Chapter 20 - Thermodynamics

- 20.1 - The Second Law of Thermodynamics: Predicting Spontaneous Change
- 20.2 - Calculating the Change in Entropy of a Reaction
- 20.3 - Entropy, Free Energy, and Work
- 20.4 - Free Energy, Equilibrium, and Reaction Direction
20.1 - The Second Law of Thermodynamics: Predicting Spontaneous Change

Figure 20.2 Spontaneous expansion of a gas

20.1 - The Second Law of Thermodynamics: Predicting Spontaneous Change

Figure 20.3
Expansion of a gas and the increase in number of microstates.

20.1 - The Second Law of Thermodynamics: Predicting Spontaneous Change

## 1877 Ludwig Boltzman

## $S=k \ln W$

where $S$ is entropy, $W$ is the number of ways of arranging the components of a system, and $k$ is a constant (the Boltzman constant), $R / N_{A}$ ( $R=$ universal gas constant, $N_{A}=$ Avogadro's number.
-A system with relatively few equivalent ways to arrange its components (smaller W) has relatively less disorder and low entropy.
-A system with many equivalent ways to arrange its components (larger W) has relatively more disorder and high entropy.

$$
\Delta S_{\text {universe }}=\Delta S_{\text {system }}+\Delta S_{\text {surroundings }}>0
$$

The second law of thermodynamics.
20.1 - The Second Law of Thermodynamics: Predicting Spontaneous Change

Figure 20.4 Random motion in a crystal

The third law of thermodynamics.

A perfect crystal has zero entropy at a temperature of absolute zero.

$$
S_{\text {system }}=0 \text { at } 0 \mathrm{~K}
$$


20.1 - The Second Law of Thermodynamics: Predicting Spontaneous Change

Figure 20.5 The increase in entropy from solid to liquid to gas.

20.1 - The Second Law of Thermodynamics: Predicting Spontaneous Change

Figure 20.6
The entropy change accompanying the dissolution of a salt.

20.1 - The Second Law of Thermodynamics: Predicting Spontaneous Change

Figure 20.7
The small increase in entropy when ethanol dissolves in water.


Ethanol


Water


Solution of ethanol and water
20.1 - The Second Law of Thermodynamics: Predicting Spontaneous Change

Figure 20.8
The large decrease in entropy when a gas dissolves in a liquid.

20.1 - The Second Law of Thermodynamics: Predicting Spontaneous Change

Figure 20.9 Entropy and vibrational motion.

20.1 - The Second Law of Thermodynamics: Predicting Spontaneous Change

## Sample Problem 20.1: Predicting Relative Entropy Values

PROBLEM: Choose the member with the higher entropy in each of the following pairs, and justify your choice [assume constant temperature, except in part (e)]:
(a) 1 mol of $\mathrm{SO}_{2}(g)$ or 1 mol of $\mathrm{SO}_{3}(g)$
(b) 1 mol of $\mathrm{CO}_{2}(s)$ or 1 mol of $\mathrm{CO}_{2}(g)$
(c) 3 mol of oxygen gas $\left(\mathrm{O}_{2}\right)$ or 2 mol of ozone gas $\left(\mathrm{O}_{3}\right)$
(d) 1 mol of $\mathrm{KBr}(s)$ or 1 mol of $\mathrm{KBr}(a q)$
(e) Seawater in midwinter at $2^{\circ} \mathrm{C}$ or in midsummer at $23^{\circ} \mathrm{C}$
(f) 1 mol of $\mathrm{CF}_{4}(g)$ or 1 mol of $\mathrm{CCl}_{4}(g)$

PLAN: In general less ordered systems have higher entropy than ordered systems and entropy increases with an increase in temperature.

