Binary Variables

- Recall that the two binary values have different names:
  - True/False
  - On/Off
  - Yes/No
  - 1/0
- We use 1 and 0 to denote the two values.

Boolean Algebra

- Invented by George Boole in 1854
- An algebraic structure defined by a set $B = \{0, 1\}$, together with two binary operators ($+$ and $\cdot$) and a unary operator **($\bar{}$)**,

Binary Logic and Gates

- Binary variables take on one of two values.
- Logical operators operate on binary values and binary variables.
- Basic logical operators are the logic functions AND, OR and NOT.
- Logic gates implement logic functions.
- Boolean Algebra: a useful mathematical system for specifying and transforming logic functions.
- We study Boolean algebra as a foundation for designing and analyzing digital systems!

Logic Gates

- In the earliest computers, switches were opened and closed by magnetic fields produced by energizing coils in *relays*. The switches in turn opened and closed the current paths.
- Later, *vacuum tubes* that open and close current paths electronically replaced relays.
- Today, *transistors* are used as electronic switches that open and close current paths.

Logical Operations

- The three basic logical operations are:
  - AND
  - OR
  - NOT
- AND is denoted by a dot (·).
- OR is denoted by a plus (+).
- NOT is denoted by an overbar (¯), a single quote mark (') after, or (~) before the variable.

Truth Tables

- Tabular listing of the values of a function for all possible combinations of values on its arguments
- Example: Truth tables for the basic logic operations:

AND		
X	Y	$Z = X \cdot Y$
0	0	0
0	1	0
1	0	0
1	1	1

OR		
X	Y	$Z = X + Y$
0	0	0
0	1	1
1	0	1
1	1	1

NOT	
X	$Z = \overline{X}$
0	1
1	0

Operator Definitions

- Operations are defined on the values "0" and "1" for each operator:

AND

$$0 \cdot 0 = 0$$
$$0 \cdot 1 = 0$$
$$1 \cdot 0 = 0$$
$$1 \cdot 1 = 1$$

OR

$$0 + 0 = 0$$
$$0 + 1 = 1$$
$$1 + 0 = 1$$
$$1 + 1 = 1$$

NOT

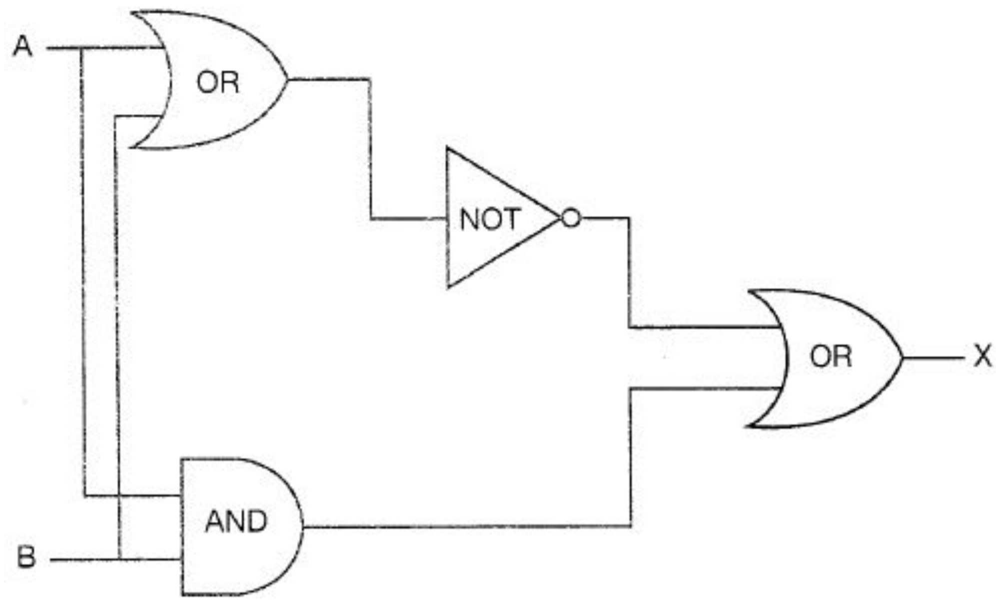
$$\overline{0} = 1$$
$$\overline{1} = 0$$

Produce a truth table I

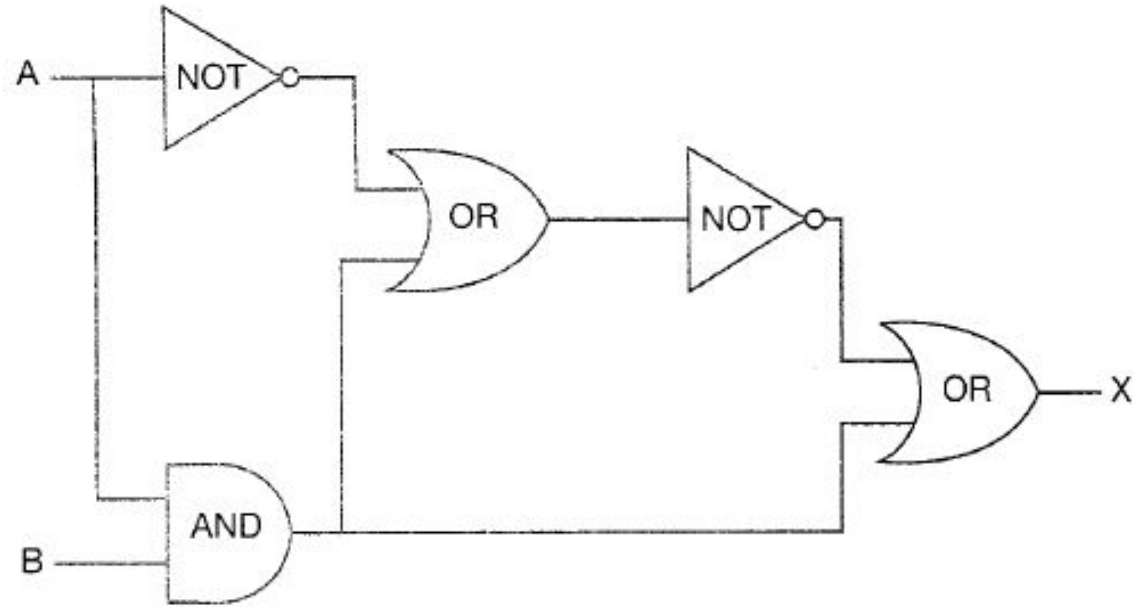
In the BooleanAlgebra, verify using truth table that $(X + Y)' = X'Y'$

In the Boolean Algebra, verify using truth table that $X + XY = X$

1. Write the boolean expression for the below circuit



2. Write the boolean expression for the below circuit



Problem 1

- A system used 3 switches A,B and C; a combination of switches determines whether an alarm, X, sounds:
- If switch A or Switch B are in the ON position and if switch C is in the OFF position then a signal to sound an alarm, X is produced.

