Confidence Interval Estimation

Types of Estimates

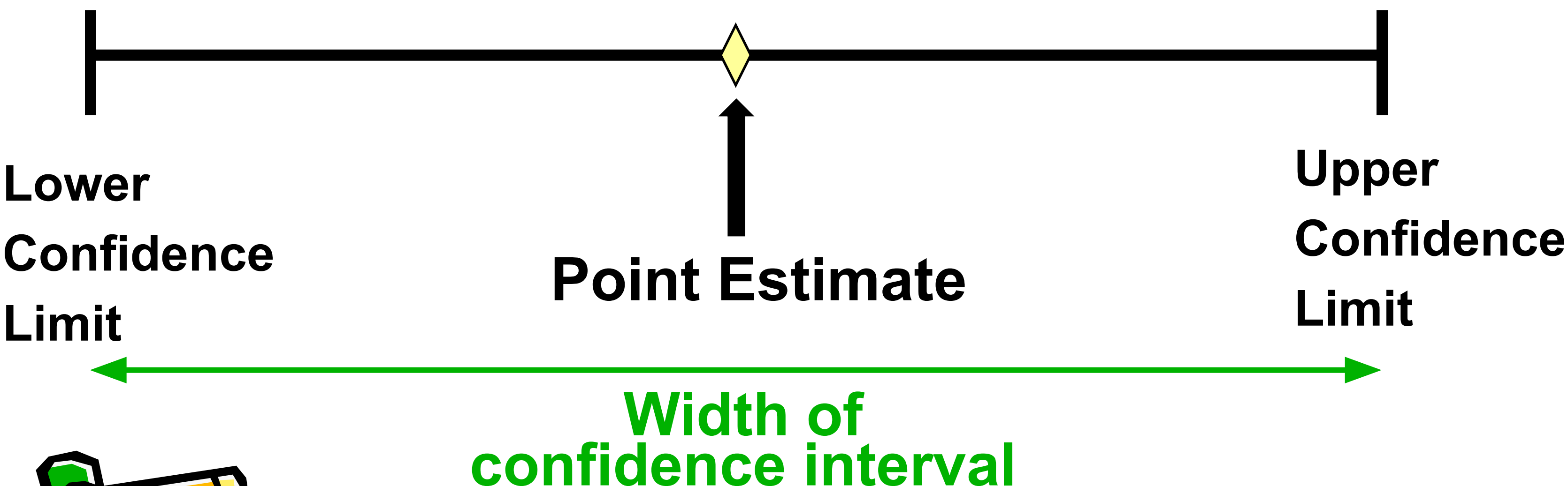
- Point Estimate
  - A single number used to estimate an unknown population parameter
- Interval Estimate
  - A range of values used to estimate a population parameter
  - Characteristics
    - Better idea of reliability of estimate
    - Decision making is facilitated

Point Estimates

Estimate Population Parameters …		with Sample Statistics
Mean	μ	$\bar{X}$
Standard deviation	σ	S
Variance	σ^2	S^2
Difference	$\mu_1 - \mu_2$	$\bar{X}_1 - \bar{X}_2$

Point and Interval Estimates

- A point estimate is a single number,
- a confidence interval provides additional information about variability