SOLUTION:
(a) 1 mol of $\mathrm{SO}_{3}(g)$ - more atoms
(d) 1 mol of $\mathrm{KBr}(a q)$ - solution $>$ solid
(b) 1 mol of $\mathrm{CO}_{2}(\mathrm{~g})$ - gas $>$ solid
(e) $23^{\circ} \mathrm{C}$ - higher temperature
(c) 3 mol of $\mathrm{O}_{2}(g)$ - larger \#mols
(f) $\mathrm{CCl}_{4}$ - larger mass
20.2 - Calculating the Change in Entropy of a Reaction

## Sample Problem 20.2: $\quad$ Calculating the Standard Entropy of Reaction, $\boldsymbol{\Delta S ^ { 0 }}{ }_{r \times n}$

PROBLEM: Calculate $\Delta \mathrm{S}^{0}{ }_{\text {rxn }}$ for the combustion of 1 mol of propane at $25^{\circ} \mathrm{C}$.

$$
\mathrm{C}_{3} \mathrm{H}_{8}(g)+5 \mathrm{O}_{2}(g) \quad \longrightarrow \mathrm{CO}_{2}(g)+4 \mathrm{H}_{2} \mathrm{O}(l)
$$

PLAN: Use summation equations. It is obvious that entropy is being lost because the reaction goes from 6 mols of gas to 3 mols of gas.

SOLUTION: Find standard entropy values in the Appendix or other table.

$$
\begin{aligned}
& \Delta S=\left[(3 \mathrm{~mol})\left(\mathrm{S}^{0} \mathrm{CO}_{2}\right)+(4 \mathrm{~mol})\left(\mathrm{S}^{0} \mathrm{H}_{2} \mathrm{O}\right)\right]-\left[(1 \mathrm{~mol})\left(\mathrm{S}^{0} \mathrm{C}_{3} \mathrm{H}_{8}\right)+(5 \mathrm{~mol})\left(\mathrm{S}^{0} \mathrm{O}_{2}\right)\right] \\
& \Delta \mathrm{S}=\left[(3 \mathrm{~mol})\left(213.7 \mathrm{~J} / \mathrm{mol}^{*} \mathrm{~K}\right)+(4 \mathrm{~mol})\left(69.9 \mathrm{~J} / \mathrm{mol}^{*} \mathrm{~K}\right)\right]-\left[(1 \mathrm{~mol})\left(269.9 \mathrm{~J} / \mathrm{mol}{ }^{*} \mathrm{~K}\right)+(5\right. \\
& \left.\mathrm{mol})\left(205.0 \mathrm{~J} / \mathrm{mol}^{*} \mathrm{~K}\right)\right]
\end{aligned}
$$

$$
\Delta \mathrm{S}=-374 \mathrm{~J} / \mathrm{K}
$$

20.2 - Calculating the Change in Entropy of a Reaction

Sample Problem 20.3: Determining Reaction Spontaneity

PROBLEM: At 298 K , the formation of ammonia has a negative $\Delta \mathrm{S}_{\text {sys }}^{0}$;

$$
\mathrm{N}_{2}(g)+3 \mathrm{H}_{2}(g) \longrightarrow \mathrm{NH}_{3}(g) \quad \Delta \mathrm{S}_{\text {sys }}^{0}=-197 \mathrm{~J} / \mathrm{K}
$$

Calculate $\Delta \mathrm{S}^{0}{ }_{\mathrm{rxn}}{ }^{\prime}$, and state whether the reaction occurs spontaneously at this temperature.

PLAN: $\quad \Delta S^{0}{ }_{\text {universe }}$ must be $>0$ in order for this reaction to be spontaneous, so $\Delta \mathrm{S}_{\text {surroundings }}^{\text {universe }}$ must be $>197 \mathrm{~J} / \mathrm{K}$. To find $\Delta \mathrm{S}_{\text {surr }}^{0}$, first find $\Delta \mathrm{H}_{\text {sys }} ; \Delta \mathrm{H}_{\text {sys }}=\Delta \mathrm{H}_{\mathrm{rxn}}$ which can be calculated using $\Delta \mathrm{H}_{\mathrm{f}}^{0}$ values from tables. $\Delta \mathrm{S}_{\text {universe }}^{0 \text { sys }} \stackrel{\text { sys }}{=} \Delta \mathrm{S}_{\text {surr }}^{0}+$ $\Delta S_{\text {sys }}^{0}$.

