G11.4A- Statistics: Standard Deviation, Error and *t*-test

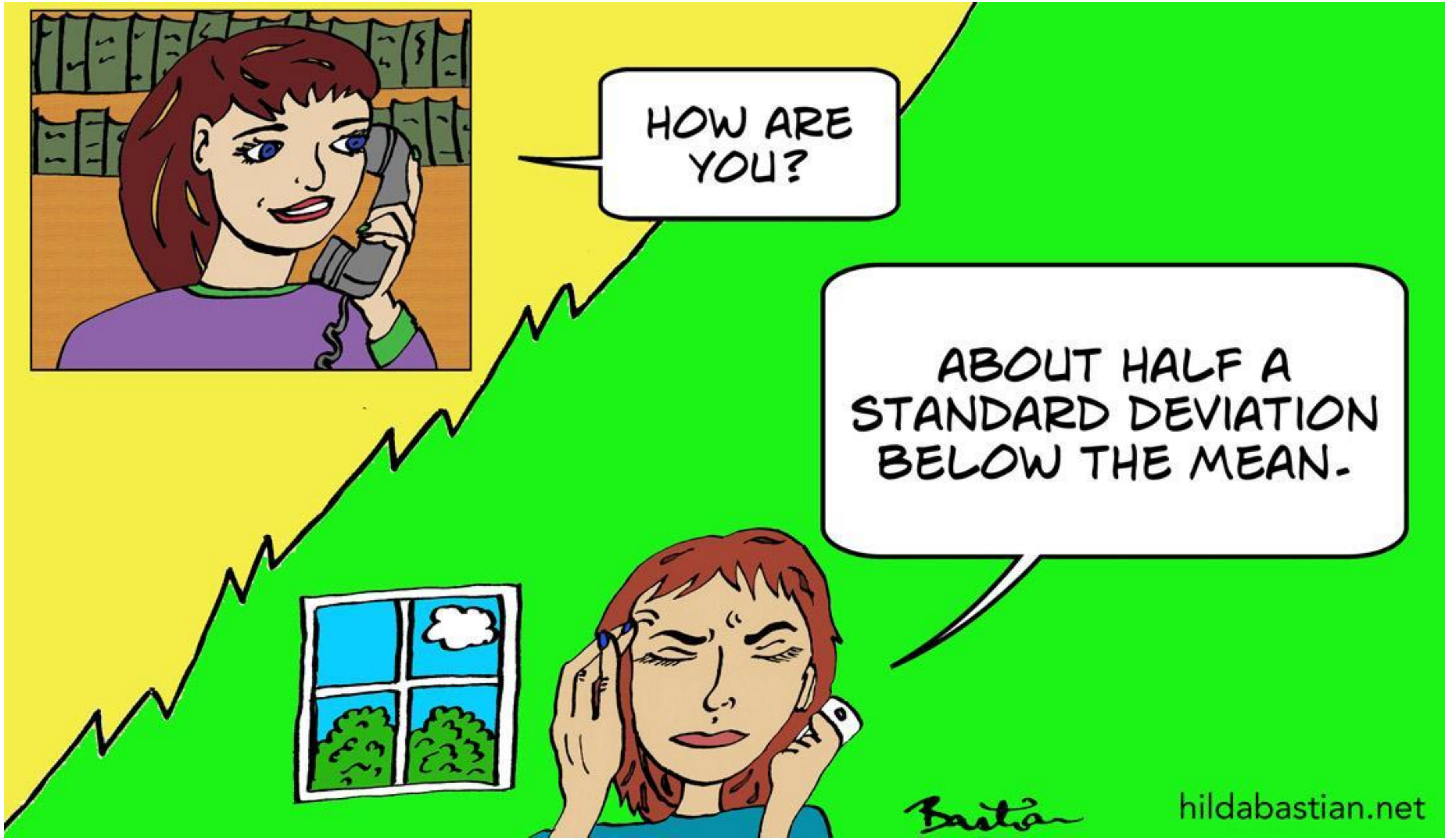
CIE Biology Jones
P497-500

Learning Objectives

11.2.4.11 explore patterns of modification variability

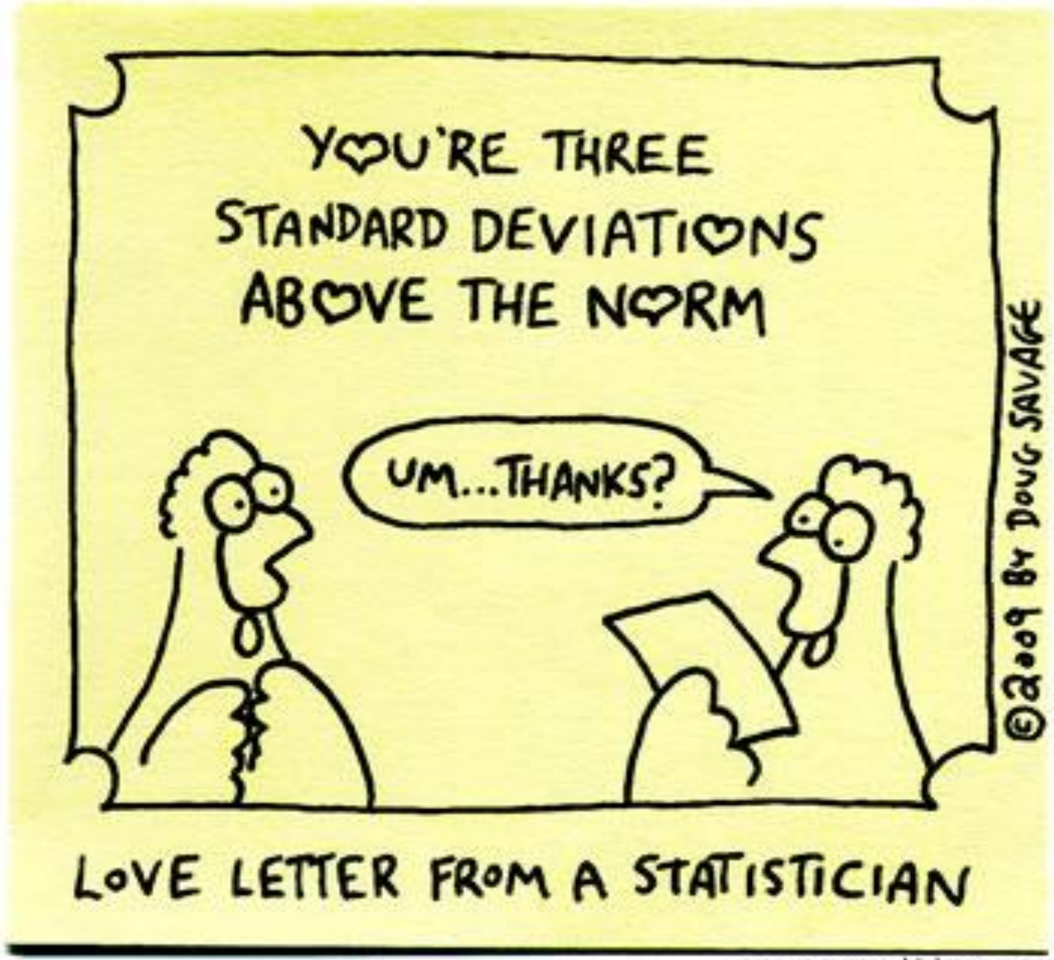
Success Criteria

1. Calculate standard deviation and error of data.
2. Explain results of standard deviation and error bars.

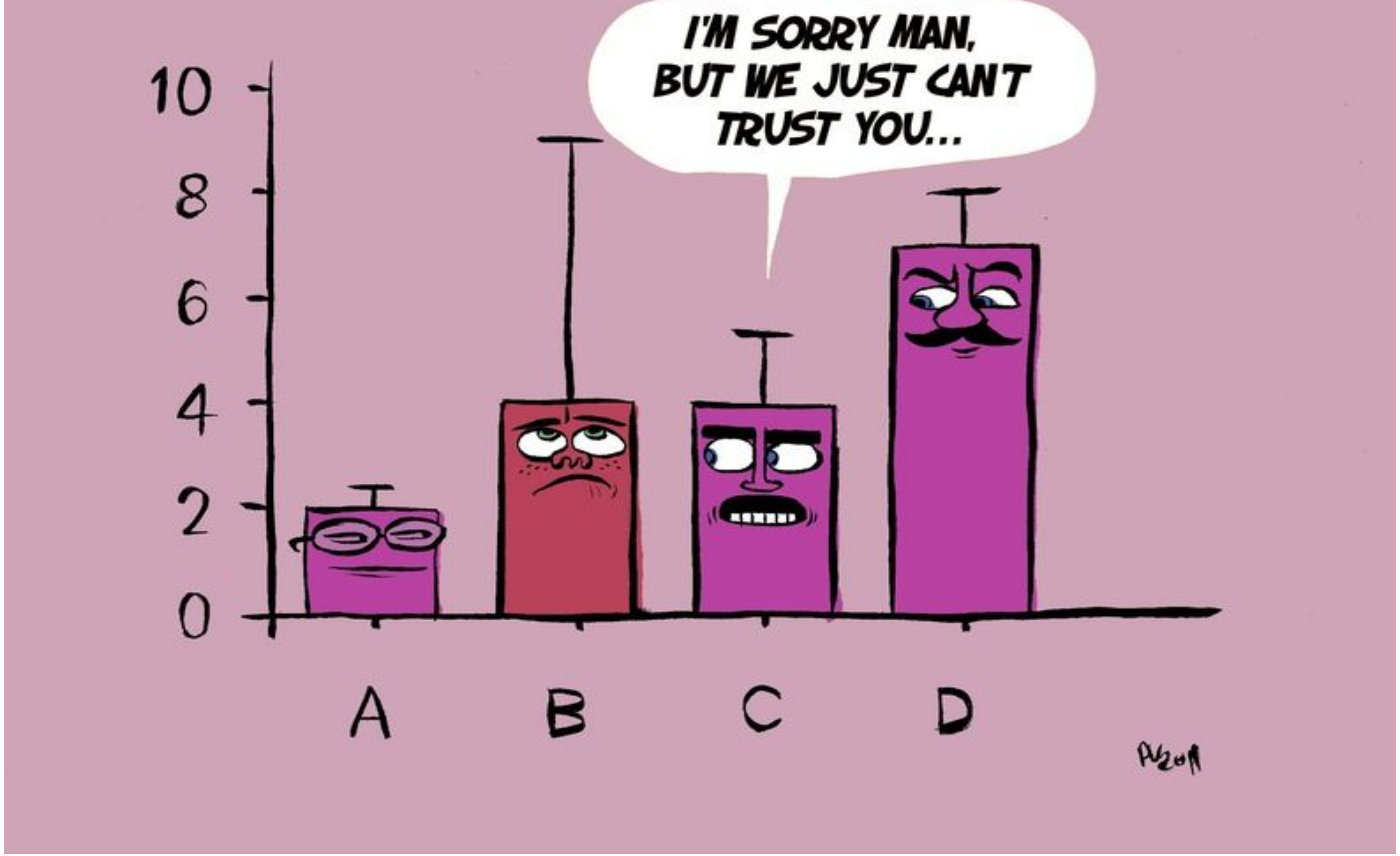

HOW ARE YOU?
ABOUT HALF A STANDARD DEVIATION BELOW THE MEAN.
hildabastian.net

Savage Chickens

by Doug Savage



www.savagechickens.com


I'M SORRY MAN,
BUT WE JUST CAN'T
TRUST YOU...
10
8
6
4
2
0
A
B
C
D

Standard Deviation

Sample Standard Deviation: $s = \sqrt{\frac{\sum(x-\bar{x})^2}{n-1}}$

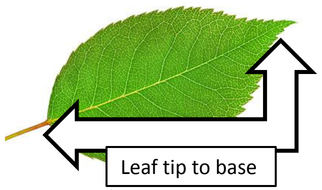
Population Standard Deviation: $\sigma = \sqrt{\frac{\sum(x-\bar{x})^2}{n}}$

$$S = \sqrt{\frac{\sum (x_i - \bar{x})}{N - 1}}$$

$$\sigma = \sqrt{\frac{\sum (x_i - \mu)^2}{N}}$$

Do you remember? Ecology Practical Term 1?