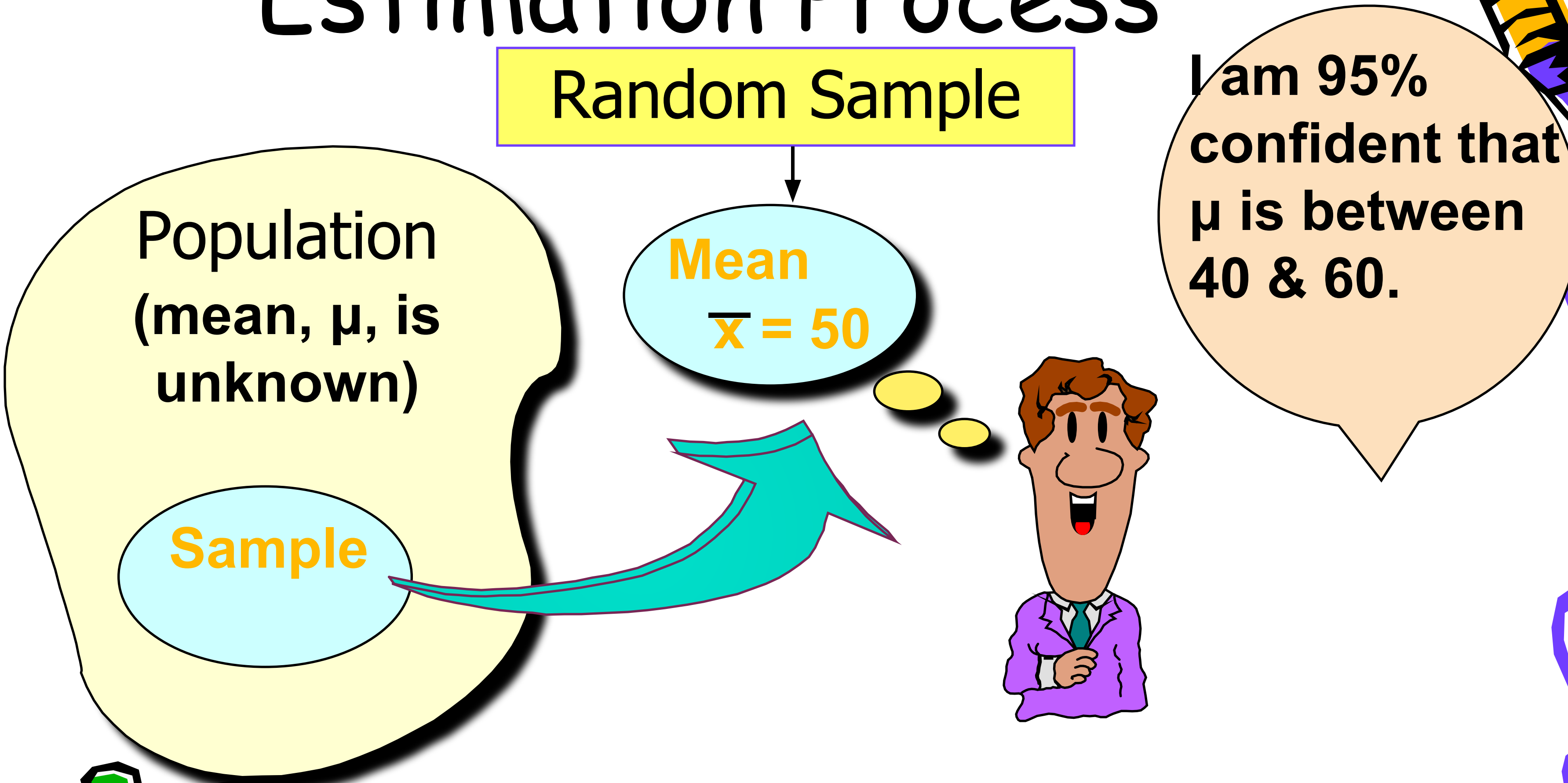
Confidence Interval Estimate

- An interval gives a range of values:
  - Takes into consideration the variation in sample statistics from sample to sample
  - Based on observation from 1 sample
  - Gives information about closeness to unknown population parameters
  - Stated in terms of level of confidence
    - Can never be 100% confident

Confidence Level, $(1-\alpha)$

- Suppose confidence level $\gamma = 95\%$
- Also written $\gamma = (1 - \alpha) = .95$
- Where α is the risk of being wrong
- A relative frequency interpretation:
  - In the long run, 95% of all the confidence intervals that can be constructed will contain the unknown parameter
- A specific interval either will contain or will not contain the true parameter
  - No probability involved in a specific interval

