# DISCOUNTED CASH FLOW APPLICATIONS

CFA Institute

NET PRESENT VALUE (NPV)

- **Net present value** is the sum of the present values of all the positive cash flows minus the sum of the present values of all the negative cash flows.

- **Interpretation**: When the discount rate applied is an appropriate hurdle rate, it measures the contribution of the project to shareholder wealth.

- **Decision rule**: Accept positive NPV projects □ they increase shareholder wealth.
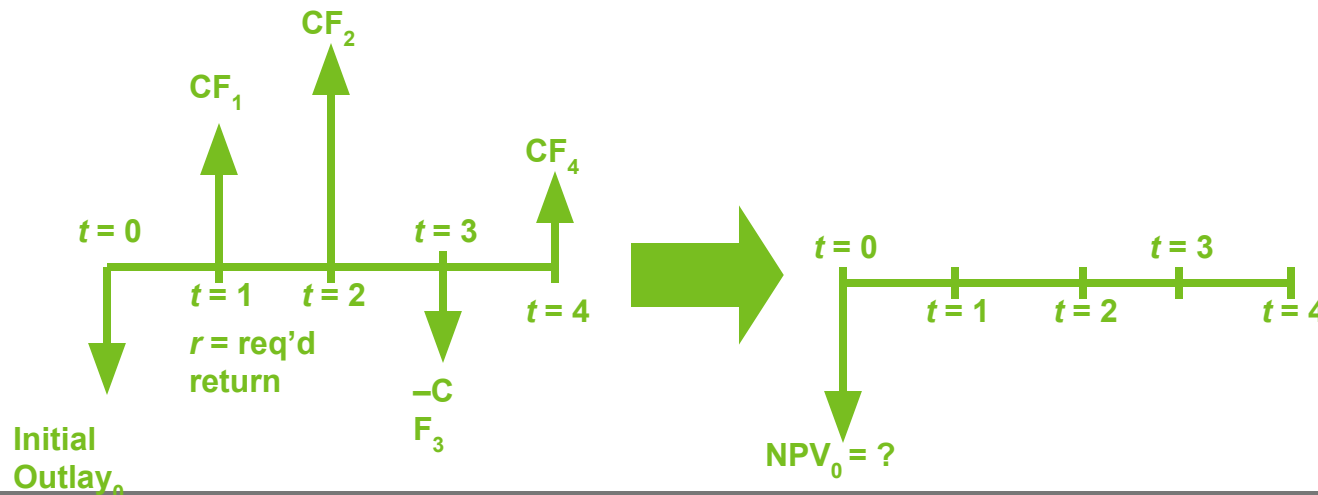
NET PRESENT VALUE (NPV)

Focus On: Calculations

Steps in calculating NPV

1. Identify all the incremental cash flows associated with the project.

2. Determine the appropriate discount rate.

3. Using that discount rate, calculate the present value of all of the inflows (positive sign) and outflows (negative sign).

4. Sum the present values together → the result is the project's NPV.

5. Apply the NPV decision rule.

   - If you have mutually exclusive projects → accept the one with the highest NPV.

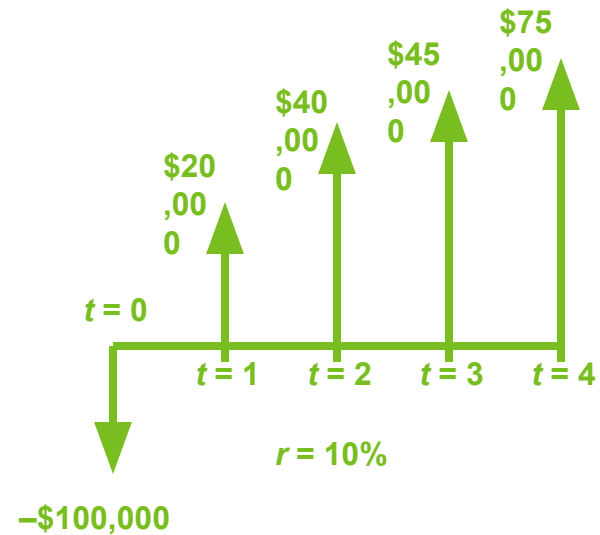$$NPV = \sum_{t=0}^{N} \frac{CF_t}{(1+r)^t}$$