T-test and Standard Deviation?



$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$$s = \sqrt{\frac{\sum(x - \bar{x})^2}{n - 1}}$$

TASK 3 – Leaves on a Tree and t-test data analysis.

Use the t-test to compare the means of two samples of measured data to see whether they are significantly different. The t-test will tell you whether there is a significant difference between the means of two samples or not.

Leaf tip to base

1 mean of sample 1
2 mean of sample 2
n_1 is number subjects in sample 1
n_2 is number subjects in sample 2
s_1 is the standard deviation of sample 1
s_2 is the standard deviation of sample 2

1 mean of sample 1
2 mean of sample 2
n_1 is number subjects in sample 1
n_2 is number subjects in sample 2
s_1 is the standard deviation of sample 1
s_2 is the standard deviation of sample 2

1. Go to one of the trees as directed by your teacher.

2. Select one branch. Measure the leaf length from tip to base, and record the length of 10 leaves on the outer portion of the branch, and 10 leaves on the inner portion of the branch. <u>T-test calculations will be done in the classroom.</u>

The *t*-test

Use this test when
- you wish to find out if there is a significant difference between two means
- the data are normally distributed
- the sample size is between **10-30**

Measurements

The investigation involved **taking measurements**

t can be calculated from the formula

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

Where $\bar{x}_1$ = mean of first sample

$\bar{x}_2$ = mean of second sample

s_1 = standard deviation of first sample

s_2 = standard deviation of second sample

n_1 = number of measurements in first sample

n_2 = number of measurements in second sample

Standard deviation formula is also needed to solve for *t*-test

$$s = \sqrt{\frac{\sum(x - \bar{x})^2}{n - 1}}$$

A table showing the critical values of t for different degrees of freedom.