Convert this problem into a logic statement.

Problem 2

A nuclear power station has a safety system based on three inputs to a logic circuit(network). A warning signal (S = 1) is produced when certain conditions in the nuclear power station occur based on these three inputs

Input	Binary Values	Description of plant status
T	1	Temperature > 115 C
	0	Temperature <= 115 C
P	1	Reactor pressure > 15 bar
	0	Reactor pressure <= 15 bar
W	1	Cooling water > 120 litres / hour
	0	Cooling water<=120 liters/hour

A warning signal (S=1) will be produced when any of the following occurs.

Either (a) Temperature > 115 C and Cooling water <=120 litres/hour

or (b) Temperature <=115 C and when Reactor pressure > 15 bar

or cooling water <= 120 litres/hour

Logic Diagrams and Expressions

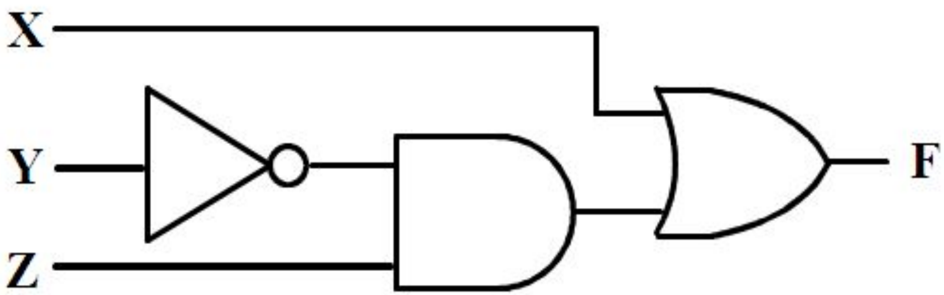
Truth Table

XYZ	$F = X + \bar{Y} \cdot Z$
0 0 0	0
0 0 1	1
0 1 0	0
0 1 1	0
1 0 0	1
1 0 1	1
1 1 0	1
1 1 1	1

Logic Equation

$$F = X + \bar{Y}\,Z$$

Logic Diagram



- **Boolean equations, truth tables and logic diagrams describe the same function!**
- **Truth tables are unique, but expressions and logic diagrams are not. This gives flexibility in implementing functions.**

Boolean Algebra

1.	$X + 0 = X$	2.	$X \cdot 1 = X$	Identity element
3.	$X + 1 = 1$	4.	$X \cdot 0 = 0$	
5.	$X + X = X$	6.	$X \cdot X = X$	Idempotence
7.	$X + \overline{X} = 1$	8.	$X \cdot \overline{X} = 0$	Complement
9.	$\overline{\overline{X}} = X$			Involution
10.	$X + Y = Y + X$	11.	$XY = YX$	Commutative
12.	$(X + Y) + Z = X + (Y + Z)$	13.	$(XY)Z = X(YZ)$	Associative
14.	$X(Y + Z) = XY + XZ$	15.	$X + YZ = (X + Y)(X + Z)$	Distributive
16.	$\overline{X + Y} = \overline{X} \cdot \overline{Y}$	17.	$\overline{X \cdot Y} = \overline{X} + \overline{Y}$	DeMorgan's

Some Properties of Boolean Algebra

- Boolean Algebra is defined in general by a set *B* that can have more than two values
- A two-valued Boolean algebra is also know as Switching Algebra. The Boolean set *B* is restricted to 0 and 1. Switching circuits can be represented by this algebra.
- The dual of an algebraic expression is obtained by interchanging + and · and interchanging 0's and 1's.
- The identities appear in dual pairs. When there is only one identity on a line the identity is self-dual, i. e., the dual expression = the original expression.
- Sometimes, the dot symbol '•' (AND operator) is

Dual of a Boolean Expression

- Example: $F = (A + C) \cdot B + 0$

  dual $F = (A \cdot C + B) \cdot 1 = A \cdot C + B$

- Example: $G = X \cdot Y + (W + Z)$

  dual $G = (X+Y) \cdot (W \cdot Z) = (X+Y) \cdot (W+Z)$

- Example: $H = A \cdot B + A \cdot C + B \cdot C$

  dual $H = (A+B) \cdot (A+C) \cdot (B+C)$

Boolean Algebraic Proof – Example 1

- $A + A \cdot B = A$ (Absorption Theorem)

<u>Proof Steps</u> <u>Justification</u>

$A + A \cdot B$

$= A \cdot 1 + A \cdot B$ Identity element: $A \cdot 1 = A$

$= A \cdot (1 + B)$ Distributive

$= A \cdot 1$ $1 + B = 1$

$= A$ Identity element

- Our primary reason for doing proofs is to learn:
  - Careful and efficient use of the identities and theorems of Boolean algebra, and
  - How to choose the appropriate identity or theorem to apply to make forward progress, irrespective of the application.

