\text{PVIFA}_{5\%, 30}) + \$1,000 (\text{PVIF}_{5\%, 30}) = \\ \$40 (15.373) + \$1,000 (.231)$$

[Table IV]

[Table II]

$$= \$614.92 + \$231.00$$

$$= \$845.92$$

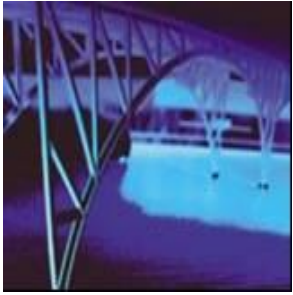


# ***Semiannual Coupon Bond Example***

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**Let us use another worksheet on your calculator to solve this problem. Assume that Bond C was purchased (settlement date) on 12-31-2004 and will be redeemed on 12-31-2019. This is identical to the 15-year period we discussed for Bond C.**

**What is its percent of par? What is the value of the bond?**



# ***Semiannual Coupon Bond Example***

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- 1. What is its percent of par?**
  - **84.628% of par (as quoted in financial papers)**
- 2. What is the value of the bond?**
  - **84.628% x \$1,000 face value = \$846.28**



# ***Preferred Stock Valuation***

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**Preferred Stock** is a type of stock that promises a (usually) fixed dividend, but at the discretion of the board of directors.

**Preferred Stock has preference over common stock in the payment of dividends and claims on assets.**



# Preferred Stock Valuation

$$V = \frac{\text{Div}_P}{(1 + k_P)^1} + \frac{\text{Div}_P}{(1 + k_P)^2} + \dots + \frac{\text{Div}_P}{(1 + k_P)^\infty}$$

$$= \sum_{t=1}^{\infty} \frac{\text{Div}_P}{(1 + k_P)^t} \quad \text{or} \quad \text{Div}_P (\text{PVIFA}_{k_P, \infty})$$

**This reduces to a *perpetuity!***

$$V = \text{Div}_P / k_P$$



# ***Preferred Stock Example***

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Stock PS has an **8%**, \$100 par value issue outstanding. The appropriate **discount rate is 10%**. What is the value of the **preferred stock**?

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$$\begin{aligned} \text{Div}_P &= \$100 ( 8\% ) = \$8.00. & k_P &= \\ 10\% \cdot V &= \text{Div}_P / k_P = \$8.00 / 10\% \\ &= \$80 \end{aligned}$$



# ***Common Stock Valuation***

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**Common stock** represents a residual ownership position in the corporation.

- **Pro rata share of future earnings after all other obligations of the firm (if any remain).**
- **Dividends may be paid out of the pro rata share of earnings.**





# ***Common Stock Valuation***

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**What cash flows will a shareholder receive when owning shares of **common stock**?**

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- (1) Future dividends**
- (2) Future sale of the common stock shares**



# ***Dividend Valuation Model***

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**Basic dividend valuation model accounts for the PV of all future dividends.**

$$V = \frac{\text{Div}_1}{(1 + k_e)^1} + \frac{\text{Div}_2}{(1 + k_e)^2} + \dots + \frac{\text{Div}_\infty}{(1 + k_e)^\infty}$$

$$= \sum_{t=1}^{\infty} \frac{\text{Div}_t}{(1 + k_e)^t}$$

**Div<sub>t</sub>**: Cash Dividend  
at time t

**k<sub>e</sub>**: Equity investor's  
required return



# ***Adjusted Dividend Valuation Model***

**The basic dividend valuation model  
adjusted for the future stock sale.**

$$V = \frac{\text{Div}_1}{(1 + k_e)^1} + \frac{\text{Div}_2}{(1 + k_e)^2} + \dots + \frac{\text{Div}_n + \text{Price}_n}{(1 + k_e)^n}$$

**$n$ :** The year in which the firm's shares are expected to be sold.

**$\text{Price}_n$ :** The expected share price in year  $n$ .



# ***Dividend Growth Pattern Assumptions***

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**The dividend valuation model requires the forecast of all future dividends. The following dividend growth rate assumptions simplify the valuation process.**

**Constant Growth**

**No Growth**

**Growth Phases**



# Constant Growth Model

The **constant growth model** assumes that dividends will grow forever at the rate  $g$ .

