LESSON 7

MOHR CIRCLES.EQUIVALENT STRESS DEFINITION AND COMPUTATION

SUBMITTED BY ASSISTANT TEACHER
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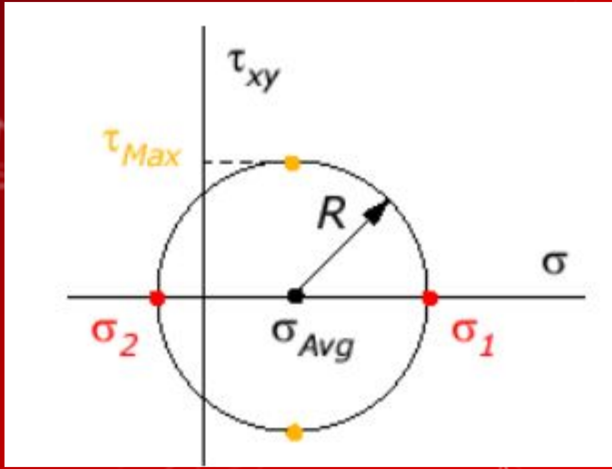
MOHR'S CIRCLE

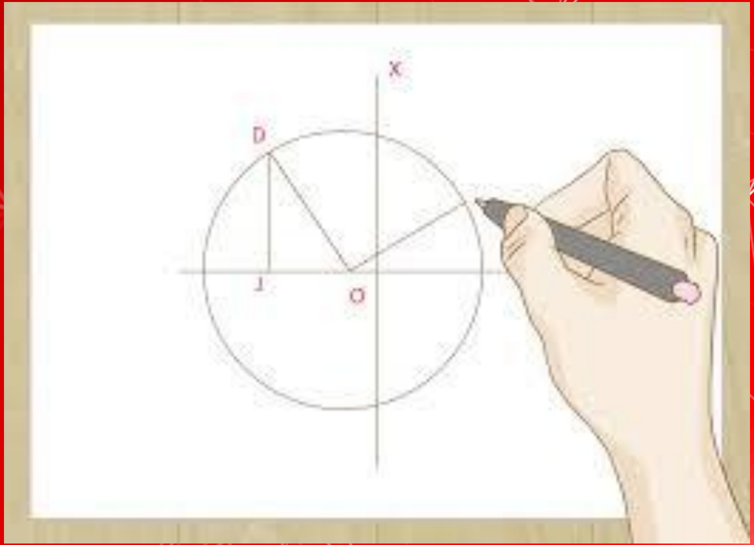
Introduced by Otto Mohr in 1882, Mohr's Circle illustrates principal stresses and stress transformations via a graphical format,



The two principal stresses are shown in **red**, and the maximum shear stress is shown in **orange**. Recall that the normal stesses equal the principal stresses when the stress element is aligned with the principal directions, and the shear stress equals the maximum shear stress when the stress element is rotated 45° away from the principal directions.

As the stress element is rotated away from the principal (or maximum shear) directions, the normal and shear stress components will always lie on Mohr's Circle.

Mohr's Circle was the leading tool used to visualize relationships between normal and shear stresses, and to estimate the maximum stresses, before hand-held calculators became popular. Even today, Mohr's Circle is still widely used by engineers all over the world.



DERIVATION OF MOHR'S CIRCLE

To establish Mohr's Circle, we first recall the stress transformation formulas for plane stress at a given location,

$$\begin{cases} \sigma_{x'} - \dfrac{\sigma_x + \sigma_y}{2} = \dfrac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \\ \tau_{x'y'} = -\dfrac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta \end{cases}$$

Using a basic trigonometric relation ($\cos^2 2\theta + \sin^2 2\theta = 1$) to combine the two above equations we have,

$$\left(\sigma_{x'} - \frac{\sigma_x + \sigma_y}{2}\right)^2 + \tau_{x'y'}{}^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2$$

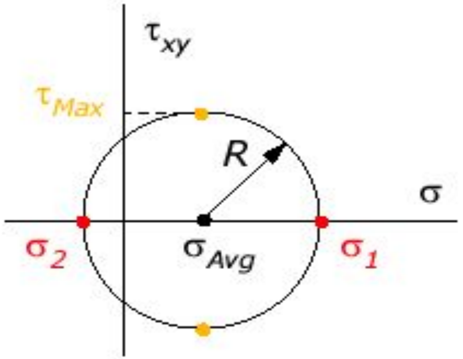
This is the equation of a circle, plotted on a graph where the abscissa is the normal stress and the ordinate is the shear stress. This is easier to see if we interpret σ_x and σ_y as being the two principal stresses, and τ_{xy} as being the maximum shear stress. Then we can define the average stress, σ_{avg}, and a "radius" R (which is just equal to the maximum shear stress),

$$\sigma_{\mathrm{Avg}} = \frac{\sigma_x + \sigma_y}{2} \qquad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

