ARCH and GARCH

Modeling Volatility Dynamics

Modeling Unequal Variability

- Equal Variability: Homoscedasticity
- Unequal Variability: Heteroscedasticity
  - Means any variability (around the mean) that is not homoscedasticity
  - Models must be developed for specific cases

What These Acronym Mean?

- ARCH
  - Autoregressive Conditional Heteroscedasticity
- GARCH
  - Generalized ARCH

Information in e^2

- Let ε_t have the mean 0 and the variance σ_t.
- Let e_t be the residual of a model fitted.
- Then:
  - e_t estimates ε_t
  - e_t^2 estimates the variance σ_t^2.

ARCH Modeling of σ_t^2.

- **ARCH(1)**

$$\sigma_t^2 = \varpi + \alpha\varepsilon_{(t-1)}^2$$

- **ARCH as AR(1) on** $\varepsilon_t^2 = \sigma_t^2 + \nu_t$

$$\varepsilon_t^2 = \varpi + \alpha\varepsilon_{(t-1)}^2 + \nu_t$$

GARCH

- **GARCH(1)**

$$\sigma_t^2 = \varpi + \alpha\varepsilon_{(t-1)}^2 + \beta\sigma_{(t-1)}^2$$

- **GARCH (1) as ARMA(1,1) on** $\varepsilon_t^2 = \sigma_t^2 + \nu_t$

$$\varepsilon_t^2 = \varpi + \left(\alpha + \beta\right)\varepsilon_{(t-1)}^2 + \nu_t - \beta\nu_{(t-1)}$$

$\varepsilon_t > 0$

Asymmetry in GARCH - TARCH

- **TARCH(1,1)**

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \gamma d\varepsilon_{t-1s}^2 + \beta\sigma_{t-1}^2$$

d = 1 if ε_t < 0, and = 0 if ε_t $\geq$ 0

Asymmetry in GARCH - EGARCH

- **EGARCH(1,1)**

$$\log\left(\sigma_t^2\right) = \omega + \beta \log \sigma_{t-1}^2 + \alpha \left| \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right| + \gamma \frac{\varepsilon_{t-1}}{\sigma_{t-1}}$$

$\sigma_t^2 > 0$

$$\gamma \neq 0 \; for \; asymmetric \; effect$$

Eviews Command

ARCH(p, q) *series_name* c