Degrees of freedom | **Critical Value**

Degrees of freedom	Critical value	Degrees of freedom	Critical value
4	2.78		
5	2.57	15	2.13
6	2.48	16	2.12
7	2.37	18	2.10
8	2.31	20	2.09
9	2.26	22	2.07
10	2.23	24	2.06
11	2.20	26	2.06
12	2.18	28	2.05
13	2.16	30	2.04
14	2.15	40	2.02

The number of degrees of freedom = $(n_1 + n_2) - 2$

T-test - Is there significant difference between two means?

Set up a chart that will help you solve for **s** and **t**.

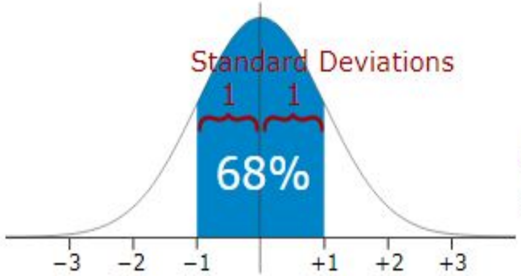
Sample Number	Sample X_1	$X_1 - \overline{X}_1$	$(X_1 - \overline{X}_1)^2$	Sample X_2	$X_2 - \overline{X}_2$	$(X_2 - \overline{X}_2)^2$
1						
2						
3						
4						
5						
6						
7						
8						
9						
10						
	$\overline{X}_1 =$		$\sum =$	$\overline{X}_2 =$		$\sum =$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

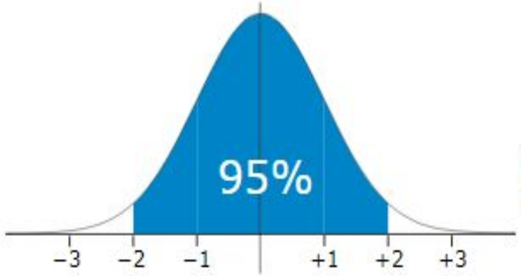
$$s = \sqrt{\frac{\sum (x - \bar{x})^2}{n - 1}}$$

Standard Deviation A graphical expression of the distance between numbers.

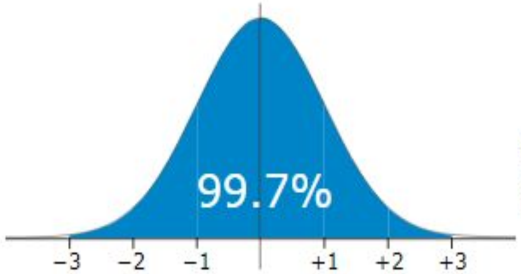
When we calculate the standard deviation we find that (generally):



68% of values are within
1 standard deviation of the mean



95% of values are within
2 standard deviations of the mean



99.7% of values are within
3 standard deviations of the mean

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$$s = \sqrt{\frac{\sum (x - \bar{x})^2}{n - 1}}$$

Standard deviation will give the **+/- error value** of a measurement.

	Female	Male
mean	21	10
SD	3	1.6
2 x SD	6	3.2
Mean + (2 x SD)	27	13
Mean - (2 x SD)	15	7

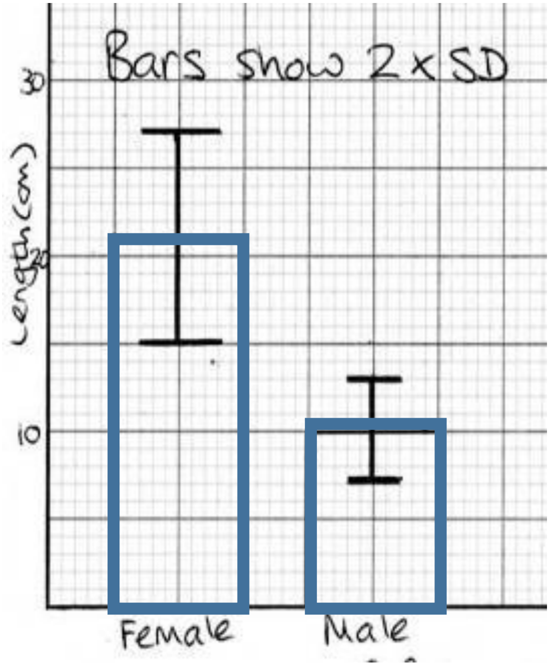
There is no overlap in the (±2 SD) bars.

This indicates that the differences in the means the size of male and females is **unlikely** to be due to chance.

Describing the results

We can draw a bar chart of the mean and plot the ± 2 Standard deviations from the mean and look at the **overlap of the bars.**

If there is NO overlap of the error bars overlap, then there is IS significant difference between the two samples.



Error Bars +/-

Note: You cannot say how 'unlikely' this is due to chance – just that it is **unlikely**!

	DATA A (mm)	DATA B (mm)
mean	32.3	48.8
SD	9.3	1.7
2 x SD	18.6	2.4
Mean + (2 x SD)	50.9	51.2
Mean - (2 x SD)	13.7	46.4

There is an overlap in the (±2 SD) bars.