The circle equation above now takes on a more familiar form,

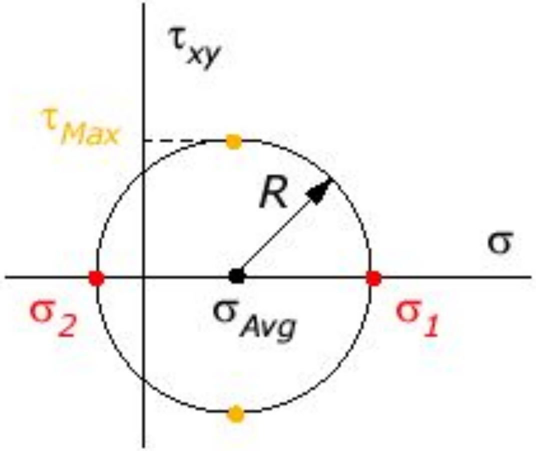
$$\left(\sigma_{x'} - \sigma_{\mathrm{Avg}}\right)^2 + \tau_{x'y'}{}^2 = R^2$$

The circle is centered at the average stress value, and has a radius R equal to the maximum shear stress, as shown in the figure below,



PRINCIPAL STRESSES FROM MOHR'S CIRCLE

A chief benefit of Mohr's circle is that the principal stresses σ_1 and σ_2 and the maximum shear stress τ_{max} are obtained immediately after drawing the circle,



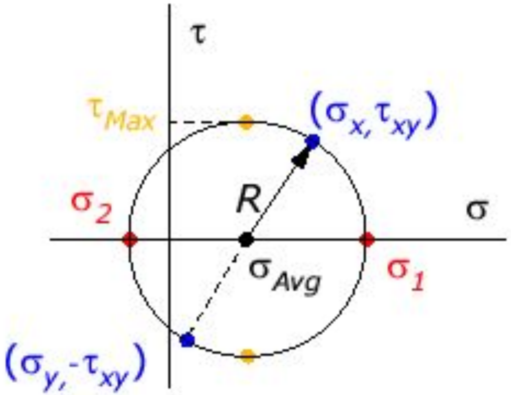
$$\begin{cases} \sigma_{1,2} = \sigma_{\text{Avg}} \pm R \\ \tau_{\text{Max}} = R \end{cases}$$

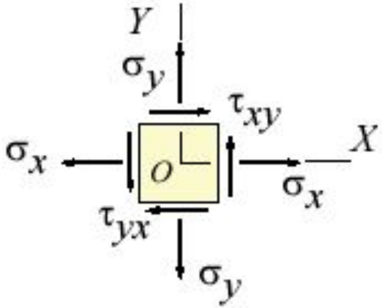
where,

$$\sigma_{\text{Avg}} = \frac{\sigma_x + \sigma_y}{2} \qquad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^{\ 2}}$$

PRINCIPAL DIRECTIONS FROM MOHR'S CIRCLE

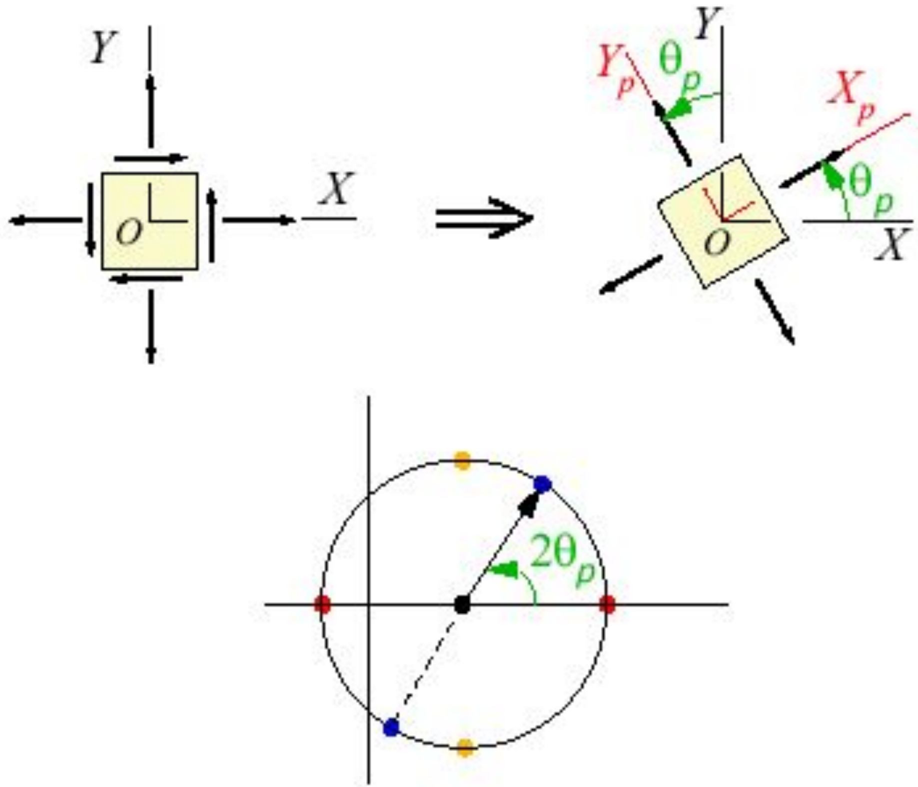
Mohr's Circle can be used to find the directions of the principal axes. To show this, first suppose that the normal and shear stresses, σ_x, σ_y, and τ_{xy}, are obtained at a given point *O* in the body. They are expressed relative to the coordinates *XY*, as shown in the stress element at right below.