NET PRESENT VALUE (NPV)

Focus On: Calculations

Consider Project A with the following cash flows:
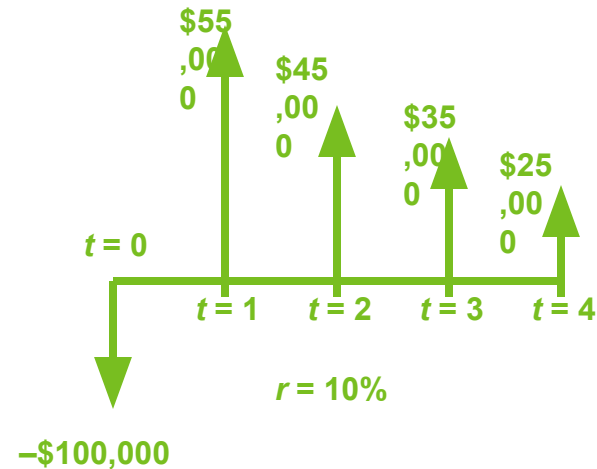
The NPV for this project is…?

Decision?

Consider Project B with the following cash flows:

The NPV for this project is…?

Decision?

Project A

$20,000 at $t = 1$
$40,000 at $t = 2$
$45,000 at $t = 3$
$75,000 at $t = 4$
$t = 0$
$r = 10\%$
−$100,000

Project B

$55,000 at $t = 1$
$45,000 at $t = 2$
$35,000 at $t = 3$
$25,000 at $t = 4$
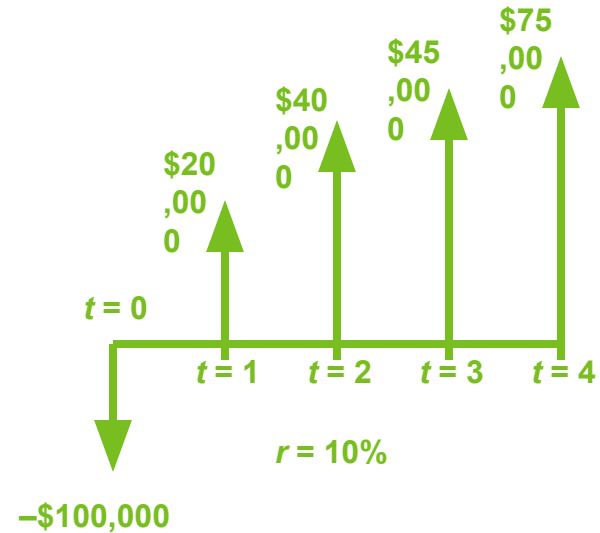$t = 0$
$r = 10\%$
−$100,000

NET PRESENT VALUE (NPV)

Focus On: Calculations

Consider Project A with the following cash flows:

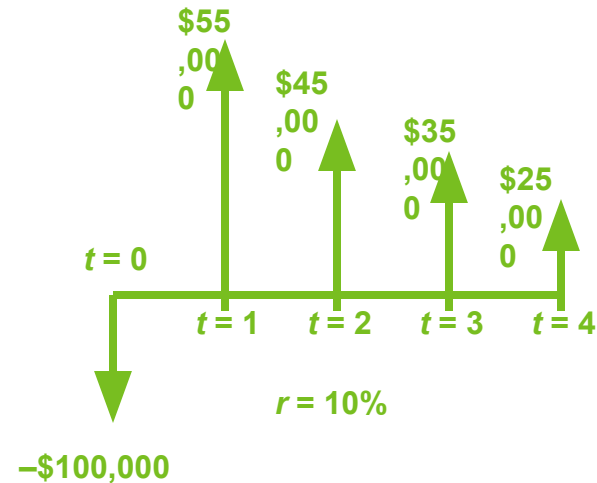The NPV for this project is $29,872.52.