Boolean Algebraic Proof – Example 2

- $AB + \overline{A}C + BC = AB + \overline{A}C$ (Consensus Theorem)

<u>Proof Steps</u>	<u>Justification</u>
$AB + \overline{A}C + BC$	
$= AB + \overline{A}C + 1 \cdot BC$	Identity element
$= AB + \overline{A}C + (A + \overline{A}) \cdot BC$	Complement
$= AB + \overline{A}C + ABC + \overline{A}BC$	Distributive
$= AB + ABC + \overline{A}C + \overline{A}CB$	Commutative
$= AB \cdot 1 + ABC + \overline{A}C \cdot 1 + \overline{A}CB$	Identity element
$= AB\,(1+C) + \overline{A}C\,(1 + B)$	Distributive
$= AB\,.\,1 + \overline{A}C\,.\,1$	$1+X = 1$
$= AB + \overline{A}C$	Identity element

Proof

$$A + \overline{A}B = A + B$$

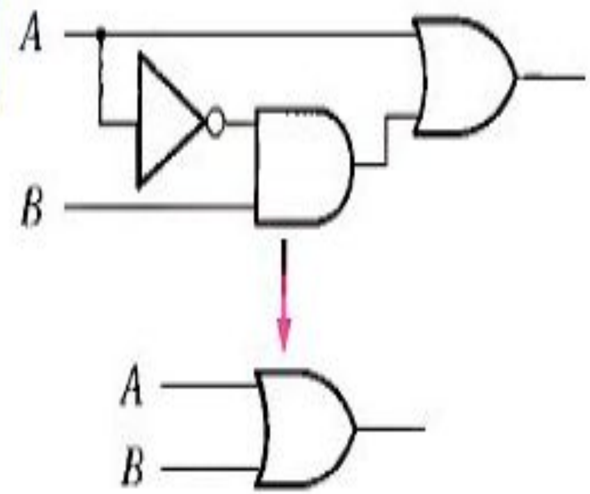
This rule can be proved as follows:

$A + \overline{A}B = (A + AB) + \overline{A}B$	$A = A + AB$
$= (AA + AB) + \overline{A}B$	$A = AA$
$= AA + AB + A\overline{A} + \overline{A}B$	adding $A\overline{A} = 0$
$= (A + \overline{A})(A + B)$	Factoring
$= 1.\ (A + B)$	$A + \overline{A} = 1$
$= A + B$	drop the 1

Minimization of Boolean Expression

A	B	$\overline{A}B$	$A+\overline{A}B$	$A+B$
0	0	0	0	0
0	1	1	1	1
1	0	0	1	1
1	1	0	1	1

equal



Simplification of Boolean Algebra

- $(A + B)(A + C) = A + BC$
- This rule can be proved as follows:
- $(A + B)(A + C) = AA + AC + AB + BC$(Distributive law)

$= A + AC + AB + BC$ ($AA = A$)

$= A(1 + C) + AB + BC$ $(1 + C = 1)$

$= A. 1 + AB + BC$

$= A(1 + B) + BC$ $(1 + B = 1)$

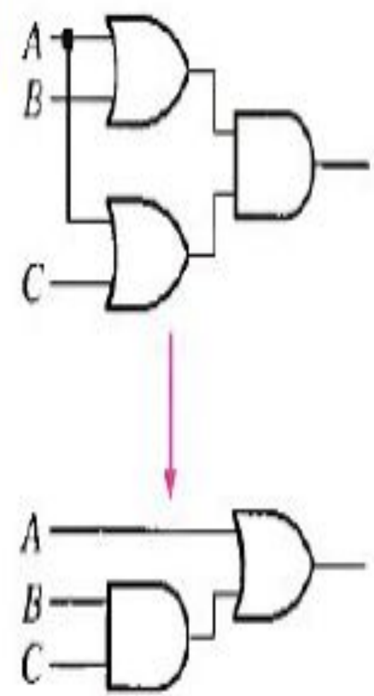
$= A. 1 + BC$ $(A . 1 = A)$

$= A + BC$

Logic Diagram

A	B	C	A + B	A + C	(A + B)(A + C)	BC	A + BC
0	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	1	1	1	1
1	0	0	1	1	1	0	1
1	0	1	1	1	1	0	1
1	1	0	1	1	1	0	1
1	1	1	1	1	1	1	1

equal



Useful Theorems

- Minimization
  $X\,Y + \overline{X}\,Y = Y$
- Minimization (dual)
  $(X+Y)(\overline{X}+Y) = Y$
- Absorption
  $X + X\,Y = X$
- Absorption (dual)
  $X \cdot (X + Y) = X$
- Simplification
  $X + \overline{X}\,Y = X + Y$
- Simplification (dual)
  $X \cdot (\overline{X} + Y) = X \cdot Y$
- DeMorgan's
- $\overline{X + Y} = \overline{X} \cdot \overline{Y}$
- DeMorgan's (dual)
- $\overline{X \cdot Y} = \overline{X} + \overline{Y}$

De morgan's Law

The complement of two or more ANDed variables is equivalent to the OR of the complements of the individual variables.

The formula for expressing this theorem for two variables is

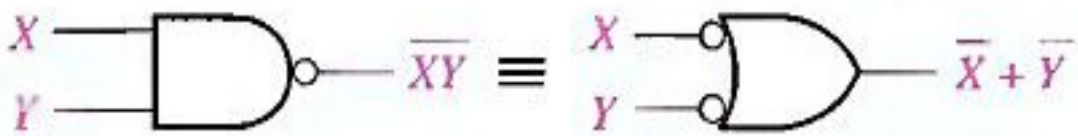
$$\overline{XY} = \overline{X} + \overline{Y}$$

The complement of two or more ORed variables is equivalent to the AND of the complements of the individual variables,

The formula for expressing this theorem for two variables is

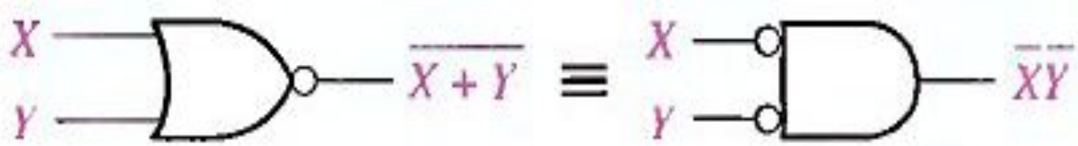
$$\overline{X + Y} = \overline{X}\,\overline{Y}$$

Gate equivalencies and the corresponding truth tables that illustrate De Morgan's theorems.