Estimation Process

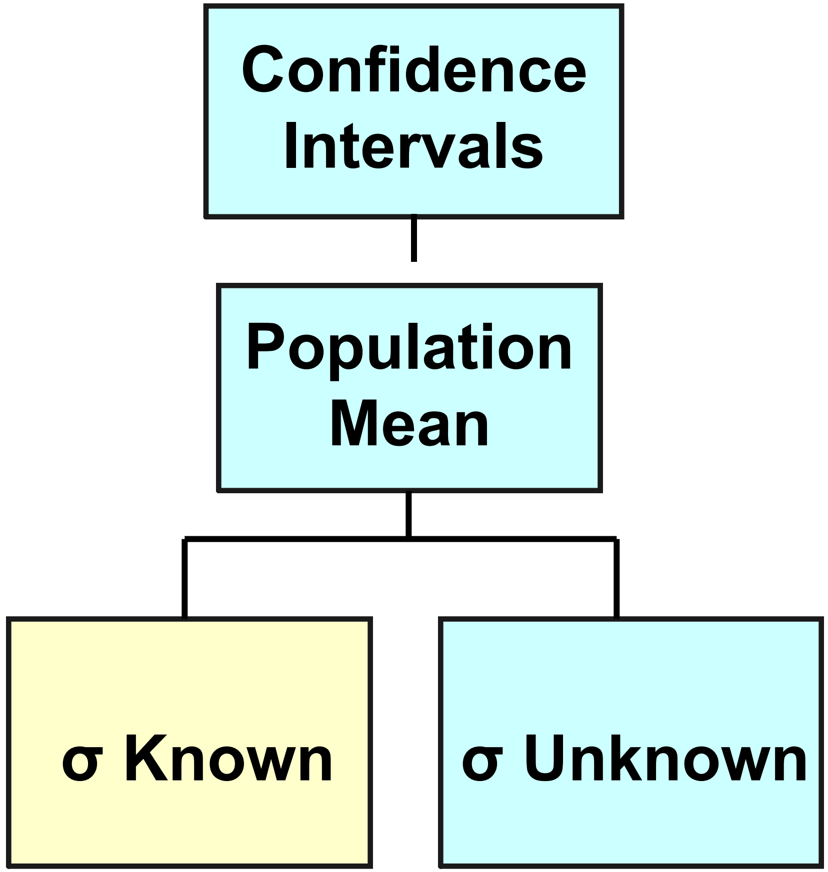


General Formula

- The general formula for all confidence intervals is:

Point Estimate ± (Critical Value)(Standard Error)

Confidence Intervals



Confidence Interval for μ (σ Known)

- Assumptions
  - Population standard deviation σ is known
  - Population is normally distributed
  - If population is not normal, use large sample
- Confidence interval estimate for μ

