LINEAR ALGEBRA

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OVERVIEW

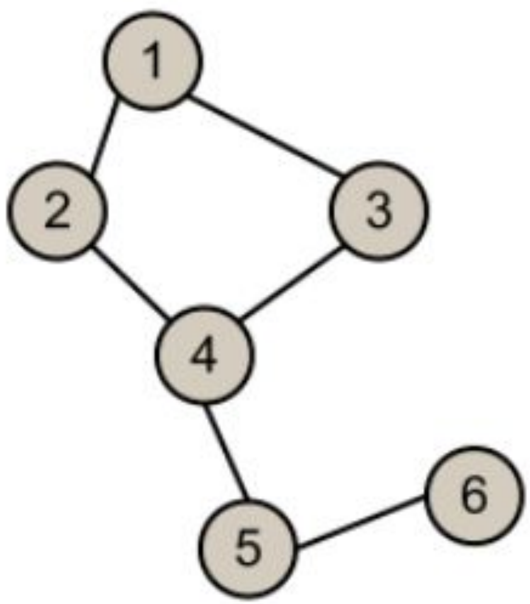
- Application of matrices
- SLEs
- Kronecker-Cappelli Theorem.

APPLICATION OF MATRICES

- *Graph theory*
- *Computer graphics*
- *Cryptography*
- *Solving SLEs*

GRAPH THEORY

Undirected Graph & Adjacency Matrix



Undirected Graph

	1	2	3	4	5	6
1	0	1	1	0	0	0
2	1	0	0	1	0	0
3	1	0	0	1	0	0
4	0	1	1	0	1	0
5	0	0	0	1	0	1
6	0	0	0	0	1	0

Adjacency Matrix

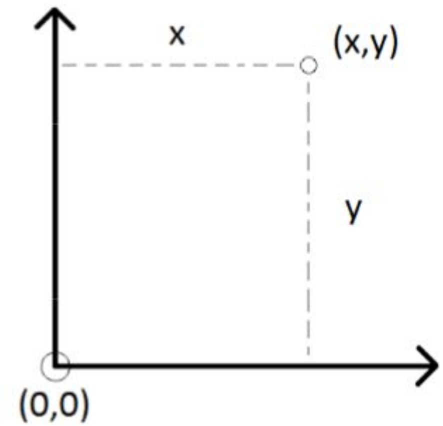
COMPUTER GRAPHICS

Point representation

- We use a column vector (a 2x1 matrix) to represent a 2D point

$$p = \begin{bmatrix} x \\ y \end{bmatrix}$$

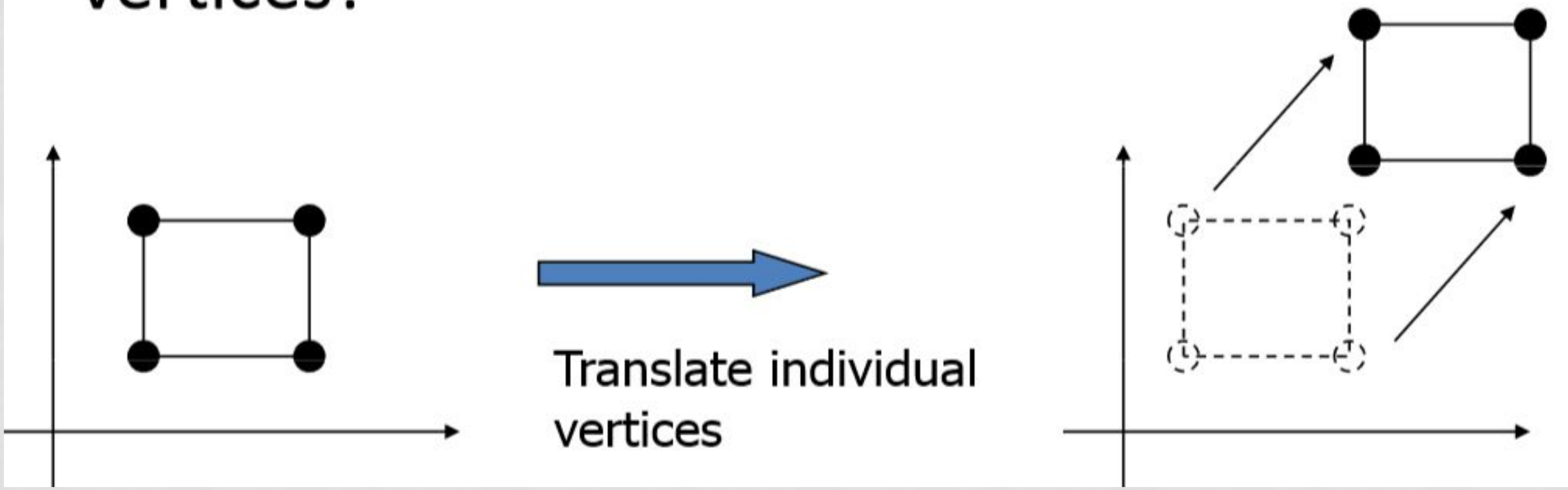
- Points are defined with respect to
  - origin (point)
  - coordinate axes (basis vectors)



COMPUTER GRAPHICS

Translation

- How to translate an object with multiple vertices?