Decision ☐ Accept the project.

Consider Project B with the following cash flows:

The NPV for this project is $27,783.12.

Decision ☐ Accept the project.

INTERNAL RATE OF RETURN (IRR)

- The **internal rate of return** is the discount rate that sets the sum of the present value of the positive cash flows equal to the sum of the present value of the negative cash flows.

  - The discount rate at which NPV = 0

- **Interpretation**: IRR is the expected compound return when all intervening cash flows in the project can be reinvested at the IRR and the investment will be held until maturity.

- **Calculation**: In practice, use a spreadsheet, financial software, or financial calculator to determine IRR.

- **Decision**: Accept projects for which IRR > hurdle rate → increases shareholder wealth.

$$\text{NPV} = \sum_{t=0}^{N} \frac{CF_t}{(1 + \text{IRR})^t} = 0$$

INTERNAL RATE OF RETURN (IRR)

Focus On: Calculations

Consider Project A with the following cash flows:

The IRR for this project is…?

Decision?

Consider Project B with the following cash flows:

The IRR for this project is…?

Decision?

$20,000 $40,000 $45,000 $75,000

t = 0

t = 1   t = 2   t = 3   t = 4

Hurdle rate = 10%

−$100,000

$55,000 $45,000 $35,000 $25,000

t = 0

t = 1   t = 2   t = 3   t = 4
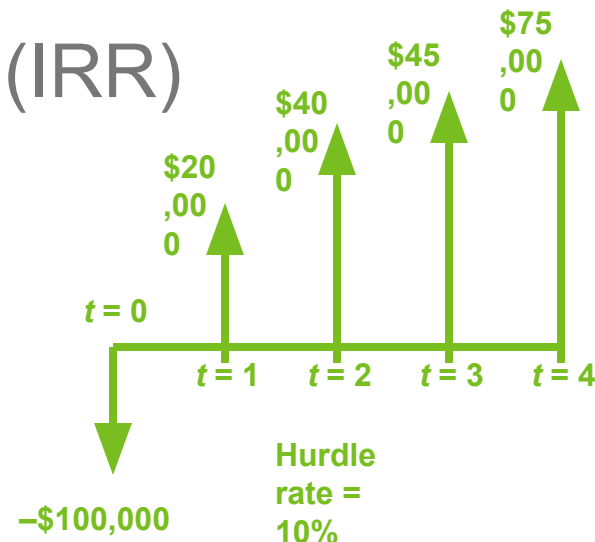
Hurdle rate = 10%

−$100,000

INTERNAL RATE OF RETURN (IRR)

Focus On: Calculations

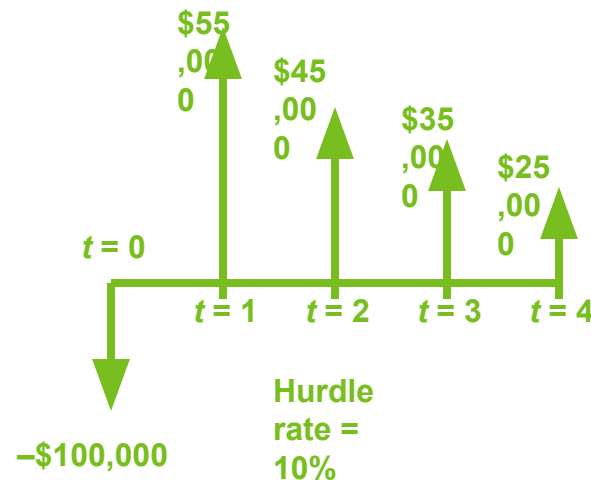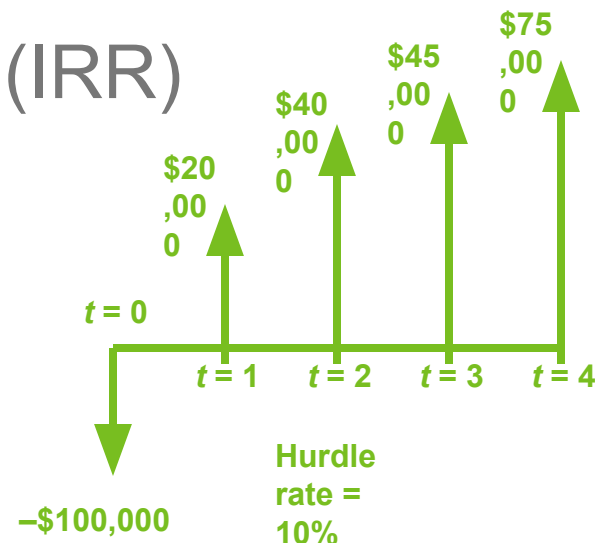Consider Project A with the following cash flows:

The IRR for this project is 21.84%.

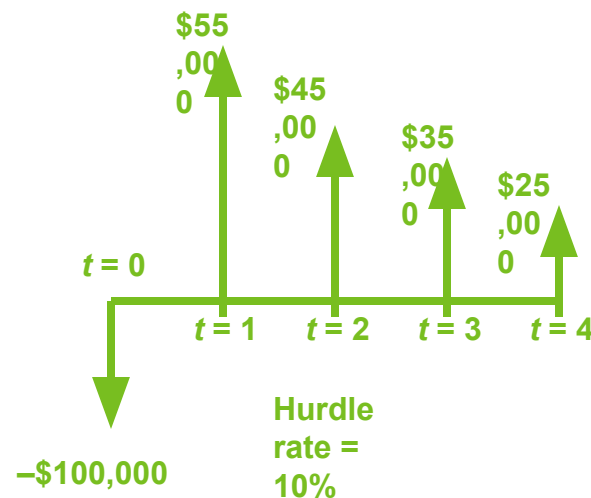Decision ☐ Accept the project.

Consider Project B with the following cash flows:

The IRR for this project is 25.62%.

Decision ☐ Accept the project.

$20,000

$40,000

$45,000

$75,000

t = 0

t = 1   t = 2   t = 3   t = 4

−$100,000

Hurdle rate = 10%

$55,000

$45,000

$35,000

$25,000

t = 0

t = 1   t = 2   t = 3   t = 4
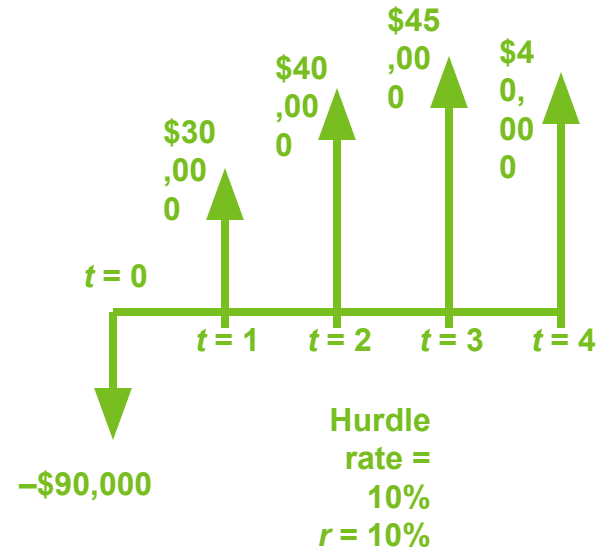
−$100,000

Hurdle rate = 10%

NPV VS. IRR

- If projects are **independent**, the decision to invest in one does not preclude investment in the other.

  - NPV and IRR will yield the same investment decisions.

- Projects are **mutually exclusive** if the selection of one project precludes the selection of another project ☐ project selection is determined by rank.

  - NPV and IRR may give different ranks when

    - The projects have different scales (sizes)

    - The timing of the cash flows differs

  - If projects have different ranks ☐ use NPV.