$$\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

Critical Value

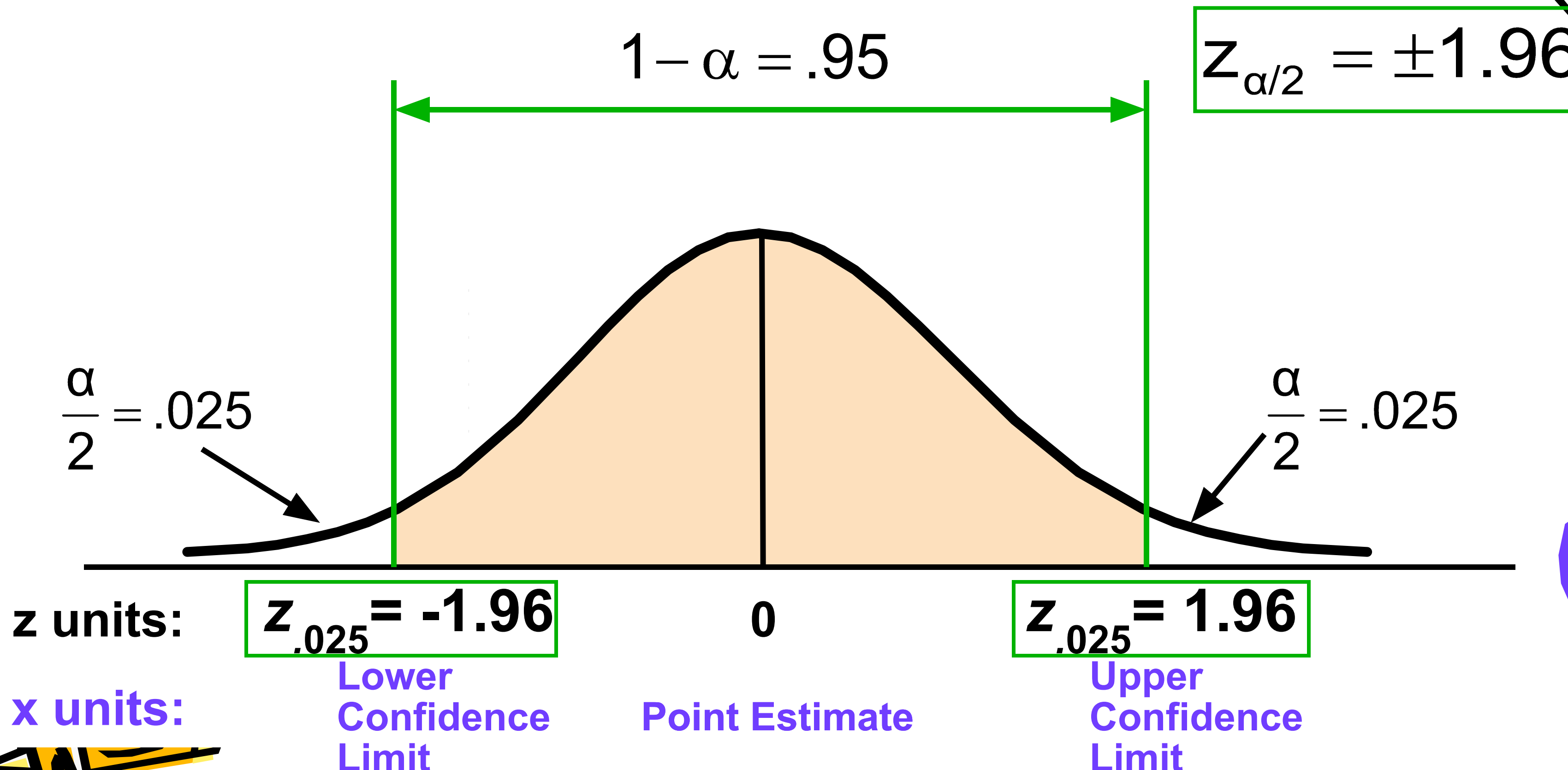
Standard error

Finding the Critical Value

Confidence Level γ	Confidence Coefficient, $\gamma = 1-\alpha$	z value, $z_{\alpha/2}$
80%	.80	1.28
90%	.90	1.645
95%	.95	1.96
98%	.98	2.33
99%	.99	2.57
99.8%	.998	3.08
99.9%	.999	3.27

Finding the Critical Value

- Consider a 95% confidence interval:



Margin of Error

- Margin of Error (e): the amount added and subtracted to the point estimate to form the confidence interval

Example: Margin of error for estimating μ, σ known:

$$\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

$$e = z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

Factors Affecting Margin of Error

$$e = z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

Intervals Extend from

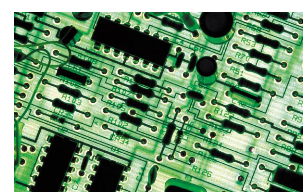
$\bar{X} - Z\sigma_{\bar{x}}$ to $\bar{X} + Z\sigma_{\bar{x}}$

- Data variation, σ : e $\downarrow$ as σ $\downarrow$
- Sample size, n : e $\downarrow$ as n $\uparrow$
- Level of confidence, 1 - α : e $\downarrow$ if γ = 1 - α $\downarrow$

Example

A sample of 11 circuits from a large normal population has a mean resistance of 2.20 ohms. We know from past testing that the population standard deviation is .35 ohms.

