

# Greedy algorithm

A **greedy algorithm** is an algorithmic paradigm that follows the problem solving heuristic of making the **locally optimal choice** at each stage with the intent of finding a **global optimum**. In many problems, a greedy strategy does not usually produce an optimal solution, but nonetheless a greedy heuristic may yield locally optimal solutions that approximate a globally optimal solution in a reasonable amount of time.

For example, a greedy strategy for the traveling salesman problem (which is of a high computational complexity) is the following heuristic: "At each step of the journey, visit the nearest unvisited city." This heuristic does not intend to find a best solution, but it terminates in a **reasonable number of steps**; finding an optimal solution to such a complex problem typically requires unreasonably many steps.

Kruskal's algorithm and Prim's algorithm are greedy algorithms for constructing minimum spanning trees of a given connected graph. They always find an optimal solution, which may not be unique in general.

The choice made by a greedy algorithm may depend on choices made so far, but not on future choices or all the solutions to the subproblem. It iteratively makes one greedy choice after another, reducing each given problem into a smaller one. In other words, a greedy algorithm never reconsiders its choices. This is the main difference from dynamic programming.

**Dynamic programming** is both a mathematical optimization method and a computer programming method. It refers to simplifying a complicated problem by breaking it down into simpler sub-problems in a recursive manner.

Greedy algorithms determine minimum number of coins to give while making change. These are the steps a human would take to emulate a greedy algorithm to represent 36 cents using only coins with values  $\{1, 5, 10, 20\}$ . The coin of the highest value, less than the remaining change owed, is the local optimum.

$$36 = 20 + 10 + 5 + 1.$$

Example for a non-canonical system: 24 cents using only coins with values  $\{1, 5, 7\}$ . A greedy algorithm gives a solution:  $7+7+7+1+1+1$ .

Is it optimal?

Backtracking is a general algorithm for finding all (or some) solutions to some computational problems, notably constraint satisfaction problems, that incrementally builds candidates to the solutions, and abandons a candidate ("backtracks") as soon as it determines that the candidate cannot possibly be completed to a valid solution.

The knapsack problem or rucksack problem is a problem in combinatorial optimization: Given a set of items, each with a weight and a value, determine the number of each item to include in a collection so that the total weight is less than or equal to a given limit and the total value is as large as possible.

1. Consider a method of cutting a rectangle with whole lengths of sides into the least number of squares. Cut the square with the largest side from the rectangle, and repeat this operation with the remaining part of the rectangle the necessary number of times. Show that this method does not always allow cutting into the smallest number of squares.

2. The cells of the copybook paper are painted in a checkerboard pattern. Draw the largest circle of radius, which lies entirely on the white fields, and explain the answer.

3. There are five pieces of chain: 3, 4, 5, 6 and 7 rings. Is it possible to make one chain of them by cutting and connecting only three rings?

4. Is it true that for each point inside the convex quadrilateral the sum of the distances from it to the vertices is less than the perimeter?



5. One tetrahedron lies inside the other. Can the sum of the lengths of the edges of the inner tetrahedron be greater than the sum of the lengths of the edges of the outer tetrahedron?

6. Is it possible to cut an isosceles right triangle into isosceles

9. Are there three natural numbers, the sum of which is equal to 201, and the product is equal to 30030?
10. At the vertices and center of a regular octagon, arrange the numbers from 1 to 9 (each one at a time) so that the sum of the numbers along all the big diagonals is the same. What values can take a number in the center?
11. How many five-digit numbers exist in which all digits are different and go (from left to right) in descending order?
12. How many five-digit numbers exist that all numbers are different and go (from left to right) in ascending order?

13. The sequence is given:  $X_1 = 1$ ,  $X_2 = 2$ , and  $X_{n+2} = X_{n+1} - X_n$  for every natural  $n$ . What is  $X_{2019}$  equal to?

14. A set of  $n > 5$  numbers is arranged in a circle. It is known that the sum of any three consecutive numbers does not exceed 3, and the sum of any five consecutive numbers does not exceed 5. Find all such sets when the sum of all numbers is  $n$ .

15. There are several cities and several one-way roads in the country. Each road connects two cities and does not pass through the rest. At the same time, no matter what two cities you can take, you can even drive from one of them to another without violating traffic rules. Prove that there is a city from which you can drive to any other without breaking traffic rules.

16. Is there an 18-digit number made up of numbers 1, 2, 3, 4, in which all two-digit numbers formed by two adjacent numbers are different?

17. The building is divided into 16 rectangular rooms. The commandant measured the perimeters of the eight rooms. Seven out of eight measurements are shown in figure, and the result of the eighth we denoted by the letter  $x$ . Find  $x$ .

<i>7</i>		<b>13</b>	
		<i>10</i>	<i>7</i>
<b>10</b>	<i>11</i>		
	<b>5</b>		<i>x</i>

18. Find all integer  $n$  for which the number  $5n^2+10$  will be an exact square, or prove that there are none.