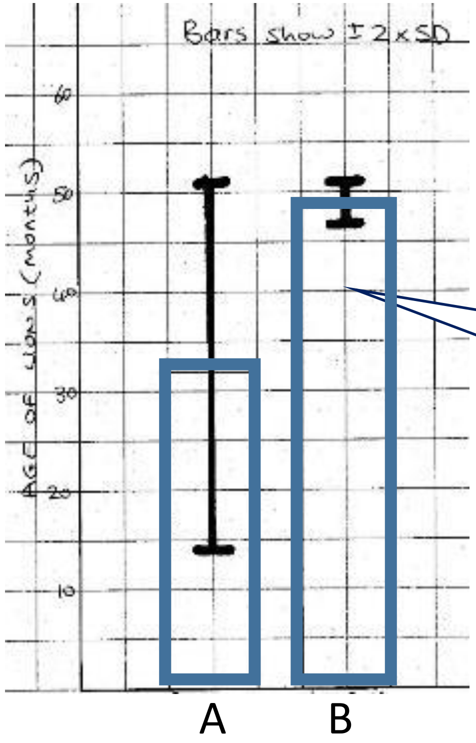
This indicates that the differences in the means between A and B are **likely** to be due to chance.

Describing the results

We can draw a bar chart of the mean and plot the **± 2** Standard deviations from the mean and look at the

overlap of the bars.

If error bars overlap, then there is **no significant difference** between the two samples.



Note: You cannot say how 'likely' this is due to chance – just that it is **likely**!

Alternative Hypothesis vs Null Hypothesis

Alternative Hypothesis (a hypothesis you want to prove)	Null hypothesis
If the flowers are counted there will be only yellow flowers.	There will be no significant difference between the expected flower color and the observed flower color.
If leaves are exposed to more sunlight, then they will be larger than leaves that receive less sunlight.	There will be no significant difference in the size of the leaves.

Acceptance or Rejection of the of the Null Hypothesis

Our calculated value of t is less than the critical value of t.

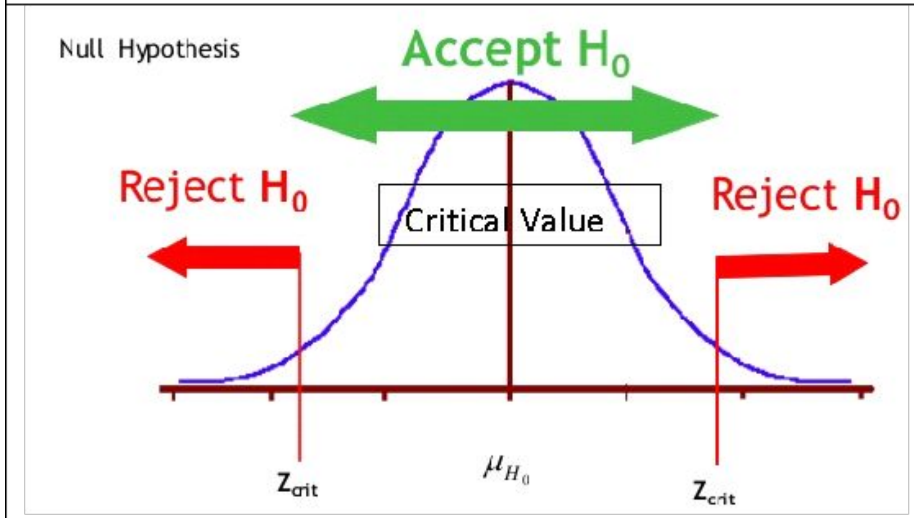
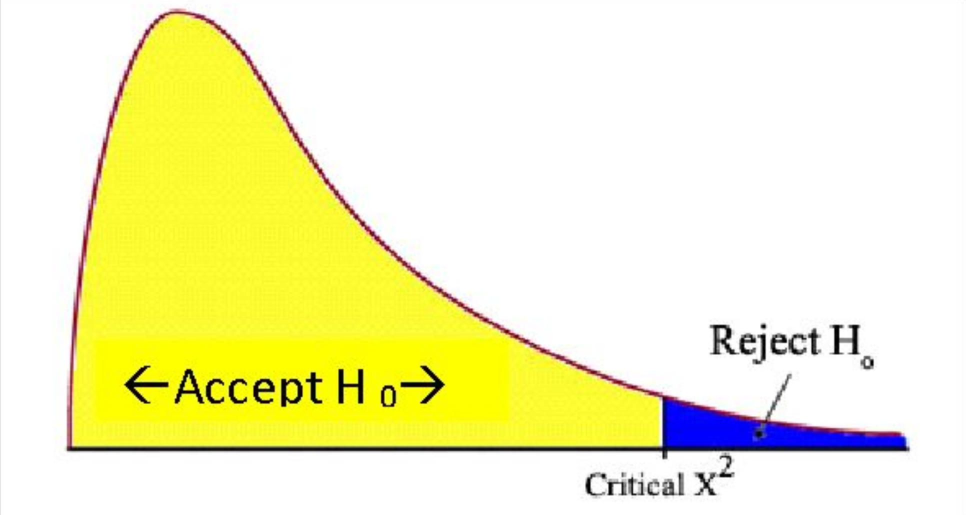
There is more than 5% probability that the differences in the means (mean of **A** and the mean of **B**) **are** due to chance.