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$$V = \frac{D_0(1+g)}{(1+k_e)^1} + \frac{D_0(1+g)^2}{(1+k_e)^2} + \dots + \frac{D_0(1+g)^\infty}{(1+k_e)^\infty}$$

$$= \frac{D_1}{(k_e - g)}$$

$D_1$ : Dividend paid at time 1.

$g$ : The constant growth rate.

$k_e$ : Investor's required return.



# ***Constant Growth Model Example***

Stock CG has an expected **dividend growth rate of 8%**. Each share of stock just received an annual **\$3.24 dividend**.

The appropriate **discount rate is 15%**.  
What is the value of the **common stock**?

$$D_1 = \$3.24 ( 1 + .08 ) = \$3.50$$

$$V_{CG} = D_1 / ( k_e - g ) = \$3.50 / ( .15 - .08 ) \\ = \$50$$



# Zero Growth Model

The **zero growth model** assumes that dividends will grow forever at the rate  $g = 0$ .

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$$V_{ZG} = \frac{D_1}{(1 + k_e)^1} + \frac{D_2}{(1 + k_e)^2} + \dots + \frac{D_\infty}{(1 + k_e)^\infty}$$

$$= \frac{D_1}{k_e}$$

$D_1$ : Dividend paid at time 1.

$k_e$ : Investor's required return.



# Zero Growth Model Example

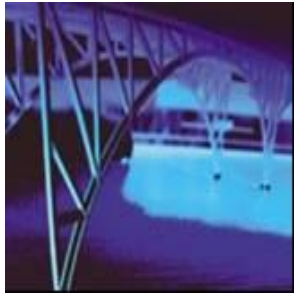
Stock ZG has an expected growth rate of 0%. Each share of stock just received an annual \$3.24 dividend per share. The appropriate discount rate is 15%. What is the value of the common stock?

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$$D_1 = \$3.24 ( 1 + 0 ) = \$3.24$$

$$V_{ZG} = D_1 / ( k_e - 0 ) = \$3.24 / ( .15 - 0 ) \\ = \$21.60$$

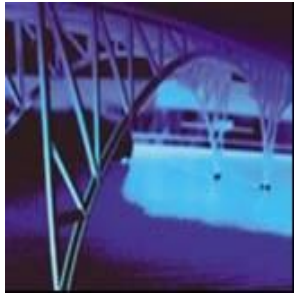




# Growth Phases Model

The **growth phases model** assumes that dividends for each share will grow at two or more *different* growth rates.

$$V = \sum_{t=1}^n \frac{D_0 (1+g_1)^t}{(1+k_e)^t} + \sum_{t=n+1}^{\infty} \frac{D_n (1+g_2)^t}{(1+k_e)^t}$$



# Growth Phases Model

Note that the second phase of the **growth phases model** assumes that dividends will grow at a constant rate  $g_2$ . We can rewrite the formula as:

$$V = \sum_{t=1}^n \frac{D_0(1+g_1)^t}{(1+k_e)^t} + \left[ \frac{1}{(1+k_e)^n} \right] \left[ \frac{D_{n+1}}{(k_e - g_2)} \right]$$