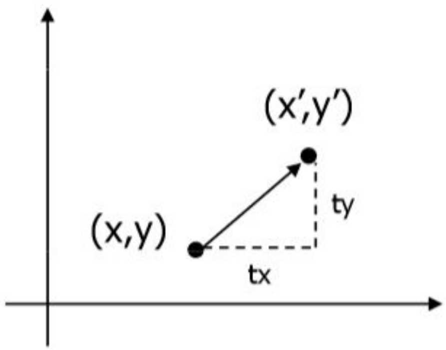
COMPUTER GRAPHICS

Translation

- Re-position a point along a straight line
- Given a point (x,y), and the translation distance or vector (tx,ty)

The new point: (x′, y′)

$$x' = x + tx$$
$$y' = y + ty$$



OR $p' = p + t$ where $p' = \begin{bmatrix} x' \\ y' \end{bmatrix}$ $p = \begin{bmatrix} x \\ y \end{bmatrix}$ $t = \begin{bmatrix} tx \\ ty \end{bmatrix}$

CRYPTOGRAPHY

- Study of encoding and decoding secret messages
- Useful in sending sensitive information so that only the intended receivers can understand the message
- A common use of cryptography is to send government secrets.

ENCODING

- First we will assign numbers to represent each letter of the alphabet.
- We then create a "plaintext" matrix that holds the message in terms of numbers.
- Then we pick an invertible square matrix, which can be multiplied with the "plaintext matrix".

A	1
B	2
C	3
D	4
E	5
F	6
G	7
H	8
I	9
J	10
K	11
L	12
M	13
N	14

O	15
P	16
Q	17
R	18
S	19
T	20
U	21
V	22
W	23
X	24
Y	25
Z	26
_	27
‘	28

Encrypting the Message

$$
\begin{bmatrix} 1 & 2 & -1 \\ 2 & 2 & 4 \\ 1 & 3 & -3 \end{bmatrix}
\times
\begin{bmatrix}
6 & 18 & 5 & 5 & 27 & 12 & 1 & 21 & 14 & 4 & 18 & 25 & 27 \\
13 & 15 & 14 & 5 & 25 & 27 & 21 & 14 & 4 & 5 & 18 & 27 & 19 \\
15 & 13 & 5 & 15 & 14 & 5 & 28 & 19 & 27 & 4 & 5 & 19 & 11
\end{bmatrix}
$$

Encoding matrix

Plaintext

$$
=
$$

$$
\begin{bmatrix}
17 & 35 & 28 & 0 & 63 & 61 & 15 & 30 & -5 & 10 & 49 & 60 & 54 \\
98 & 118 & 58 & 80 & 160 & 98 & 156 & 146 & 144 & 34 & 92 & 180 & 136 \\
0 & 24 & 32 & -25 & 60 & 78 & -20 & 6 & -55 & 7 & 57 & 49 & 51
\end{bmatrix}
$$

Ciphertext

DECIPHERING THE MESSAGE

- In order to decode the message, we would have to take the inverse of the encoding matrix to obtain the decoding matrix.

- Multiplying the decoding matrix with the ciphertext would result in the plaintext version.

- Then the arbitrarily assigned number scheme can be used to retrieve the message.

A 1
B 2
C 3
D 4
E 5
F 6
G 7
H 8
I 9
J 10
K 11
L 12
M 13
N 14

Decrypting the Message

$$\begin{bmatrix} 9 & -3/2 & -5 \\ -5 & 1 & 3 \\ -2 & 1/2 & 1 \end{bmatrix} \mathbf{X}$$

$$\begin{bmatrix} 17 & 35 & 28 & 0 & 63 & 61 & 15 & 30 & -5 & 10 & 49 & 60 & 54 \\ 98 & 118 & 58 & 80 & 160 & 98 & 156 & 146 & 144 & 34 & 92 & 180 & 136 \\ 0 & 24 & 32 & -25 & 60 & 78 & -20 & 6 & -55 & 7 & 57 & 49 & 51 \end{bmatrix}$$

$$=$$

$$\begin{bmatrix} 6 & 18 & 5 & 5 & 27 & 12 & 1 & 21 & 14 & 4 & 18 & 25 & 27 \\ 13 & 15 & 14 & 5 & 25 & 27 & 21 & 14 & 4 & 5 & 18 & 27 & 19 \\ 15 & 13 & 5 & 15 & 14 & 5 & 28 & 19 & 27 & 4 & 5 & 19 & 11 \end{bmatrix}$$

O 15
P 16
Q 17
R 18
S 19
T 20
U 21
V 22
W 23
X 24
Y 25
Z 26
_ 27
‘ 28

YOU MIGHT WANT TO READ THIS.