Determine a 95% confidence interval for the true mean resistance of the population.



Solution –

$$\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

$$= 2.20 \pm 1.96\,(.35/\sqrt{11})$$

$$= 2.20 \pm .2068$$

$$(1.9932\ \ .;\ 2.4068\ )$$

To get a *Z* value use the **NORMSINV** function with

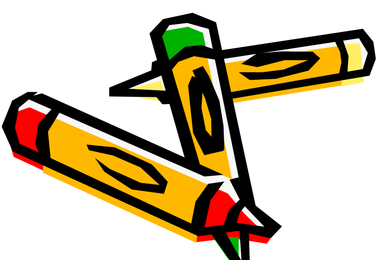
p=γ+ alpha/2

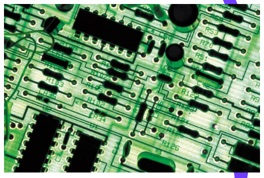
for 95% confidence use 0.975

= NORMSINV(0,975)

=NORM.S.INV(0,975)

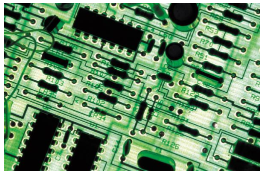
Result =1.96





Interpretation

- We are γ=95% confident that the true mean resistance is between 1.9932 and 2.4068 ohms



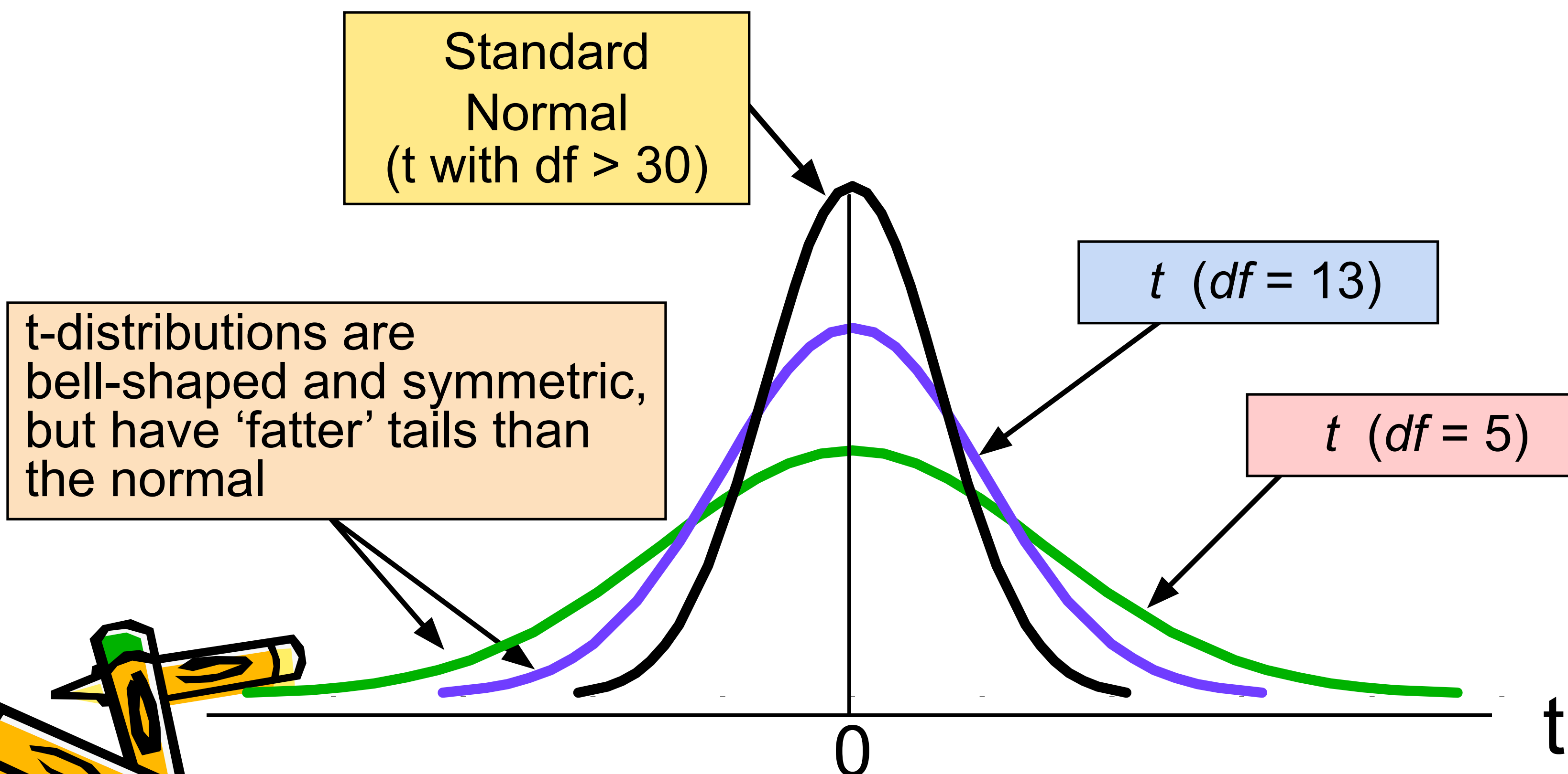
Confidence Interval for μ (σ Unknown)