We **accept** our null hypothesis.

Our calculated value of Chi-squared is much larger than the critical value of Chi-squared.

There is less than 5% probability that the differences (between the observed and expected data) **are** due to chance.

We **reject** our null hypothesis.

H_0 = Null Hypothesis	
t-test t > critical value reject H_0 t < critical value accept H_0	**Chi Square** χ^2 > critical value reject H_0 χ^2 < critical value accept H_0
	
Our calculated value of t is **greater than** the critical value of t. There is **more than 5% probability** that the differences in the means (mean mass of A and B) **are not** due to chance. We **reject** our null hypothesis.	Our calculated value of Chi-squared is much **larger** than the critical value of Chi-squared. There is **less than 5% probability** that the differences (between the observed and expected data) are due to chance. We **reject** our null hypothesis.
Our calculated value of t is **less than** the critical value of t. There is **more than 5% probability** that the differences in the means (mean mass A and B) **are** due to chance. We **accept** our null hypothesis.	Our calculated value of Chi-squared is **smaller** than the critical value of Chi-squared. There is **more than 5% probability** that the differences (between the observed and expected data) are due to chance. We **accept** our null hypothesis.

Statistical test	When to use it	Criteria for using the test	Examples of use	How to interpret the value you calculate
t-test	You want to know if two sets of continuous data are significantly different from one another.	• You have two sets of continuous, quantitative data (page 494). • You have more than 10 but less than 30 readings for each set of data. • Both sets of data come from populations that have normal distributions. • The standard deviations for the two sets of data are very similar.	Are the surface areas of the leaves on the north-facing side of a tree significantly different from the surface areas on the south-facing side? Are the reaction times of students who have drunk a caffeine-containing drink significantly different from students who have drunk water?	Use a *t*-test table to look up your value of *t*. If this value is greater than the *t* value for a probability of 0.05 (the critical value), then you can say that your two populations are significantly different.
χ^2 test	You want to know if your observed results differ significantly from your expected results.	• You have two or more sets of quantitative data, which belong to two or more discontinuous categories (i.e. they are nominal data – page 494)	Are the numbers of offspring of different phenotypes obtained in a genetic cross significantly different from the expected numbers?	Use a χ^2 table to look up your value of χ^2. If this value is greater than the χ^2 value for a probability of 0.05, then you can say that your observed results differ significantly from your expected results.

The formula for calculating standard deviation is:

$$s = \sqrt{\frac{\sum(x - \bar{x})^2}{n - 1}}$$

where:

- $\bar{x}$ is the mean
- $\sum$ stands for 'sum of'
- x refers to the individual values in a set of data
- n is the total number of observations (individual values, readings or measurements) in one set of data
- s is standard deviation
- $\sqrt{}$ is the symbol for square root

Woodland vs garden petals

Standard deviation

A useful statistic to know about data that have an approximately normal distribution is how far they spread out on either side of the mean value. This is called the **standard deviation**. The larger the standard deviation, the wider the variation from the mean (Figure P2.6).

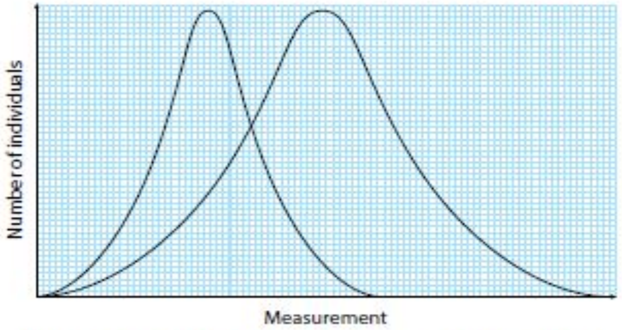


Figure P2.6 Normal distribution curves with small and large standard deviations.

A student measured the length of 21 petals from flowers of a population of a species of plant growing in woodland. These were the results:

Woodland

Petal lengths in woodland population / mm

3.1	3.2	2.7	3.1	3.0	3.2	3.3
3.1	3.1	3.3	3.3	3.2	3.2	3.3
3.2	2.9	3.4	2.9	3.0	2.9	3.2

Standard deviation formula

The formula for calculating standard deviation is:

$$s = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n-1}}$$

where:

$\bar{x}$ is the mean
Σ stands for 'sum of'
x refers to the individual values in a set of data
n is the total number of observations (individual values, readings or measurements) in one set of data
s is standard deviation
$\sqrt{\ }$ is the symbol for square root

You may have a calculator that can do all the hard work for you – you just key in the individual values and it will calculate the standard deviation. However, you do need to know how to do the calculation yourself. The best way is to set your data out in a table, and work through it step by step.