NPV VS. IRR

Focus On: Calculations

- Consider Project C with the following cash flows:
  - The NPV is $28,600.26.
  - The IRR is 24.42%.

$45,000 at t=3, $40,000 at t=2, $40,000 at t=4, $30,000 at t=1

t = 0

t = 1   t = 2   t = 3   t = 4

Hurdle rate = 10%
r = 10%

−$90,000

	Project A	**Project B**	**Project C**
NPV	$29,872.52	$27,783.12	$28,600.26
IRR	21.84%	25.62%	24.42%
Decision	Accept	Accept	Accept

- If the projects are independent, you accept all three.

- If the projects are mutually exclusive, you accept Project A even though it has the smallest IRR.

- If Projects B and C are mutually exclusive, you accept Project C.

IRR CHALLENGES

IRR is a very appealing measure because it is intuitive; we all understand (or think we do) rates of return.

- Unfortunately, IRR has several shortcomings.

  - We will only realize the IRR as calculated if we

  a) can reinvest all the project cash flows at that IRR, and

  b) hold the investment to maturity.

  - IRR and NPV *can* give different rankings when

    - The scale of the projects being compared is different

    - The timing of the cash flows is different

- **Conclusion**: NPV should be preferred to IRR.

PORTFOLIO RETURN MEASUREMENT

- **Performance measurement** is the calculation of the return on an asset or a portfolio of assets.

  - This is particularly challenging when funds flow into or out of the portfolio or when the composition of the assets changes over time.

  - Holding period returns are one fundamental building block of performance measurement.

$$\text{HPR} = \frac{P_1 - P_0 + D_1}{P_0}$$

- **Performance appraisal** is the comparison of the rate of return on an asset or a portfolio of assets to an appropriate required (desired) rate of return.

  - The rate against which the performance is being assessed is often known as a benchmark rate and may be a rate on a reference set of assets, such as an index.

HOLDING PERIOD RETURN

Focus On: Calculations

- You recently purchased shares of a dividend-paying utility, Old Dominion Co., for $23.50. Today, ODC has just paid a $1.34 dividend, and you have decided to sell your shares. Your market sell order is executed at $24.36. What is your holding period return?

HPR = ?

$P_1 = \$24.36$
$D_1 = \$1.34$

$P_0 = \$23.50$

$$\text{HPR} = \frac{P_1 - P_0 + D_1}{P_0}$$

$$\text{HPR} = \frac{\$24.36 - \$23.50 + \$1.34}{\$23.50} = 0.0936 = 9.36\%$$

MONEY-WEIGHTED RATE OF RETURN

The internal rate of return calculated on a portfolio is called the "money-weighted rate of return."
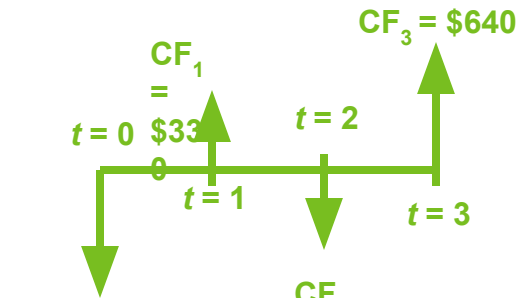
- Money-weighted rate of return accounts for the timing and amount of dollar flows into and out of the portfolio.

  - More weight is placed on periods in which more is invested.

- Consider the following cash flows for a portfolio over three years.

	Time			
	0	1	2	3
Share Price	$300	$325	$315	$320
Activity	Buy 2 shr	Div = $2.50/shr Sell 1 shr after	Div = $2.50/shr Buy 1 shr	Sell 2 shrs
Inflows:	--	$5.00 + $325	$2.50	$640
Outflows	−$600	--	−$315	--
Net CF	−$600	$330	−$312.50	$640

MONEY-WEIGHTED RATE OF RETURN

Focus On: Calculations

- Recall: Money-weighted return (MWR below) is inherently an IRR.