SOLUTION:

$$
\begin{aligned}
& \Delta \mathrm{H}_{r x}^{0}=\left[(2 \mathrm{~mol})\left(\Delta \mathrm{H}_{\mathrm{f}}^{0} \mathrm{NH}_{3}\right)\right]-\left[(1 \mathrm{~mol})\left(\Delta \mathrm{H}_{\mathrm{f}}^{0} \mathrm{~N}_{2}\right)+(3 \mathrm{~mol})\left(\Delta \mathrm{H}_{\mathrm{f}}^{0} \mathrm{H}_{2}\right)\right] \\
& \Delta \mathrm{H}_{\mathrm{rx}}^{0}=-91.8 \mathrm{~kJ} \\
& \Delta \mathrm{~S}_{\text {surr }}^{0}=-\Delta \mathrm{H}_{\text {sys }}^{0} / \mathrm{T}=\quad-\left(-91.8 \times 10^{3} \mathrm{~J} / 298 \mathrm{~K}\right) \quad=308 \mathrm{~J} / \mathrm{K} \\
& \Delta \mathrm{~S}_{\text {universe }}^{0}=\Delta \mathrm{S}_{\text {surr }}^{0}+\Delta \mathrm{S}_{\text {sys }}^{0} \quad=308 \mathrm{~J} / \mathrm{K}+(-197 \mathrm{~J} / \mathrm{K})=111 \mathrm{~J} / \mathrm{K} \\
& \Delta \mathrm{~S}_{\text {universe }}^{0}>0 \text { so the reaction is spontaneous. }
\end{aligned}
$$

20.3 - Entropy, Free Energy, and Work

Sample Problem 20.4: $\quad$ Calculating $\Delta \mathbf{G}^{0}$ from Enthalpy and Entropy Values

PROBLEM: Potassium chlorate, a common oxidizing agent in fireworks and matchheads, undergoes a solid-state disproportionation reaction when heated.

Note that the oxidation number of Cl in the reactant is higher in one of the products and lower in the other (disproportionation).

$$
\stackrel{+5}{4 \mathrm{KClO}_{3}(s)} \xrightarrow{\Delta} 3 \mathrm{KClO}_{4}^{+7}(s)+\mathrm{KCl}(s)^{-1}
$$

Use $\Delta H^{0}{ }_{f}$ and $S^{0}$ values to calculate $\Delta G^{0}{ }_{\text {sys }}\left(\Delta G_{r x n}^{0}\right)$ at $25^{\circ} \mathrm{C}$ for this reaction.
PLAN: Use Appendix B values for thermodynamic entities; place them into the Gibbs Free Energy equation and solve.

SOLUTION:

$$
\begin{aligned}
& \Delta \mathrm{H}_{\mathrm{rxn}}^{0}=\Sigma m \Delta \mathrm{H}_{\text {products }}^{0}-\Sigma n \Delta \mathrm{H}_{\text {reactants }}^{0} \\
& \Delta \mathrm{H}_{\mathrm{rxn}}^{0}=(3 \mathrm{~mol})(-432.8 \mathrm{~kJ} / \mathrm{mol})+(1 \mathrm{~mol})(-436.7 \mathrm{~kJ} / \mathrm{mol})-
\end{aligned}
$$