NAND Negative-OR

Inputs		Output	
X	Y	$\overline{XY}$	$\overline{X} + \overline{Y}$
0	0	1	1
0	1	1	1
1	0	1	1
1	1	0	0



NOR Negative-AND

Inputs		Output	
X	Y	$\overline{X + Y}$	$\overline{X}\,\overline{Y}$
0	0	1	1
0	1	0	0
1	0	0	0
1	1	0	0

Truth Table to Verify De Morgan's

$$\overline{X + Y} = \overline{X} \cdot \overline{Y} \qquad \overline{X \cdot Y} = \overline{X} + \overline{Y}$$

X	Y	$X \cdot Y$	$X+Y$	$\overline{X}$	$\overline{Y}$	$\overline{X+Y}$	$\overline{X} \cdot \overline{Y}$	$\overline{X \cdot Y}$	$\overline{X}+\overline{Y}$
0	0	0	0	1	1	1	1	1	1
0	1	0	1	1	0	0	0	1	1
1	0	0	1	0	1	0	0	1	1
1	1	1	1	0	0	0	0	0	0

- Generalized DeMorgan's Theorem:

$$\overline{X_1 + X_2 + \ldots + X_n} = \overline{X_1} \cdot \overline{X_2} \cdot \ldots \cdot \overline{X_n}$$

$$\overline{X_1 \cdot X_2 \cdot \ldots \cdot X_n} = \overline{X_1} + \overline{X_2} + \ldots + \overline{X_n}$$

Simplification-Example

- Using Boolean algebra techniques, simplify this expression:

  AB + A(B + C) + B(B + C)

- Step 1: Apply the distributive law to the second and third terms in the expression, as follows:

  AB + AB + AC + BB + BC

- Step 2: Apply (BB = B) to the fourth term.

  AB + AB + AC + B + BC

- Step 3: Apply (AB + AB = AB) to the first two terms.

  AB + AC + B + BC

- Step 4: Apply (B + BC = B) to the last two terms.

  AB + AC + B

- Step 5: Apply (AB + B = B) to the first and third terms.

- Used to evaluate any logic function
- Consider $F(X, Y, Z) = X\,Y + \overline{Y}\,Z$

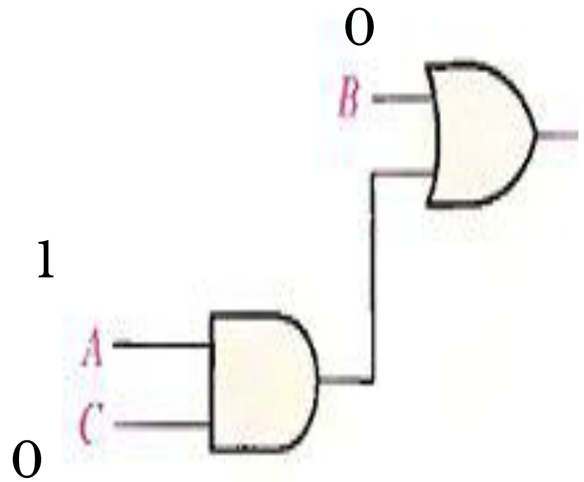
X	Y	Z	$X\,Y$	$\overline{Y}$	$\overline{Y}\,Z$	$F = X\,Y + \overline{Y}\,Z$
0	0	0				
0	0	1				
0	1	0				
0	1	1				
1	0	0				
1	0	1				
1	1	0				
1	1	1				

Truth Tables – Cont'd

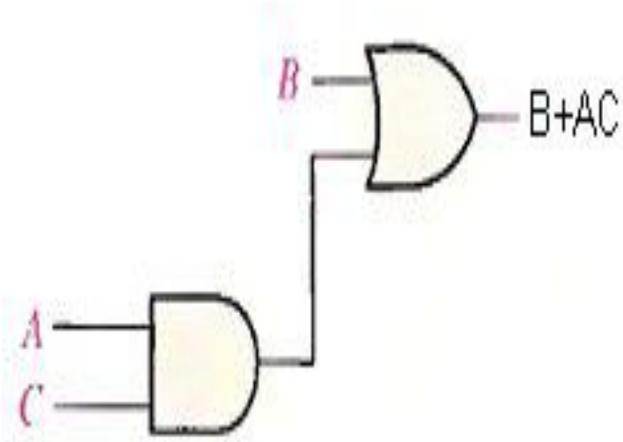
- Used to evaluate any logic function
- Consider $F(X, Y, Z) = X\,Y + \overline{Y}\,Z$

X	Y	Z	$X\,Y$	$\overline{Y}$	$\overline{Y}\,Z$	$F = X\,Y + \overline{Y}\,Z$
0	0	0	0	1	0	0
0	0	1	0	1	1	1
0	1	0	0	0	0	0
0	1	1	0	0	0	0
1	0	0	0	1	0	0
1	0	1	0	1	1	1
1	1	0	1	0	0	1
1	1	1	1	0	0	1