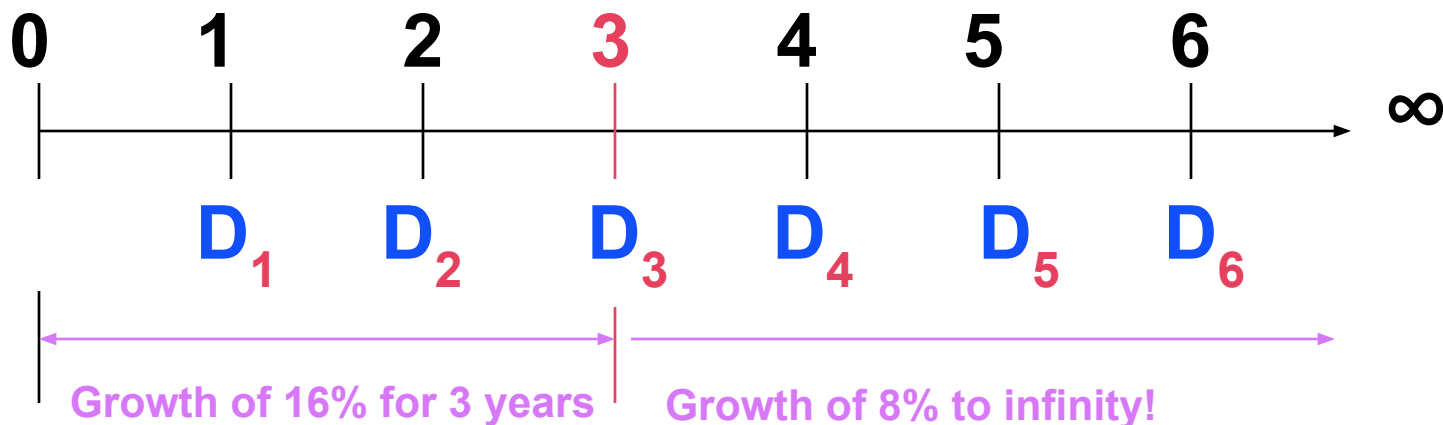
# ***Growth Phases Model Example***

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**Stock GP has an expected growth rate of 16% for the first 3 years and 8% thereafter. Each share of stock just received an annual \$3.24 dividend per share. The appropriate discount rate is 15%. What is the value of the common stock under this scenario?**



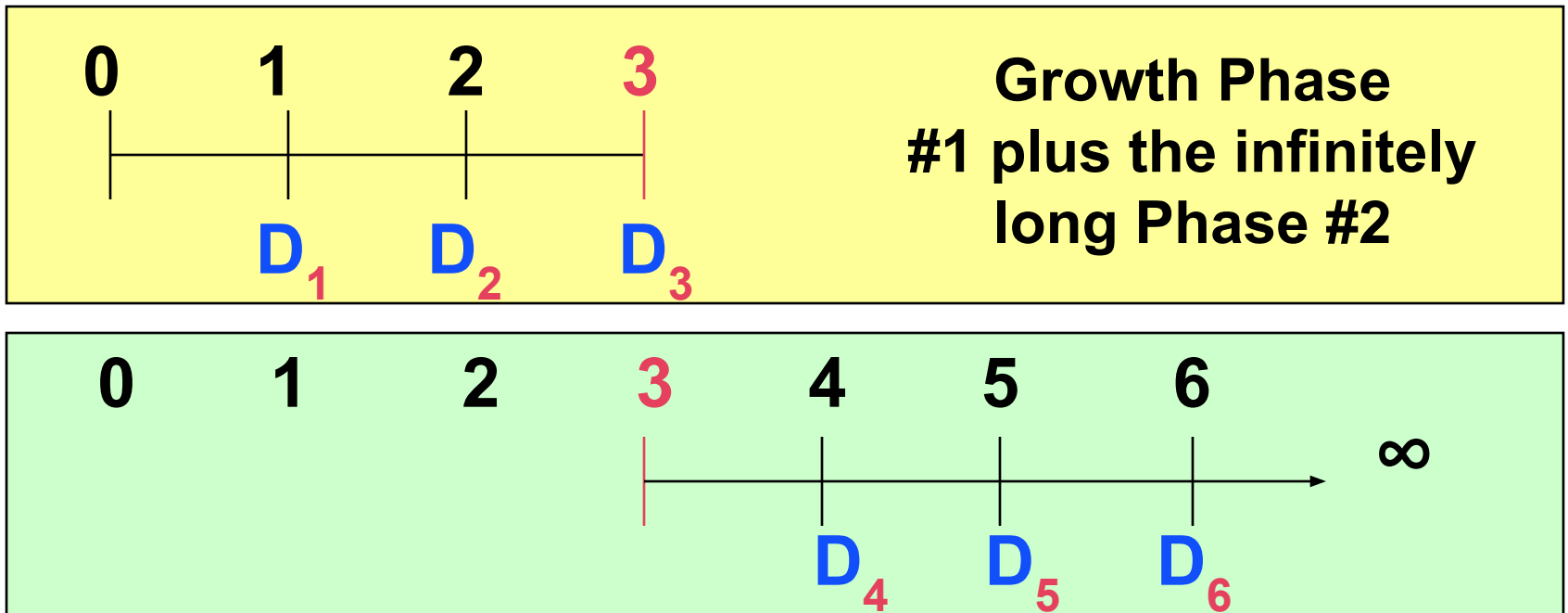
# ***Growth Phases Model Example***



Stock GP has two phases of growth. The first, 16%, starts at time  $t=0$  for 3 years and is followed by 8% thereafter starting at time  $t=3$ . We should view the time line as two separate time lines in the valuation.