1 List the measurement for each petal in the first column of a table like Table P2.1.
2 Calculate the mean for the petal length by adding all the measurements and dividing this total by the number of measurements.
3 Calculate the difference from the mean for each observation. This is $(x - \bar{x})$.
4 Calculate the squares of each of these differences from the mean. This is $(x - \bar{x})^2$.
5 Calculate the sum of the squares. This is $\Sigma(x - \bar{x})^2$.
6 Divide the sum of the squares by $n - 1$.
7 Find the square root of this. The result is the standard deviation, s, for that data set.

Table P2.1 shows the calculation of the standard deviation for petals from plants in the woodland.

garden

QUESTION

P2.5 The student measured the petal length from a second population of the same species of plant, this time growing in a garden. These are the results:

Petal lengths in garden population / mm

2.8	3.1	2.9	3.2	2.9	2.7	3.0
2.8	2.9	3.0	3.2	3.1	3.0	3.2
3.0	3.1	3.3	3.2	2.9		

Show that the standard deviation for this set of data is 0.16.

Calculating Standard Deviation

Step 1: Find the mean.

Step 2: For each data point, find the square of its distance to the mean.

Step 3: Sum the values from Step 2.

Step 4: Divide by the number of data points.

Step 5: Take the square root

3 Calculate the difference between each observation and the mean, $x - \bar{x}$.

1 List each observation, x.

x	$(x - \bar{x})$	$(x - \bar{x})^2$
3.1	−0.02	0.001
3.2	0.08	0.006
2.7	−0.42	0.180
3.1	−0.02	0.001
3.0	−0.12	0.015
3.2	0.08	0.006
3.3	0.18	0.031
3.1	−0.02	0.001
3.1	−0.02	0.001
3.3	0.18	0.031
3.3	0.18	0.031
3.2	0.08	0.006
3.2	0.08	0.006
3.3	0.18	0.031
3.2	0.08	0.006
2.9	−0.22	0.050
3.4	0.28	0.076
2.9	−0.22	0.050
3.0	−0.12	0.015
2.9	−0.22	0.050
3.2	0.08	0.006
Σx = 65.6		$\Sigma(x - \bar{x})^2$ = 0.600
n = 21		$n - 1$ = 20
$\bar{x}$ = 3.12		$\frac{\Sigma(x-\bar{x})^2}{n-1}$ = 0.03
		s = 0.17

2 Calculate the mean, $\bar{x}$.

4 Calculate the square of each difference, $(x - \bar{x})^2$.

5 Calculate the sum of the squares of each difference, $\Sigma(x - \bar{x})^2$.

6 Divide the sum of the squares by $n - 1$.

7 Find the square root. This is the standard deviation.

Table P2.1 Calculation of standard deviation for petal length in a sample of plants from woodland. All lengths are in mm.

Calculate Standard Error

Standard error

The 21 petals measured were just a sample of all the thousands of petals on the plants in the wood and in the garden. If we took another sample, would we get the same value for the mean petal length? We cannot be certain without actually doing this, but there is a calculation that we can do to give us a good idea of how close our mean value is to the true mean value for all of the petals in the wood. The calculation works out the **standard error** (S_M) for our data.

Once you have worked out the standard deviation, s, then the standard error is very easy to calculate. The formula is:

$$S_M = \frac{s}{\sqrt{n}}$$

where S_M = standard error

s = standard deviation

n = the sample size (in this case, the number of petals in the sample)

So, for the petals in woodland:

$$S_M = \frac{0.17}{\sqrt{21}} = \frac{0.17}{4.58} = 0.04$$

What does this value tell us?

The standard error tells us how certain we can be that our mean value is the true mean for the population that we have sampled.

We can be 95% certain that – if we took a second sample from the same population – the mean for that second sample would lie within 2 × our value of S_M from the mean for our first sample.

So here, we can be 95% certain that the mean petal length of a second sample would lie within 2 × 0.04 mm of our mean value for the first sample.

QUESTION

P2.6 Show that the standard error for the lengths of the sample of petals taken from the garden (Question P2.5) is also 0.04. Show each step in your working.

Error bars

The standard error can be used to draw error bars on a graph. Figure P2.7 shows the means for the two groups of petals, plotted on a bar chart.

The bars drawn through the tops of the plotted bars are called error bars.