- If the population standard deviation σ is unknown, we can substitute the sample standard deviation, s as an estimate

In these case the **t-distribution** is used instead of the normal distribution

Student's t Distribution

Note: t ⟶ Normal as n increases



Confidence Interval for μ (σ Unknown)

- Assumptions
  - Population standard deviation is unknown
  - Population is not highly skewed
  - Population is normally distributed or the sample size is large (>30)
- Use Student's t Distribution

- Confidence Interval Estimate:

$$\bar{x} \pm t_{\gamma} \frac{s}{\sqrt{n}}$$

- **where t is the critical value of the t-distribution with n-1 degrees of freedom and an area of α/2 in each tail)**

Define t_γ from equation

$$P\left(|T| < t_\gamma\right) = \gamma$$

γ – ***Confidence Coefficient.***

t_γ – obtain with using *Excel function* TINV.

t_γ = TINV(*1 – γ; n–1*)

=T.INV.2T *(1 – γ; n–1)*

Example

A random sample of $n = 25$ has $\overline{X} = 50$ and $S = 8$. Form 95% confidence interval for μ

- **degrees of freedom** = $n - 1 = 24$,
- $\gamma = 0{,}95$.

To get a **t - value** use the **TINV** function.

The value of ***alpha =(1-confidence)*** and **n-1** degrees of freedom are the inputs needed.

For **95%** confidence use ***alpha*** **=0.05** and for a sample size of **25** use **24** df

$$t_{\gamma} = \text{TINV}(0{,}05;\ 24) = 2{,}0639$$

$$t_{\gamma} = \text{T.INV.2T}(0{,}05;24) = 2{,}0639$$

$$\bar{X} \pm t_{\alpha/2,\, n-1} \frac{S}{\sqrt{n}} = 50 \pm (2.0639) \frac{8}{\sqrt{25}}$$

46.698 ….. μ ….. 53.302

$46.698 \leq \mu \leq 53.302$

Confidence Interval on the Variance and Standard Deviation of a Normal Distribution