Logic Diagram

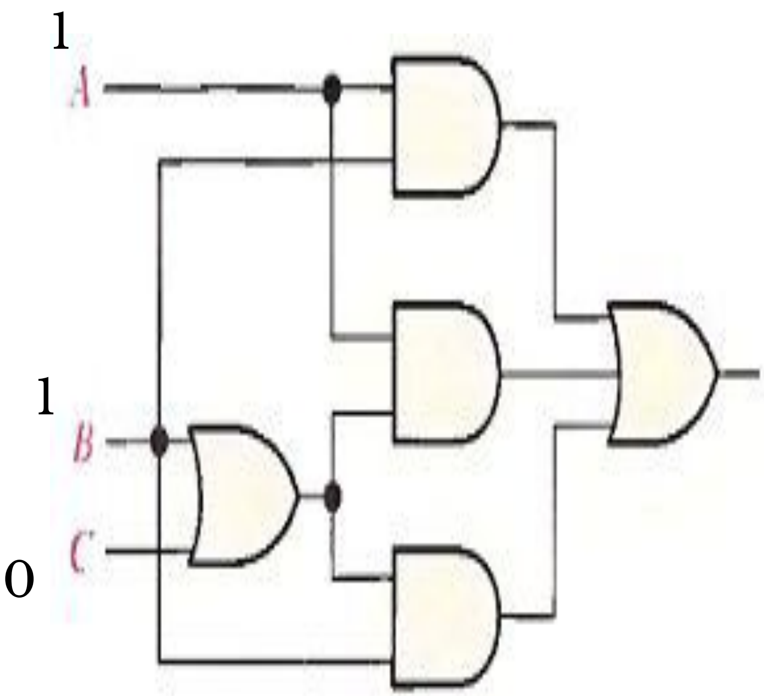


Logic Diagram

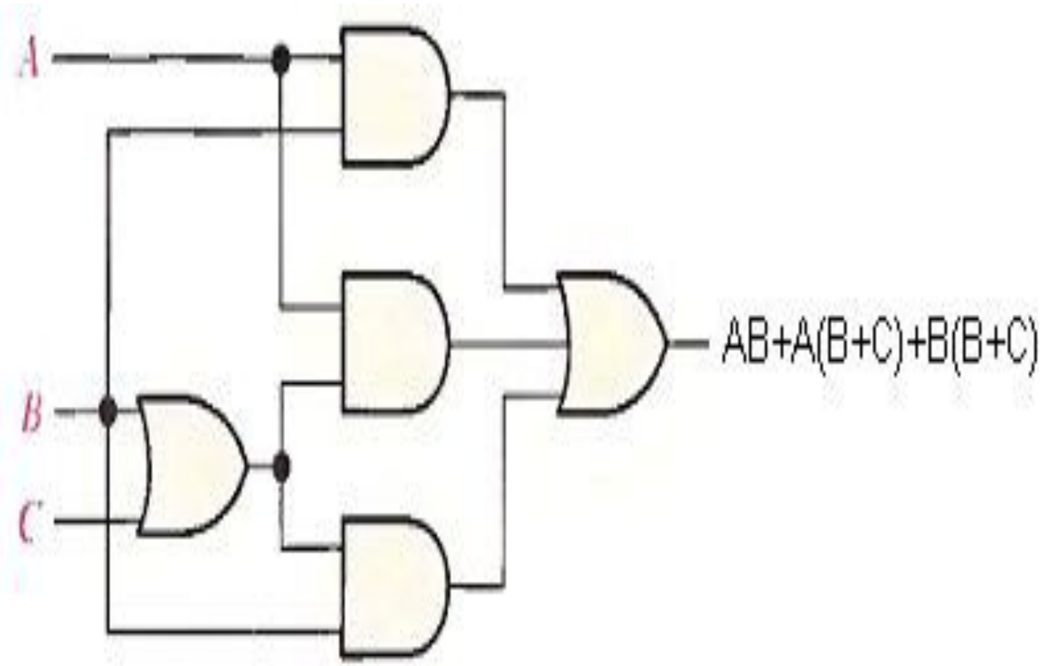


(b)

Logic Diagram



Logic Diagram



Expression Simplification

- An application of Boolean algebra
- Simplify to contain the smallest number of literals (variables that may or may not be complemented)

$$AB + \overline{A}CD + \overline{A}BD + \overline{A}C\overline{D} + ABCD$$

$$= AB + ABCD + \overline{A}CD + \overline{A}C\overline{D} + \overline{A}BD$$

$$= AB + AB(CD) + \overline{A}C(D + \overline{D}) + \overline{A}BD$$

$$= AB + \overline{A}C + \overline{A}BD = B(A + \overline{A}D) + \overline{A}C$$

$$= B(A + D) + \overline{A}C \text{ (has only 5 literals)}$$

Canonical Forms…..

- Minterms and Maxterms
- Sum-of-products (SOP) Canonical Form
- Product-of-sum (POS) Canonical Form
- Representation of Complements of Functions
- Conversions between Representations

Minterms

- **Minterms** are AND terms with every variable present in either true or complemented form.
- Given that each binary variable may appear normal (e.g., x) or complemented (e.g., $\overline{x}$), there are 2^n minterms for n variables.
- **Example:** Two variables (X and Y) produce 2 x 2 = 4 combinations:

  XY (both normal)

  $X\overline{Y}$ (X normal, Y complemented)

  $\overline{X}Y$ (X complemented, Y normal)

  $\overline{X}\,\overline{Y}$ (both complemented)

- Thus there are four minterms of two variables.

Maxterms

- **Maxterms** are OR terms with every variable in true or complemented form.
- Given that each binary variable may appear normal (e.g., x) or complemented (e.g., $\overline{x}$), there are 2^n maxterms for n variables.
- **Example:** Two variables (X and Y) produce 2 x 2 = 4 combinations:

  $X + Y$ (both normal)

  $X + \overline{Y}$ (x normal, y complemented)

  $\overline{X} + Y$ (x complemented, y normal)

  $\overline{X} + \overline{Y}$ (both complemented)

Minterms & Maxterms for 2 variables

- **Two variable minterms and maxterms.**

x	y	Index	Minterm	Maxterm
0	0	0	$m_0 = \overline{x}\,\overline{y}$	$M_0 = x + y$
0	1	1	$m_1 = \overline{x}\,y$	$M_1 = x + \overline{y}$
1	0	2	$m_2 = x\,\overline{y}$	$M_2 = \overline{x} + y$
1	1	3	$m_3 = x\,y$	$M_3 = \overline{x} + \overline{y}$

- **The minterm m_i should evaluate to 1 for each combination of x and y.**
- **The maxterm is the complement of the minterm**