$$
(4 \mathrm{~mol})(-397.7 \mathrm{~kJ} / \mathrm{mol})
$$

$$
\Delta \mathrm{H}_{\mathrm{rxn}}^{0}=-144 \mathrm{~kJ}
$$

20.3 - Entropy, Free Energy, and Work

## Sample Problem 20.4: <br> Calculating $\Delta \mathbf{G}^{\mathbf{0}}$ from Enthalpy and Entropy Values

## continued

$$
\begin{aligned}
& \Delta \mathrm{S}_{\mathrm{rxn}}^{0}=\Sigma m \Delta \mathrm{~S}_{\text {products }}^{0}-\sum n \Delta \mathrm{~S}_{\text {reactants }}^{0} \\
& \Delta \mathrm{~S}_{\mathrm{rxn}}^{0}=(3 \mathrm{~mol})\left(151 \mathrm{~J} / \mathrm{mol}^{*} \mathrm{~K}\right)+(1 \mathrm{~mol})\left(82.6 \mathrm{~J} / \mathrm{mol}^{*} \mathrm{~K}\right)- \\
& \quad(4 \mathrm{~mol})(143.1 \mathrm{~J} / \mathrm{mol} * \mathrm{~K}) \\
& \Delta \mathrm{S}_{\mathrm{rxn}}^{0}=-36.8 \mathrm{~J} / \mathrm{K} \\
& \Delta \mathrm{G}_{\mathrm{rxn}}^{0}=\Delta \mathrm{H}_{\mathrm{rxn}}^{0}-\mathrm{T} \Delta \mathrm{~S}_{\mathrm{rxn}}^{0} \\
& \Delta \mathrm{G}_{\mathrm{rxn}}^{0}=-144 \mathrm{~kJ}-(298 \mathrm{~K})(-36.8 \mathrm{~J} / \mathrm{K})\left(\mathrm{kJ} / 10^{3} \mathrm{~J}\right) \\
& \Delta \mathrm{G}_{\mathrm{rxn}}^{0}=-133 \mathrm{~kJ}
\end{aligned}
$$

20.3 - Entropy, Free Energy, and Work

## Sample Problem 20.5: $\quad$ Calculating $\Delta \mathbf{G}^{0}{ }_{r \times n}$ from $\Delta \mathbf{G}^{\mathbf{0}}{ }_{f}$ Values

PROBLEM: Use $\Delta G_{f}^{0}$ values to calculate $\Delta G_{r x n}$ for the reaction in Sample Problem 20.4:

$$
4 \mathrm{KClO}_{3}(s) \xrightarrow{\Delta} \nVdash \mathrm{ClO}_{4}(s)+\mathrm{KCl}(s)
$$

PLAN: Use the $\Delta \mathrm{G}$ summation equation.
SOLUTION: $\quad \Delta \mathrm{G}^{0}{ }_{\mathrm{rxn}}=\Sigma m \Delta \mathrm{G}_{\text {products }}^{0}-\Sigma n \Delta \mathrm{G}_{\text {reactants }}^{0}$

$$
\Delta G^{0}{ }_{r \times n}=(3 \mathrm{~mol})(-303.2 \mathrm{~kJ} / \mathrm{mol})+(1 \mathrm{~mol})(-409.2 \mathrm{~kJ} / \mathrm{mol})-
$$

(4mol)(-296.3kJ/mol)

$$
\Delta G_{r x n}^{0}=-134 \mathrm{~kJ}
$$

## Sample Problem 20.6: Determining the Effect of Temperature on $\Delta \mathbf{G}^{\mathbf{0}}$

PROBLEM: An important reaction in the production of sulfuric acid is the oxidation of $\mathrm{SO}_{2}(g)$ to $\mathrm{SO}_{3}(g)$ :

$$
2 \mathrm{SO}_{2}(g)+\mathrm{O}_{2}(g) \quad \longrightarrow \mathrm{SO}_{3}(g)
$$

At $298 \mathrm{~K}, \Delta \mathrm{G}^{0}=-141.6 \mathrm{~kJ} ; \Delta \mathrm{H}^{0}=-198.4 \mathrm{~kJ} ;$ and $\Delta \mathrm{S}^{0}=-187.9 \mathrm{~J} / \mathrm{K}$
(a) Use the data to decide if this reaction is spontaneous at $25^{\circ} \mathrm{C}$, and predict how $\Delta \mathrm{G}^{0}$ will change with increasing $T$.
(b) Assuming $\Delta H^{0}$ and $\Delta S^{0}$ are constant with increasing $T$, is the reaction spontaneous at $900 .{ }^{\circ} \mathrm{C}$ ?