$CF_3 = \$640$

$CF_1 = \$330$

$t = 0$ $t = 2$

$t = 1$ $t = 3$

$CF_0 = -\$600$

$CF_2 = -\$312.50$

$$0 = -\$600 + \frac{\$330}{(1 + \text{MWR})^1} + \frac{-\$312.50}{(1 + \text{MWR})^2} + \frac{\$640}{(1 + \text{MWR})^3}$$

$$\text{MWR} = ?$$

MONEY-WEIGHTED RATE OF RETURN

Focus On: Calculations

- Recall: Money-weighted return (MWR below) is inherently an IRR.

$CF_3 = \$640$

$CF_1 = \$330$

$t = 0$

$t = 1$

$t = 2$

$t = 3$

$CF_0 = -\$600$

$CF_2 = -\$312.50$
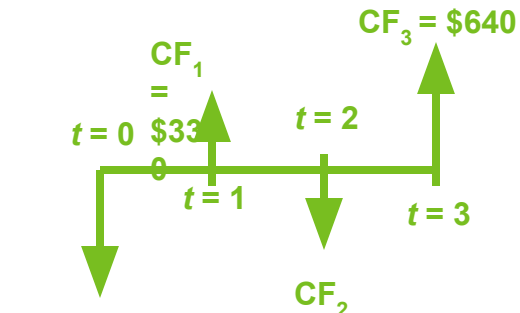
$$0 = -\$600 + \frac{\$330}{(1 + \text{MWR})^1} + \frac{-\$312.50}{(1 + \text{MWR})^2} + \frac{\$640}{(1 + \text{MWR})^3}$$

$$\text{MWR} = 3.8095\%$$

TIME-WEIGHTED RATE OF RETURN

Focus On: Calculations

	Time			
	0	1	2	3
Share Price	$300	$325	$315	$320
Activity	Buy 2 shr	Div = $2.50/shr Sell 1 shr after	Div = $2.50/shr Buy 1 shr	Sell 2 shrs
Change in Value		$650 + $5.00 – $600	$315 + $2.50 – $325	$640 – 630
Initial Value		$600	$325	$630
HPR		0.09167	–0.02307	0.01587

$$\text{TWR} = $$

TWR =
?

TIME-WEIGHTED RATE OF RETURN

Focus On: Calculations

	Time			
	0	1	2	3
Share Price	$300	$325	$315	$320
Activity	Buy 2 shr	Div = $2.50/shr Sell 1 shr after	Div = $2.50/shr Buy 1 shr	Sell 2 shrs
Change in Value		$650 + $5.00 – $600	$315 + $2.50 – $325	$640 – 630
Initial Value		$600	$325	$630
HPR		0.09167	–0.02307	0.01587

$$\text{TWR} = \sqrt[3]{(1.09167)(0.97693)(1.01587)} - 1$$

TWR = 2.706 %

TWR VS. MWR

- Money-weighted returns place greater weight on those periods in which investment is higher and, therefore, give a "better" picture of the actual investor experience.

- Time-weighted returns remove the effect of inflows and outflows to the portfolio and are, therefore, a better indicator of managerial skill.

- In our example, the fund had significantly more money invested during the positive return years, from $t = 0$ to $t = 1$ and from $t = 2$ to $t = 3$, causing the money-weighted return to be much higher than the time-weighted return.

DIFFERING MONEY MARKET YIELDS

Instruments that mature in less than a year are known as money market instruments.

- There are a number of different conventions for calculating yields on money market instruments.

  - Bank discount yield

  - Holding period yield

  - Effective annual yield

  - Money market yield

BANK DISCOUNT YIELD

By convention, Treasury instruments of less than a year in original maturity (T-bills) have yield quoted on a bank discount basis.