Minterms & Maxterms for 3 variables

x	y	z	Index	Minterm	Maxterm
0	0	0	0	$m_0 = \bar{x}\,\bar{y}\,\bar{z}$	$M_0 = x + y + z$
0	0	1	1	$m_1 = \bar{x}\,\bar{y}\,z$	$M_1 = x + y + \bar{z}$
0	1	0	2	$m_2 = \bar{x}\,y\,\bar{z}$	$M_2 = x + \bar{y} + z$
0	1	1	3	$m_3 = \bar{x}\,y\,z$	$M_3 = x + \bar{y} + \bar{z}$
1	0	0	4	$m_4 = x\,\bar{y}\,\bar{z}$	$M_4 = \bar{x} + y + z$
1	0	1	5	$m_5 = x\,\bar{y}\,z$	$M_5 = \bar{x} + y + \bar{z}$
1	1	0	6	$m_6 = x\,y\,\bar{z}$	$M_6 = \bar{x} + \bar{y} + z$
1	1	1	7	$m_7 = x\,y\,z$	$M_7 = \bar{x} + \bar{y} + \bar{z}$

Maxterm M_i is the complement of minterm m_i

$$M_i = \overline{m_i} \text{ and } m_i = \overline{M_i}$$

The Standard SOP Form

- A standard SOP expression is one in which *all* the variables in the domain appear <u>in each product term</u> in the expression. $A\overline{B}CD + \overline{A}\,\overline{B}C\overline{D} + ABC\overline{D}$
  - Example:

- Standard SOP expressions are important in:
  - Constructing truth tables
  - The Karnaugh map simplification method

Converting Product Terms to Standard SOP (example)

- Convert the following Boolean expression into standard SOP form:

$$A\overline{B}C + \overline{A}\,\overline{B} + AB\overline{C}D$$

$$A\overline{B}C = A\overline{B}C(D+\overline{D}) = \boxed{A\overline{B}CD + A\overline{B}C\overline{D}}$$

$$\overline{A}\,\overline{B} = \overline{A}\,\overline{B}(C+\overline{C}) = \overline{A}\,\overline{B}C + \overline{A}\,\overline{B}\,\overline{C}$$

$$\overline{A}\,\overline{B}C(D+\overline{D}) + \overline{A}\,\overline{B}\,\overline{C}(D+\overline{D}) = \boxed{\overline{A}\,\overline{B}CD + \overline{A}\,\overline{B}C\overline{D} + \overline{A}\,\overline{B}\,\overline{C}D + \overline{A}\,\overline{B}\,\overline{C}\,\overline{D}}$$

$$A\overline{B}C + \overline{A}\,\overline{B} + AB\overline{C}D = \boxed{A\overline{B}CD + A\overline{B}C\overline{D}} + \boxed{\overline{A}\,\overline{B}CD + \overline{A}\,\overline{B}C\overline{D} + \overline{A}\,\overline{B}\,\overline{C}D + \overline{A}\,\overline{B}\,\overline{C}\,\overline{D}} + AB\overline{C}D$$

Sum-Of- Product (SOP)

- Sum-Of-Minterm (SOM) canonical form:
  Sum of minterms of entries that evaluate to '**1**'

x	y	z	F	Minterm
0	0	0	0	
0	0	1	**1**	$m_1 = \bar{x}\,\bar{y}\,z$
0	1	0	0	
0	1	1	0	
1	0	0	0	
1	0	1	0	
1	1	0	**1**	$m_6 = x\,y\,\bar{z}$
1	1	1	**1**	$m_7 = x\,y\,z$

Focus on the '**1**' entries

$$F = m_1 + m_6 + m_7 = \sum (1, 6, 7) = \bar{x}\,\bar{y}\,z + x\,y\,\bar{z} + x\,y\,z$$

Sum-Of-Minterm Examples

- $F(a,b,c,d) = \sum(2,3,6,10,11)$
- $F(a,b,c,d) = \mathrm{m}_2 + \mathrm{m}_3 + \mathrm{m}_6 + \mathrm{m}_{10} + \mathrm{m}_{11}$

$$\bar{a}\,\bar{b}\,c\,\bar{d} + \bar{a}\,\bar{b}\,c\,d + \bar{a}\,b\,c\,\bar{d} + a\,\bar{b}\,c\,\bar{d} + a\,\bar{b}\,c\,d$$

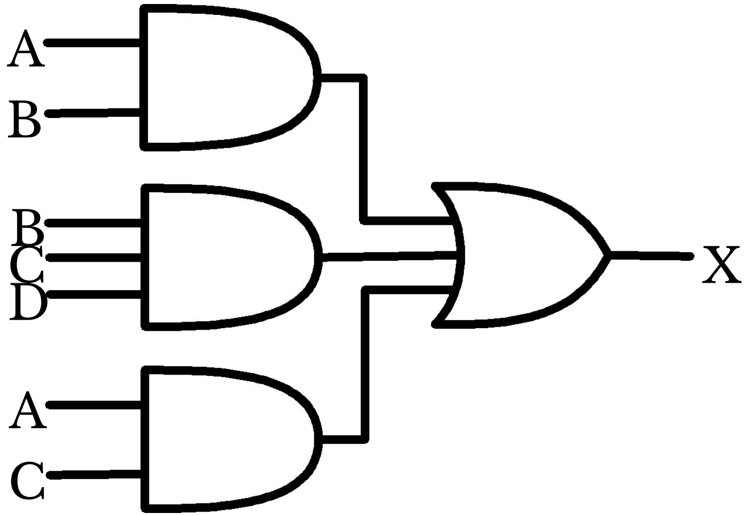
- $G(a,b,c,d) = \sum(0,1,12,15)$
- $G(a,b,c,d) = \mathrm{m}_0 + \mathrm{m}_1 + \mathrm{m}_{12} + \mathrm{m}_{15}$

$$\bar{a}\,\bar{b}\,\bar{c}\,\bar{d} + \bar{a}\,\bar{b}\,\bar{c}\,d + a\,b\,\bar{c}\,\bar{d} + a\,b\,c\,d$$