PLAN: The sign of $\Delta G^{0}$ tells us whether the reaction is spontaneous and the signs of $\Delta \mathrm{H}^{0}$ and $\Delta \mathrm{S}^{0}$ will be indicative of the $T$ effect. Use the Gibbs free energy equation for part (b).

SOLUTION: (a) The reaction is spontaneous at $25^{\circ} \mathrm{C}$ because $\Delta \mathrm{G}^{0}$ is (-). Since $\Delta H^{0}$ is (-) but $\Delta S^{0}$ is also (-), $\Delta G^{0}$ will become less spontaneous as the temperature increases.
20.3 - Entropy, Free Energy, and Work

## Sample Problem 20.6: Determining the Effect of Temperature on $\Delta \mathbf{G}^{\mathbf{0}}$

## continued

(b) $\Delta \mathrm{G}^{0}{ }_{\mathrm{rxn}}=\Delta \mathrm{H}_{\mathrm{rxn}}^{0}-\mathrm{T} \Delta \mathrm{S}_{\mathrm{rxn}}^{0}$

$$
\Delta \mathrm{G}_{\mathrm{rxn}}^{0}=-198.4 \mathrm{~kJ}-(1173 \mathrm{~K})\left(-187.9 \mathrm{~J} / \mathrm{mol}^{*} \mathrm{~K}\right)\left(\mathrm{kJ} / 10^{3} \mathrm{~J}\right)
$$

$\Delta \mathrm{G}_{\mathrm{rxn}}^{0}=22.0 \mathrm{~kJ}$; the reaction will be nonspontaneous at $900 .{ }^{\circ} \mathrm{C}$
20.3 - Entropy, Free Energy, and Work

## Figure B20.3

The coupling of a nonspontaneous reaction to the hydrolysis of ATP.

20.3 - Entropy, Free Energy, and Work

Figure B20.4 The cycling of metabolic free enery through ATP

20.3 - Entropy, Free Energy, and Work

Figure B20.5 Why is ATP a high-energy molecule?


20.4 - Free Energy, Equilibrium, and Reaction Direction

## Free Energy, Equilibrium and Reaction Direction

-If $Q / K<1$, then $\ln Q / K<0$; the reaction proceeds to the right ( $\Delta \mathrm{G}<0$ )

- If $Q / K>1$, then $\ln \mathrm{Q} / \mathrm{K}>0$; the reaction proceeds to the left ( $\Delta \mathrm{G}>0$ )
-If $Q / K=1$, then $\ln Q / K=0$; the reaction is at equilibrium $(\Delta G=0)$

$$
\Delta G=R T \ln Q / K=R T \ln Q-R T \ln K
$$

Under standard conditions ( 1 M concentrations, 1atm for gases), $\mathrm{Q}=1$ and In Q = 0 so

$$
\Delta G^{0}=-R T \operatorname{In} K
$$

20.4 - Free Energy, Equilibrium, and Reaction Direction

Table 20.2 The Relationship Between $\Delta G^{0}$ and $K$ at $\mathbf{2 5}^{\circ} \mathrm{C}$


## Sample Problem 20.7: $\quad$ Calculating $\Delta G$ at Nonstandard Conditions

PROBLEM: The oxidation of $\mathrm{SO}_{2}$, which we considered in Sample Problem 20.6

$$
2 \mathrm{SO}_{2}(g)+\mathrm{O}_{2}(g) \quad \longrightarrow \mathrm{SO}_{3}(g)
$$