Definition

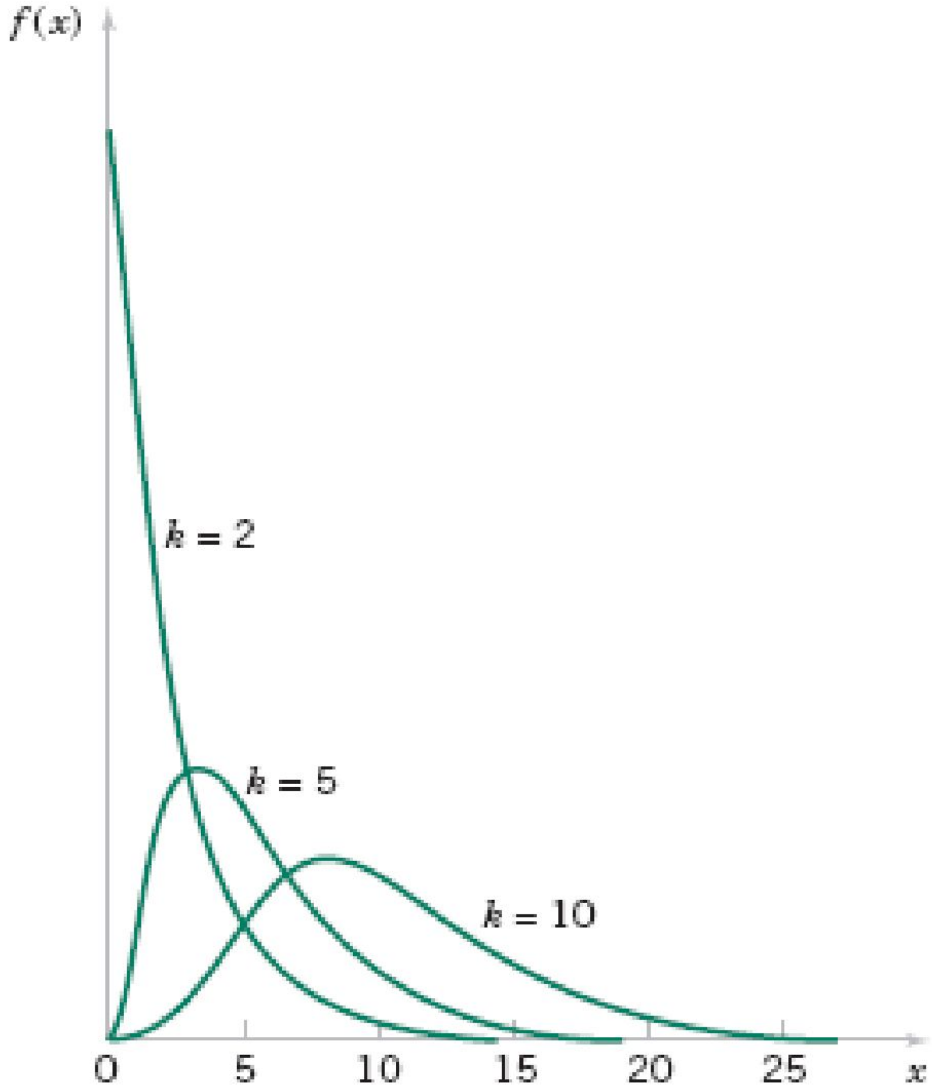
Let $X_1, X_2, \ldots, X_n$ be a random sample from a normal distribution with mean μ and variance σ^2, and let S^2 be the sample variance. Then the random variable