- **Bank discount yield** is a percentage of face value instead of price and assumes 360 days in a year.

$$r_{BD} = \frac{D}{F}\left(\frac{360}{t}\right) = \frac{Par - Price}{Par}\left(\frac{360}{t}\right)$$

  - For a $10,000 par value T-bill trading at $9,945.70 with 67 days to maturity, the bank discount yield is

$$r_{BD} = \frac{\$10,000 - \$9,945.70}{\$10,000}\left(\frac{360}{67}\right) = 0.0292 = 2.92\%$$

  - Problems: ignores compounding, is a percentage of face value instead of price, and uses 360 days instead of 365.

HOLDING PERIOD YIELD (HPY)

Calculated the same way as a holding period return.

- HPY measures the (unannualized) return over the remaining life of the investment when P_1 is the return of face value.

$$\text{HPY} = \frac{P_1 - P_0 + D_1}{P_0}$$

- For a $10,000 par value T-bill trading at $9,945.70 with 67 days to maturity, the HPY is

$$\text{HPY} = \frac{\$10,000 - \$9,945.70}{\$9,945.70} = 0.00546 = 0.546\%$$

- Important aside: Make sure the prices used include any accrued interest.

EFFECTIVE ANNUAL YIELD (EAY)

Calculated the same way as equivalent annual yield using the holding period yield.

- The annualized expression of the holding period yield is

$$EAY = (1 + HPY)^{365/t} - 1$$

- For a $10,000 par value T-bill trading at $9,945.70 with 67 days to maturity, the EAY is

$$EAY = (1 + 0.00546)^{365/67} - 1 = 0.03011 = 3.011\%$$

MONEY MARKET YIELD

This convention makes the quoted yield on a T-bill comparable to yield quotations on interest-bearing money market instruments that pay interest on a 360-day basis.

- Also known as the CD equivalent yield.

- Generally, the money market yield will be equivalent to the holding period yield adjusted to a 360-day basis.

$$r_{MM} = \frac{360(r_{BD})}{360 - t(r_{BD})}$$

- For a $10,000 par value T-bill trading at $9,945.70 with 67 days to maturity, the money market yield is

$$r_{MM} = \frac{360(0.0292)}{360 - 67(0.0292)} = 0.02936 - 2.936\%$$

YIELD CONVERSIONS

We can convert back and forth using holding period yield, which is common to all of the calculations.

- Bank discount yield is always less than effective annual yield or money market yield.

- Given any one of HPY, r_{MM}, or r_{BD}, we can calculate the others.

$$\text{HPY} = \left(\frac{360}{t}\right) = r_{MM} = r_{BD}\frac{F}{P_0}$$

- Recall for our bond that $r_{MM} = 0.02936$.

- If HPY = 0.00546, then $r_{MM} = 0.00546\left(\frac{360}{67}\right) = 0.02934$.

- If $r_{BD} = 0.0292$, then $r_{MM} = 0.0292\frac{\$10,000}{\$9,945.70} = 0.02936$.

BOND-EQUIVALENT YIELD (BEY)

An annualized periodic yield wherein the process of annualizing is accomplished by multiplying the periodic yield by the number of periods in a year.

- By convention, the yield on most bonds is expressed as a bond-equivalent yield instead of as an equivalent annual yield.

- Problem: This process ignores intraperiod compounding.

- Calculating a BEY:

  - Compute the period yield to maturity (YTM).

  - Multiply the periodic YTM by the number of periods in a year.

    - For example: YTM x 2 for semiannual, YTM x 4 for quarterly

SUMMARY

- The process of evaluating projects incorporates two primary decision criteria known as net present value (NPV) and internal rate of return (IRR).

  - When projects are independent, NPV and IRR can be used interchangeably.

  - When projects are mutually exclusive, analysts should use the NPV criteria.

- Portfolio performance measurement can be determined in several ways, including money-weighted rate of return and time-weighted rate of return.

  - When the investor controls additions and withdrawals from the portfolio, the money-weighted rate of return may be the appropriate measure.

  - The time-weighted rate of return is the standard in the investment management industry because it better represents managerial skill.

- By tradition, money market instrument yields are calculated using different techniques for different instruments.