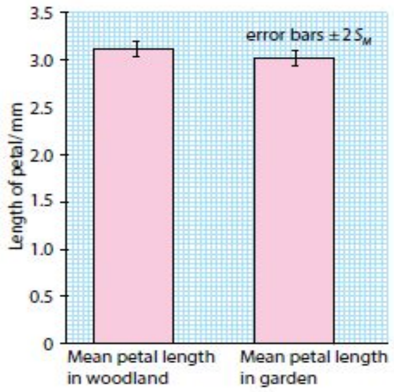


Figure P2.7 Mean petal length of plants in woodland and garden.

QUESTION

P2.7 From the data in Figure P2.7, is there strong evidence that the lengths of the petals in the woodland are significantly different from the lengths of the petals in the garden? Explain your answer.

If we draw an error bar that extends two standard errors above the mean and two standard errors below it, then we can be 95% certain that the true value of the mean lies within this range.

We can use these error bars to help us to decide whether or not there is a significant difference between the petal length in the woodland and the garden. If the error bars overlap, then the difference between the two groups is definitely not significant. If the error bars do not overlap, we still cannot be sure that the difference is significant – but at least we know it is possible that it is. You can also add error bars to line graphs, where your individual points represent mean values.

To find out whether the difference is significant, we can do a further statistical calculation, called a t-test.

Calculate *t*-test

The *t*-test

The *t*-test is used to assess whether or not the means of two sets of data with roughly normal distributions, are significantly different from one another.

For this example, we will use data from another investigation.

The corolla (petal) length of two populations of gentian were measured in mm.

Corolla lengths of population A:
13, 16, 15, 12, 18, 13, 13, 16, 19, 15, 18, 15, 15, 17, 15,

Corolla lengths of population B:
16, 14, 16, 18, 13, 17, 19, 20, 17, 15, 16, 16, 19, 21, 18,

The formula for the *t*-test is:

$$t = \frac{|\bar{x}_1 - \bar{x}_2|}{\sqrt{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)}}$$

$\bar{x}_1$ is the mean of sample 1
$\bar{x}_2$ is the mean of sample 2
s_1 is the standard deviation of sample 1
s_2 is the standard deviation of sample 2
n_1 is the number of individual measurements in sample 1
n_2 is the number of individual measurements in sample 2

1. For each set of data, calculate the mean.
2. Calculate the differences from the mean of all observations in each data set. This is $x - \bar{x}$.
3. Calculate the squares of these. This is $(x - \bar{x})^2$.
4. Calculate the sum of the squares. This is $\sum(x - \bar{x})^2$.
5. Divide this by $n_1 - 1$ for the first set and $n_2 - 1$ for the second set.
6. Take the square root of this. The result is the standard deviation for each set of data.
   For population A, $s_1 = 4.24$.
   For population B, $s_2 = 4.86$.
7. Square the standard deviation and divide by the number of observations in that sample, for both samples.
8. Add these values together for the two samples and take the square root of this.
9. Divide the difference in the two sample means with the value from step 8. This is *t* and, in this case, is 1.93.
10. Calculate the total degrees of freedom for all the data (v).
    $v = (n_1 - 1) + (n_2 - 1) = 28$
11. Refer to the table of *t* values for 28 degrees of freedom and a value of $t = 1.93$ (Table P2.2).

Degrees of freedoms for t-test

Degrees of freedom	Value of *t*			
1	6.31	12.7	63.7	63.6
2	2.92	4.30	9.93	31.6
3	2.35	3.18	5.84	12.9
4	2.13	2.78	4.60	8.61
5	2.02	2.57	4.03	6.87
6	1.94	2.45	3.71	5.96
7	1.90	2.37	3.50	5.41
8	1.86	2.31	3.36	5.04
9	1.83	2.26	3.25	4.78
10	1.81	2.23	3.17	4.59
11	1.80	2.20	3.11	4.44
12	1.78	2.18	3.06	4.32
13	1.77	2.16	3.01	4.22
14	1.76	2.15	2.98	4.14
15	1.75	2.13	2.95	4.07
16	1.75	2.12	2.92	4.02
17	1.74	2.11	2.90	3.97
18	1.73	2.10	2.88	3.92
19	1.73	2.09	2.86	3.88
20	1.73	2.09	2.85	3.85
22	1.72	2.07	2.82	3.79
24	1.71	2.06	2.80	3.75
26	1.71	2.06	2.78	3.71
28	1.70	2.05	2.76	3.67
30	1.70	2.04	2.75	3.65
>30	1.64	1.96	2.58	3.29
Probability that chance could have produced this value of *t*	0.10	0.05	0.01	0.001
Confidence level	10%	5%	1%	0.1%

Table P2.2 Values of *t*.

Using the table of probabilities in the *t*-test

In statistical tests that compare samples, it is the convention to start off by making the assumption that there is no significant difference between the samples. You assume that they are just two samples from an identical population. This is called the null hypothesis. In this case, the null hypothesis would be:

There is no difference between the corolla length in population **A** and population **B**.