$$X^2 = \frac{(n-1)S^2}{\sigma^2} \qquad (8\text{-}19)$$

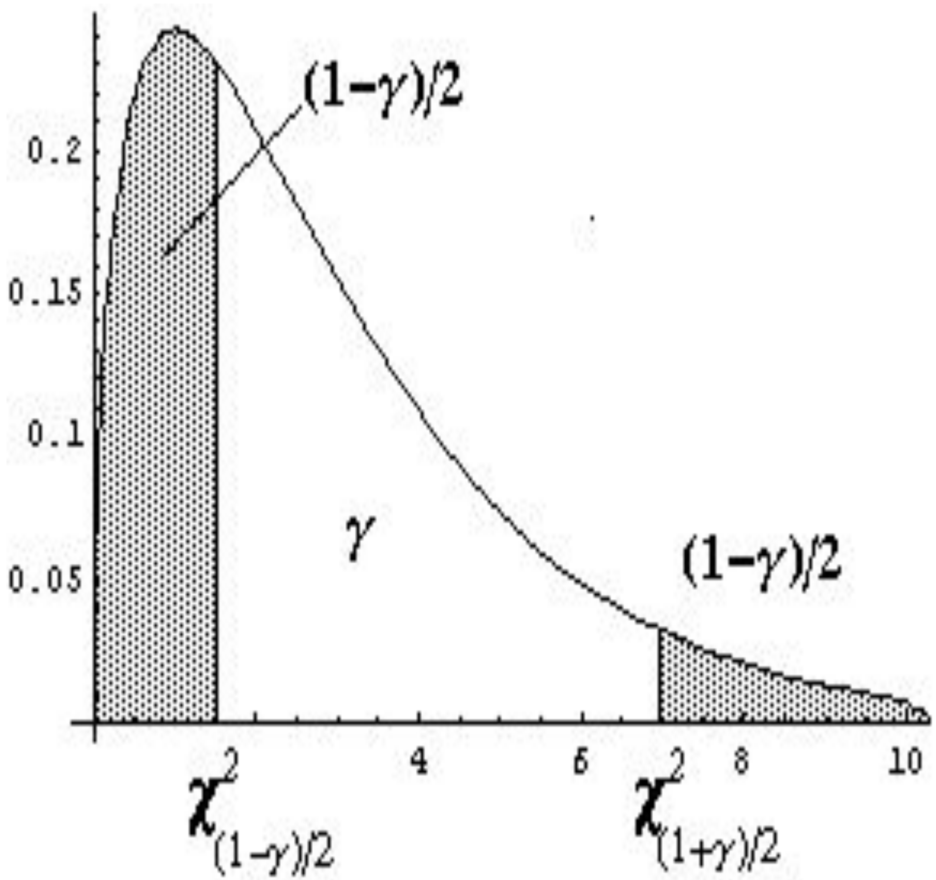
has a chi-square (χ^2) distribution with $n - 1$ degrees of freedom.

Confidence Interval on the Variance and Standard Deviation of a Normal Distribution

Probability density functions of several χ^2 distributions.



The confidence interval is chosen so that the area under the lower tail will equal area under upper tail , i.e. the probabilities are equal of hitting the right and left area .



Confidence Interval on the Variance and Standard Deviation of a Normal Distribution

$$P\left(\frac{\sqrt{n-1}S}{\sqrt{\chi_2^2}} < \sigma < \frac{\sqrt{n-1}S}{\sqrt{\chi_1^2}}\right) = \gamma$$

where χ_1^2 is the critical value of the χ^2 distribution with n-1 degrees of freedom and $a_1 = (1+\gamma)/2$;

χ_2^2 is the critical value of the χ^2 distribution with n-1 d.f. and $a_2 = (1-\gamma)/2$.

We can use χ^2 –Table for solving next equation

$$P\left(\chi^2 \geq \chi_q^2\right) = q$$

Or EXCEL function ***CHIINV (q; n-1),***

=CHISQ.INV.RT(q;n-1).

EXAMPLE

According to the 20 measurements found standard deviation S = 0,12. Find precision measurements with reliability 0.98.

With using ***CHIINV (q; n-1)*** ***we*** obtain χ_1^2 i χ_2^2 .

For degrees of freedom *n - 1=19* and probability *α2=(1-0,98)/2=0,01* define

$$\chi_2^2 = 36{,}2,$$

after that for *n - 1=19* and probability *α1=(1+0,98)/2=0,99* define $\chi_1^2 = 7{,}63$.

χ_2^2 = *CHIINV(0,01; 19)*=36,2 ;

=CHISQ.INV.RT(0,01;19).

χ_1^2 = *CHIINV(0,99;19)*=7,63.

=CHISQ.INV.RT(0,01;19).

calculate

$$\frac{\sqrt{20-1}*0,12}{\sqrt{36,2}}=\frac{4,36*0,12}{6,06}=0,09; \quad \frac{\sqrt{20-1}*0,12}{\sqrt{7,63}}=\frac{4,36*0,12}{2,76}=0,19.$$

Confidence Interval for σ is *(0,09;0,19).*