# ***Growth Phases Model Example***



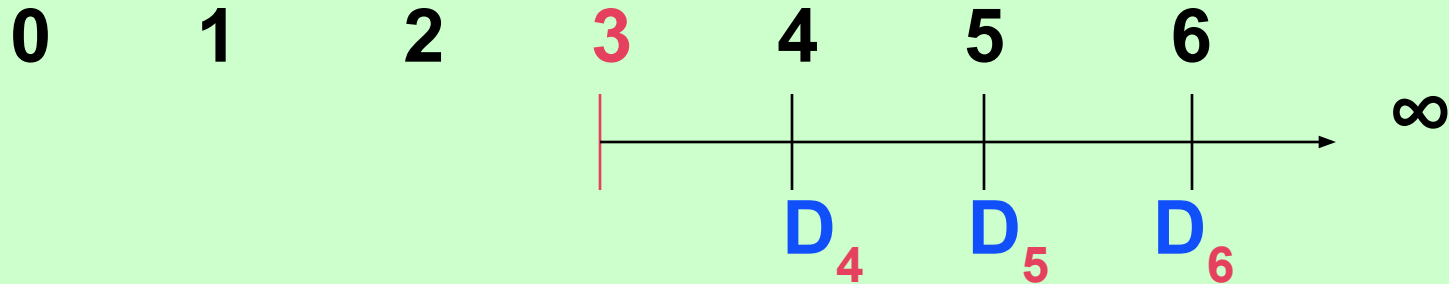
**Note that we can value Phase #2 using the  
*Constant Growth Model***



# ***Growth Phases Model Example***

$$V_3 = \frac{D_4}{k-g}$$

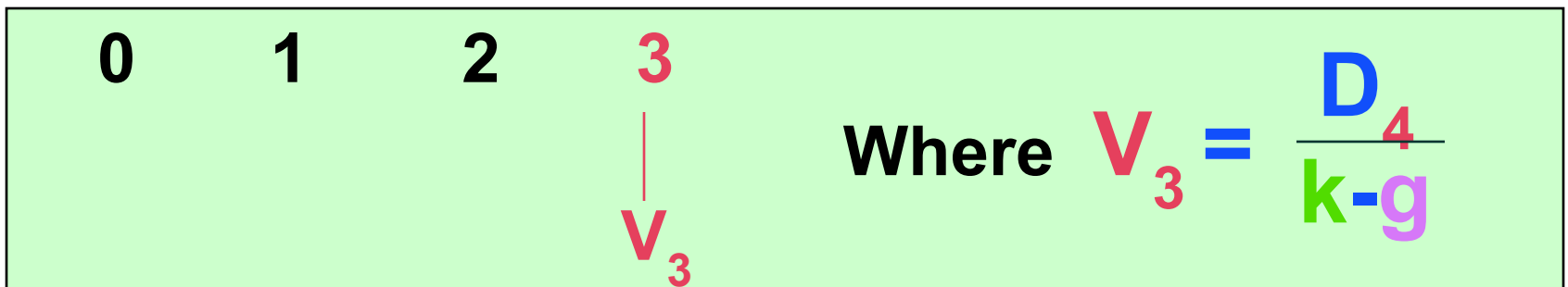
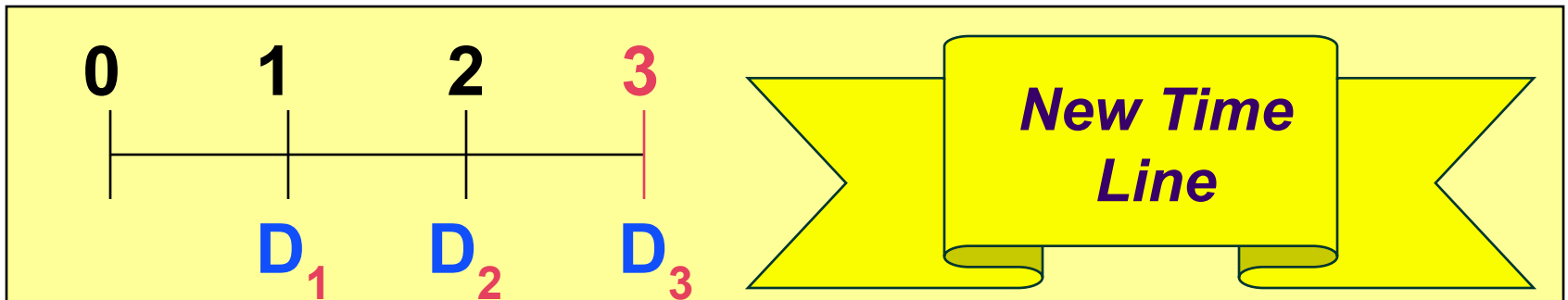
We can use this model because dividends grow at a constant 8% rate beginning at the end of Year 3.