A 1
B 2
C 3
D 4
E 5
F 6
G 7
H 8
I 9
J 10
K 11
L 12
M 13
N 14

F	*R*	*E*	*E*	_	*L*	*A*	*U*	*N*	*D*	*R*	*Y*	_
6	18	5	5	27	12	1	21	14	4	18	25	27
M	*O*	*N*	*E*	*Y*	_	*U*	*N*	*D*	*E*	*R*	_	*S*
13	15	14	5	25	27	21	14	4	5	18	27	19
O	*M*	*E*	*O*	*N*	*E*	'	*S*	_	*D*	*E*	*S*	*K*
15	13	5	15	14	5	28	19	27	4	5	19	11

O 15
P 16
Q 17
R 18
S 19
T 20
U 21
V 22
W 23
X 24
Y 25
Z 26
_ 27
‘ 28

SOLVING SLES

EXAMPLE 2

Avg. football tix = $50.02
Avg. baseball tix = $19.82

In recent Major League Baseball and National Football League seasons, based on average ticket prices, three baseball tickets and two football tickets would have cost $159.50, while two baseball tickets and one football ticket would have cost $89.66. What were the average ticket prices for the tickets for the two sports?

(Source: Team Marketing Report, Chicago.)

$2(3b + 2f = 159.50)$

$-3(2b + 1f = 89.66)$

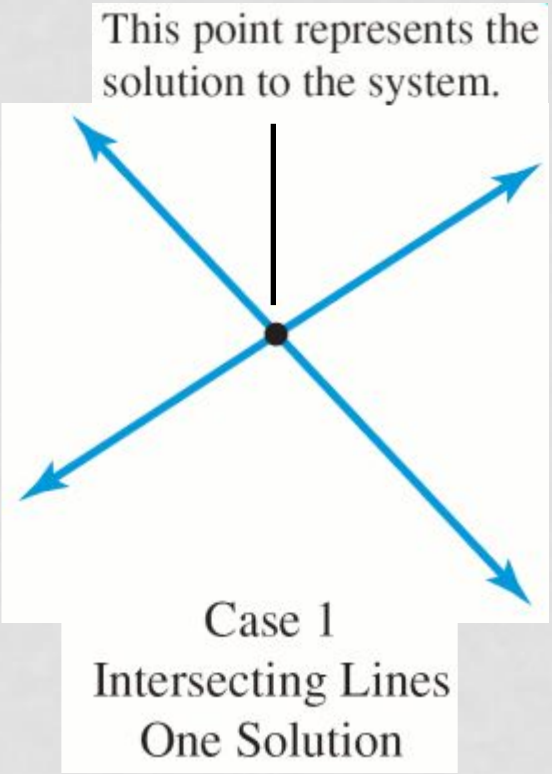
KRONECKER-CAPPELLI THEOREM

- Kronecker-Cappelli Theorem. A linear system has solutions if and only if the rank of the matrix of the system A is equal with the rank of the augmented matrix A'.

- 1. If rk(A) != rk(A'), a linear system is inconsistent (it doesn't have a solution)
- 2. If rk(A) = rk(A') < n, a linear system has infinite solution
- 3. If rk(A) = rk(A') = n, a linear system has only one solution

Systems of Linear Equations

Graphing a system of two linear equations in two unknowns gives one of three possible situations:



Case 1: Lines intersecting in a single point. The ordered pair that represents this point is the *unique solution* for the system.

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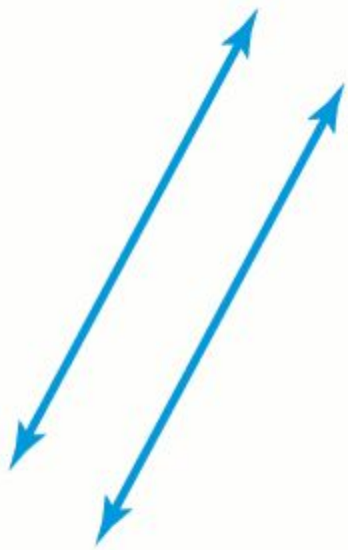
Systems of Linear Equations



Case 2
Parallel Lines
No Solutions

Case 2: Lines that are distinct parallel lines and therefore don’t intersect at all. Because the lines have no common points, this means that the system has *no solutions.*

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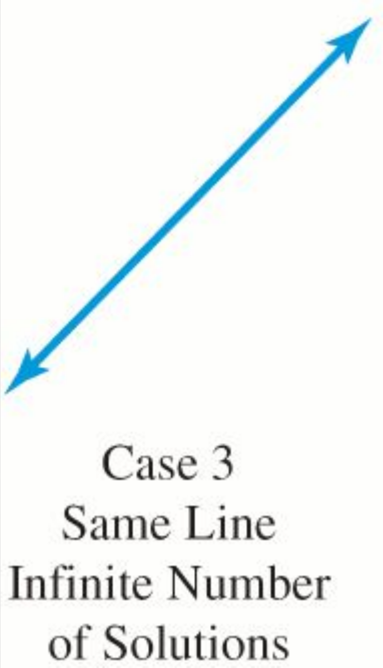
Systems of Linear Equations

Case 3: Two lines that are the same line. The lines have an infinite number of points in common, so the system will have *an infinite number of solutions.*



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