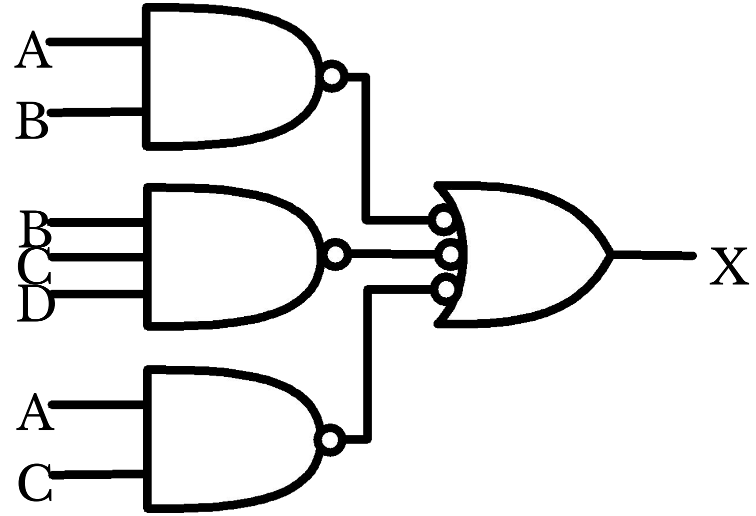
Implementation of an SOP

$X = AB + BCD + AC$

- AND/OR implementation



- NAND/NAND implementation



The Standard POS Form

- A standard POS expression is one in which *all* the variables in the domain appear <u>in each sum term</u> in the expression
  - Example: $(\overline{A}+\overline{B}+\overline{C}+\overline{D})(A+\overline{B}+C+D)(A+B+\overline{C}+D)$
- Standard POS expressions are important in:
  - Constructing truth tables
  - The Karnaugh map simplification method

Converting a Sum Term to Standard POS (example)

- Convert the following Boolean expression into standard POS form:

$$(A+\overline{B}+C)(\overline{B}+C+\overline{D})(A+\overline{B}+\overline{C}+D)$$

$$A+\overline{B}+C = A+\overline{B}+C+D\overline{D} = \boxed{(A+\overline{B}+C+D)(A+\overline{B}+C+\overline{D})}$$

$$\overline{B}+C+\overline{D} = \overline{B}+C+\overline{D}+A\overline{A} = \boxed{(A+\overline{B}+C+\overline{D})(\overline{A}+\overline{B}+C+\overline{D})}$$

$$(A+\overline{B}+C)(\overline{B}+C+\overline{D})\boxed{(A+\overline{B}+\overline{C}+D)} =$$

$$\boxed{(A+\overline{B}+C+D)(A+\overline{B}+C+\overline{D})}\boxed{(A+\overline{B}+C+\overline{D})(\overline{A}+\overline{B}+C+\overline{D})}\boxed{(A+\overline{B}+\overline{C}+D)}$$

Product-Of-Maxterm (POM)

- Product-Of-Maxterm (POM) canonical form:
  Product of maxterms of entries that evaluate to '**0**'

x	y	z	F	Maxterm
0	0	0	1	
0	0	1	1	
0	1	0	**0**	$M_2 = (x + \overline{y} + z)$
0	1	1	1	
1	0	0	**0**	$M_4 = (\overline{x} + y + z)$
1	0	1	1	
1	1	0	**0**	$M_6 = (\overline{x} + \overline{y} + z)$
1	1	1	1	

Focus on the '**0**' entries

$$F = M_2 \cdot M_4 \cdot M_6 = \prod (2, 4, 6) = (x+\overline{y}+z)\ (\overline{x}+y+z)\ (\overline{x}+\overline{y}+z)$$

Product-Of-Maxterm Examples

- $F(a,b,c,d) = \prod(1,3,6,11)$
- $F(a,b,c,d) = \mathbf{M}_1 \cdot \mathbf{M}_3 \cdot \mathbf{M}_6 \cdot \mathbf{M}_{11}$

  $(a+b+c+\bar{d})\,(a+b+\bar{c}+\bar{d})\,(a+\bar{b}+\bar{c}+d)\,(\bar{a}+b+\bar{c}+\bar{d})$
- $G(a,b,c,d) = \prod(0,4,12,15)$
- $G(a,b,c,d) = \mathbf{M}_0 \cdot \mathbf{M}_4 \cdot \mathbf{M}_{12} \cdot \mathbf{M}_{15}$

  $(a+b+c+d)\,(a+\bar{b}+c+d)\,(\bar{a}+\bar{b}+c+d)\,(\bar{a}+\bar{b}+\bar{c}+\bar{d})$

Converting to Sum-of-Minterms Form

- A function that is not in the Sum-of-Minterms form can be converted to that form by means of a truth table
- Consider $F = \overline{y} + \overline{x}\,\overline{z}$

x	y	z	F	Minterm
0	0	0	**1**	$m_0 = \overline{x}\,\overline{y}\,\overline{z}$
0	0	1	**1**	$m_1 = \overline{x}\,\overline{y}\,z$
0	1	0	**1**	$m_2 = \overline{x}\,y\,\overline{z}$
0	1	1	0	
1	0	0	**1**	$m_4 = x\,\overline{y}\,\overline{z}$
1	0	1	**1**	$m_5 = x\,\overline{y}\,z$
1	1	0	0	
1	1	1	0	

$$F = \sum(0, 1, 2, 4, 5) =$$

$$m_0 + m_1 + m_2 + m_4 + m_5 =$$

$$\overline{x}\,\overline{y}\,\overline{z} + \overline{x}\,\overline{y}\,z + \overline{x}\,y\,\overline{z} +$$

$$x\,\overline{y}\,\overline{z} + x\,\overline{y}\,z$$

Converting to Product-of-Maxterms Form

- A function that is not in the Product-of-Minterms form can be converted to that form by means of a truth table
- Consider again: $F = \bar{y} + \bar{x}\,\bar{z}$

x	y	z	F	Minterm
0	0	0	1	
0	0	1	1	
0	1	0	1	
0	1	1	**0**	$M_3 = (x+\bar{y}+\bar{z})$
1	0	0	1	
1	0	1	1	
1	1	0	**0**	$M_6 = (\bar{x}+\bar{y}+z)$
1	1	1	**0**	$M_7 = (\bar{x}+\bar{y}+\bar{z})$