Note that we can now replace all dividends from **year 4 to infinity** with the *value* at time **t=3**,  $V_3$ ! Simpler!!



# ***Growth Phases Model Example***



**Now we only need to find the first four dividends to calculate the necessary cash flows.**



# ***Growth Phases Model Example***

**Determine the annual dividends.**

$$D_0 = \$3.24 \text{ (this has been paid already)}$$

$$D_1 = D_0(1+g_1)^1 = \$3.24(1.16)^1 = \$3.76$$

$$D_2 = D_0(1+g_1)^2 = \$3.24(1.16)^2 = \$4.36$$

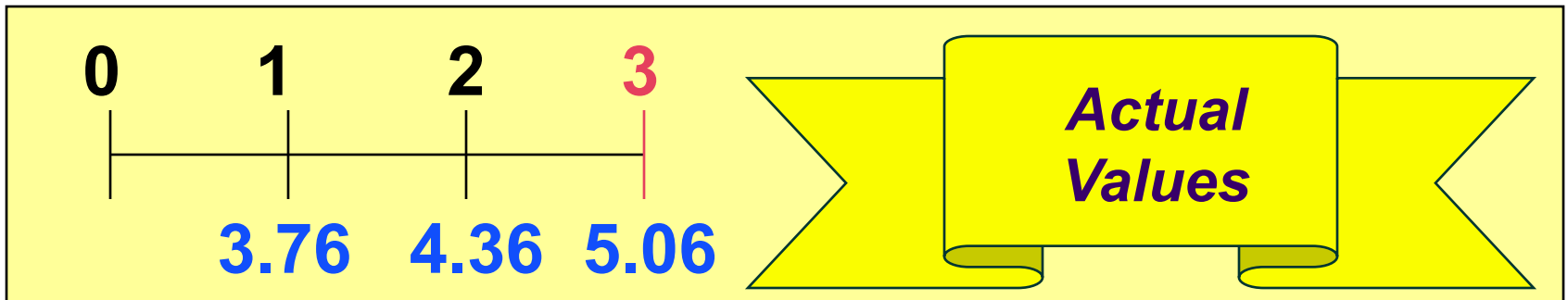
$$D_3 = D_0(1+g_1)^3 = \$3.24(1.16)^3 = \$5.06$$

$$D_4 = D_3(1+g_2)^1 = \$5.06(1.08)^1 = \$5.46$$





# ***Growth Phases Model Example***



Now we need to find the present value  
of the cash flows.



# ***Growth Phases Model Example***

**We determine the PV of cash flows.**

$$\text{PV}(D_1) = D_1 (\text{PVIF}_{15\%, 1}) = \$3.76 (.870) = \underline{\$3.27}$$

$$\text{PV}(D_2) = D_2 (\text{PVIF}_{15\%, 2}) = \$4.36 (.756) = \underline{\$3.30}$$

$$\text{PV}(D_3) = D_3 (\text{PVIF}_{15\%, 3}) = \$5.06 (.658) = \underline{\$3.33}$$

$$P_3 = \$5.46 / (.15 - .08) = \$78 \text{ [CG Model]}$$

$$\text{PV}(P_3) = P_3 (\text{PVIF}_{15\%, 3}) = \$78 (.658) = \underline{\$51.32}$$



# Growth Phases Model Example

Finally, we calculate the *intrinsic value* by summing all of cash flow present values.

$$V = \$3.27 + \$3.30 + \$3.33 + \$51.32$$

$$V = \$61.22$$

$$V = \sum_{t=1}^3 \frac{D_0(1+.16)^t}{(1+.15)^t} + \left[ \frac{1}{(1+.15)^n} \right] \left[ \frac{D_4}{(.15-.08)} \right]$$