is too slow at 298 K to be useful in the manufacture of sulfuric acid. To overcome this low rate, the process is conducted at an elevated temperature.
(a) Calculate K at 298 K and at $973 \mathrm{~K} .\left(\Delta \mathrm{G}^{0}{ }_{298}=-141.6 \mathrm{~kJ} / \mathrm{mol}\right.$ of reaction as written using $\Delta \mathrm{H}^{0}$ and $\Delta \mathrm{S}^{0}$ values at $973 \mathrm{~K} . \Delta \mathrm{G}^{0}{ }_{973}=-12.12 \mathrm{~kJ} / \mathrm{mol}$ of reaction as written.)
(b) In experiments to determine the effect of temperature on reaction spontaneity, two sealed containers are filled with 0.500atm of $\mathrm{SO}_{2}, 0.0100$ atm of $\mathrm{O}_{2}$, and 0.100 atm of $\mathrm{SO}_{3}$ and kept at $25^{\circ} \mathrm{C}$ and at $700 .{ }^{\circ} \mathrm{C}$. In which direction, if any, will the reaction proceed to reach equilibrium at each temperature?
(c) Calculate $\Delta \mathrm{G}$ for the system in part (b) at each temperature.
20.4 - Free Energy, Equilibrium, and Reaction Direction

## Sample Problem 20.7: $\quad$ Calculating $\Delta G$ at Nonstandard Conditions

## continued (2 of 3)

SOLUTION: (a) Calculating K at the two temperatures:

$$
\Delta G^{0}=-R T \ln K \text { so } \quad K=e^{-\left(\Delta G^{\circ} / R T\right)}
$$

At 298, the exponent is $-\Delta G^{0} / R T=-\frac{(-141.6 \mathrm{~kJ} / \mathrm{mol})\left(10^{3} \mathrm{~J} / \mathrm{kJ}\right)}{\left(8.314 \mathrm{~J} / \mathrm{mol}{ }^{*} \mathrm{~K}\right)(298 \mathrm{~K})}=57.2$

$$
\mathrm{K}=\mathrm{e}^{-\left(\Delta \mathrm{G}^{\circ} / \mathrm{RT}\right)}=\mathrm{e}^{57.2}=7 \times 10^{24}
$$

At 973, the exponent is $-\Delta \mathrm{G}^{0} / \mathrm{RT} \quad \frac{(-12.12 \mathrm{~kJ} / \mathrm{mol})\left(10^{3} \mathrm{~J} / \mathrm{kJ}\right)}{\left(8.314 \mathrm{~J} / \mathrm{mol}{ }^{*} \mathrm{~K}\right)(973 \mathrm{~K})}=1.50$

$$
K=e^{-\left(\Delta G^{\circ} / R T\right)}=e^{1.50}=4.5
$$

20.4 - Free Energy, Equilibrium, and Reaction Direction

## Sample Problem 20.7:

## continued (3 of 3)

(b) The value of $\mathrm{Q}=\frac{\mathrm{pSO}_{3}{ }^{2}}{\left(\mathrm{pSO}_{2}\right)^{2}\left(\mathrm{pO}_{2} \mathrm{O}\right.}=\frac{(0.100)^{2}}{(0.500)^{2}(0.0100)}=4.00$

Since $Q$ is < K at both temperatures the reaction will shift right; for 298 K there will be a dramatic shift while at 973 K the shift will be slight.
(c) The nonstandard $\Delta G$ is calculated using $\Delta G=\Delta G^{0}+R T \operatorname{lnQ}$

$$
\begin{aligned}
\Delta G_{298} & =-141.6 \mathrm{~kJ} / \mathrm{mol}+\left(8.314 \mathrm{~J} / \mathrm{mol}^{*} \mathrm{~K}\right)\left(\mathrm{kJ} / 10^{3} \mathrm{~J}\right)(298 \mathrm{~K})(\ln 4.00) \\
\Delta \mathrm{G}_{298} & =-138.2 \mathrm{~kJ} / \mathrm{mol} \\
\Delta G_{973} & =-12.12 \mathrm{~kJ} / \mathrm{mol}+\left(8.314 \mathrm{~J} / \mathrm{mol}^{*} \mathrm{~K}\right)\left(\mathrm{kJ} / 10^{3} \mathrm{~J}\right)(973 \mathrm{~K})(\ln 4.00) \\
\Delta \mathrm{G}_{298} & =-0.9 \mathrm{~kJ} / \mathrm{mol}
\end{aligned}
$$