The Mohr's Circle for this general stress state is shown at left above. Note that it's centered at σ_{avg} and has a radius *R*, and that the two points $\{\sigma_x, \tau_{xy}\}$ and $\{\sigma_y, -\tau_{xy}\}$ lie on opposites sides of the circle. The line connecting σ_x and σ_y will be defined as L_{xy}.

The **angle** between the current axes (*X* and *Y*) and the **principal axes** is defined as **θ_p**, and is equal to one half the angle between the line L_{xy} and the σ-axis as shown in the schematic below,



ROTATION ANGLE ON MOHR'S CIRCLE

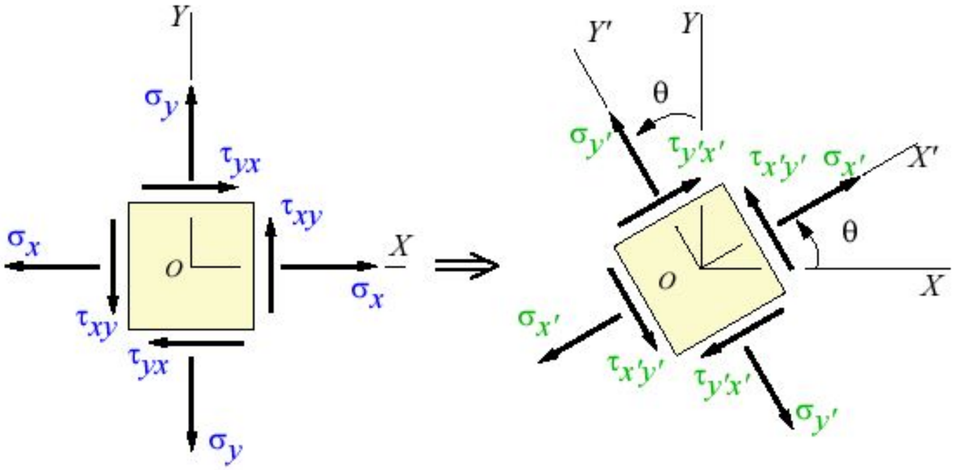
Note that the coordinate rotation angle θ_p is defined positive when starting at the *XY* coordinates and proceeding to the X_pY_p coordinates. In contrast, on the Mohr's Circle θ_p is defined positive starting on the principal stress line (i.e. the σ-axis) and proceeding to the *XY* stress line (i.e. line L_{xy}). The angle θ_p has the opposite sense between the two figures, because on one it starts on the *XY* coordinates, and on the other it starts on the principal coordinates.

Some books avoid this dichotomy between θ_p on Mohr's Circle and θ_p on the stress element by locating (σ_x, $-\tau_{xy}$) instead of (σ_x, τ_{xy}) on Mohr's Circle. This will switch the polarity of θ_p on the circle. Whatever method you choose, the bottom line is that an *opposite* sign is needed either in the interpretation or in the plotting to make Mohr's Circle physically meaningful.

STRESS TRANSFORM BY MOHR'S CIRCLE

Mohr's Circle can be used to transform stresses from one coordinate set to another, similar that that described on the plane stress page.

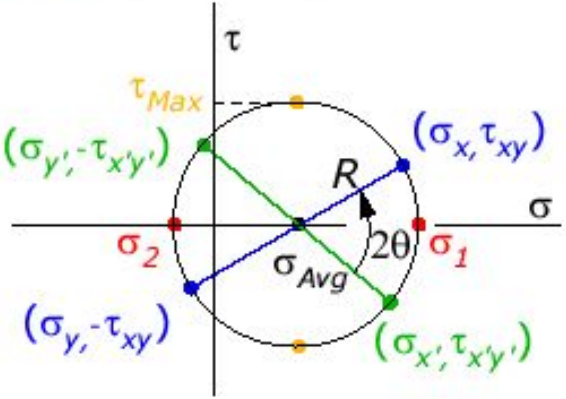
Suppose that the normal and shear stresses, σ_x, σ_y, and τ_{xy}, are obtained at a point *O* in the body, expressed with respect to the coordinates *XY*. We wish to find the stresses expressed in the new coordinate set *X'Y'*, rotated an angle θ from *XY*, as shown below:



Stresses at given coordinate system   Stresses transformed to another coordinate

To do this we proceed as follows:

- Draw Mohr's circle for the **given stress state** (σ_x, σ_y, and τ_{xy}; shown below).
- Draw the line L_{xy} across the circle from (σ_x, τ_{xy}) to (σ_y, $-\tau_{xy}$).
- Rotate the line L_{xy} by 2*θ (twice as much as the angle between *XY* and *X'Y'*) and in the *opposite* direction of θ.
- The **stresses in the new coordinates** ($\sigma_{x'}$, $\sigma_{y'}$, and $\tau_{x'y'}$) are then read off the circle.



THANKS!

Any questions?

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