$$F = \prod(3, 6, 7) =$$

$$M_3 \cdot M_6 \cdot M_7 =$$

$$(x+\bar{y}+\bar{z})\ (\bar{x}+\bar{y}+z)\ (\bar{x}+\bar{y}+\bar{z})$$

Conversions Between Canonical Forms

x	y	z	F	Minterm	Maxterm
0	0	0	0		$M_0 = (x + y + z)$
0	0	1	1	$m_1 = \bar{x}\,\bar{y}\,z$	
0	1	0	1	$m_2 = \bar{x}\,y\,\bar{z}$	
0	1	1	1	$m_3 = \bar{x}\,y\,z$	
1	0	0	0		$M_4 = (\bar{x} + y + z)$
1	0	1	1	$m_5 = x\,\bar{y}\,z$	
1	1	0	0		$M_6 = (\bar{x} + \bar{y} + z)$
1	1	1	1	$m_7 = x\,y\,z$	

$$F = m_1 + m_2 + m_3 + m_5 + m_7 = \sum(1, 2, 3, 5, 7) =$$

$$\bar{x}\,\bar{y}\,z + \bar{x}\,y\,\bar{z} + \bar{x}\,y\,z + x\,\bar{y}\,z + x\,y\,z$$

$$F = M_0 \cdot M_4 \cdot M_6 = \prod(0, 4, 6) = (x+y+z)(\bar{x}+y+z)(\bar{x}+\bar{y}+z)$$

Standard Sum-of-Products (SOP)

- A Simplification Example:

  $F(A,B,C) = \sum(1,4,5,6,7)$

- Writing the minterm expression:

  $F = \overline{A}\,\overline{B}\,C + A\,\overline{B}\,\overline{C} + A\,\overline{B}\,C + AB\overline{C} + ABC$

- Simplifying:

  $F = \overline{A}\,\overline{B}\,C + A\,(\overline{B}\,\overline{C} + \overline{B}\,C + B\,\overline{C} + B\,C)$

  $F = \overline{A}\,\overline{B}\,C + A\,(\overline{B}\,(\overline{C} + C) + B\,(\overline{C} + C))$

  $F = \overline{A}\,\overline{B}\,C + A\,(\overline{B} + B)$

  $F = \overline{A}\,\overline{B}\,C + A$

  $F = \overline{B}\,C + A$

- Simplified F contains 3 literals compared to 15

Three-way light control

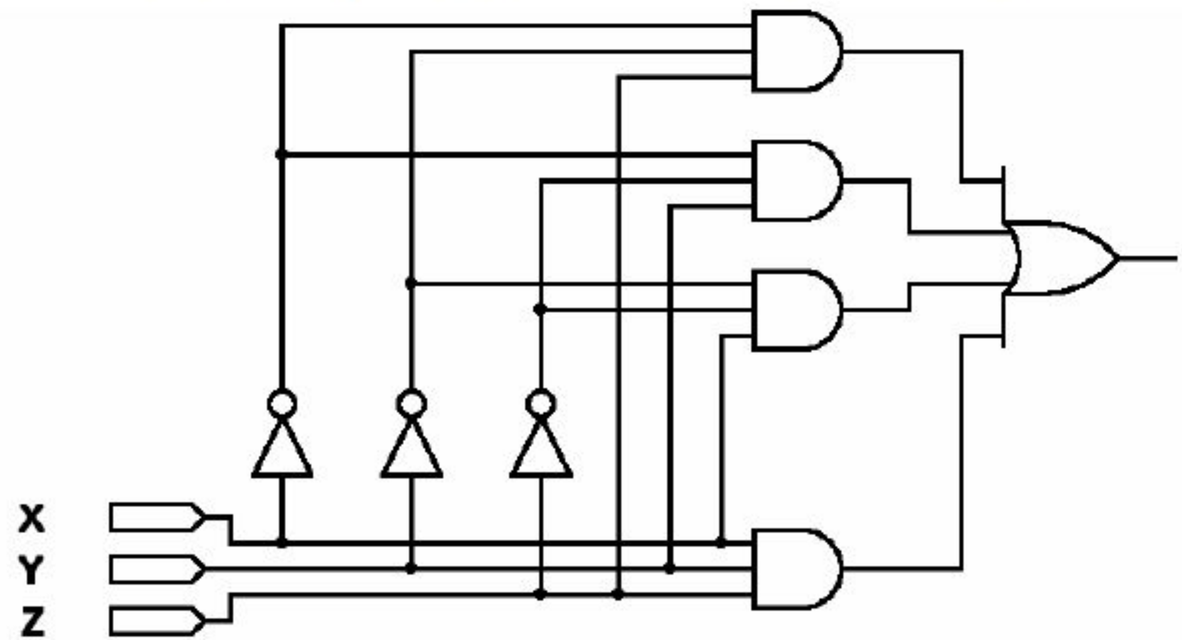
- Assume a room has three doors and a switch by each door controls a single light in the room.
  - Let x, y, and z denote the state of the switches
  - Assume the light is off if all switches are open
  - Closing any switch turns the light on. Closing another switch will have to turn the light off.
  - Light is on if any one switch is closed and off if two (or no) switches are closed.
  - Light is on if all three switches are closed

Three-way light control

x	y	z	f
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

$$f(x,y,z)=m_1+m_2+m_4+m_7$$
$$f(x,y,z)=x'y'z+x'yz'+xy'z'+xyz$$

This is the simplest sum-of-products form.



Car safety alarm

- Design a car safety alarm considering four inputs
  - Door closed (D)
  - Key in (K)
  - Seat pressure (S)
  - Seat belt closed (B)
- The alarm (A) should sound if
  - The key is in and the door is not closed, or
  - The door is closed and the key is in and the driver is in the seat and the seat belt is not closed

Car safety alarm

D	K	S	B	A
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	1
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	1
1	1	1	1	0

$A(D,K,S,B)=\Sigma m(4,5,6,7,14)$

$A(D,K,S,B)=D'KS'B'+D'KS'B+D'KSB'+D'KSB+DKSB'$

$=D'KS'+D'KS+KSB'$

$=D'K+KSB'$