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# Forward and futures contracts and cash flows engineering

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# Lesson objectives

- Introduce the concept of futures and forward contracts.
- Consider differences between futures and forwards.
- Analyze futures and forwards payoffs and cash flows.
- Consider examples of cash flow engineering with futures and forwards.

# Introduction

- Forward and future contracts represent one of the basic types of financial derivatives.
- Both futures and forwards can fix the future selling or buying price which allows to use them for arbitraging hedging  and pricing purposes.
- In both cases counterparties commit to buy or sell the asset.
- However, futures  differ from forwards in terms of flexibility, cash flows calculation, counterparty risk etc.

# Futures vs forwards

| FUTURES | FORWARDS |
|---|---|
| Traded on exchanges | OTC |
| Standardized, highly liquid | Customized |
| Low counterparty risk | High counterparty risk |
| Initial margin payment | No initial payment |
| Regulated | Unregulated |
| Marked-to-market daily | Net gain/loss at expiration |

# Example of commodity futures contract

- NYMEX crude oil futures with delivery in Dec 2008 traded in Sep 12 2008 at a price $101.18 per barrel.

a) 1000 barrels for each contract
c) Initial margin: $ 4050
d) Maintenance margin : $3000
e) Contract price: 0
f) Buyer has a “long” position
g) Seller has a “short” position

# Futures contract mechanism 1

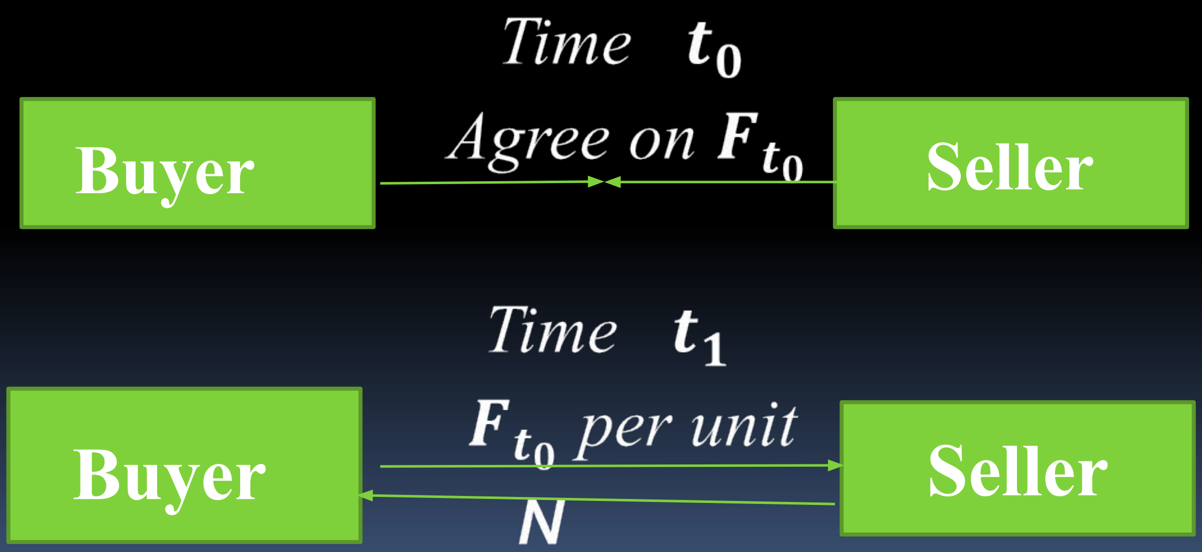
- Example: futures contract for 1000 ounces of gold concluded on Dec 12 with expiration on Dec 15
- Agreed price : $500/oz
- Dec 12 settlement: $495
- Dec 13 settlement: $491
- Dec 14 settlement : $497
- Dec 15 settlement: $498

# Futures contract mechanism 2

# Forward contract definition

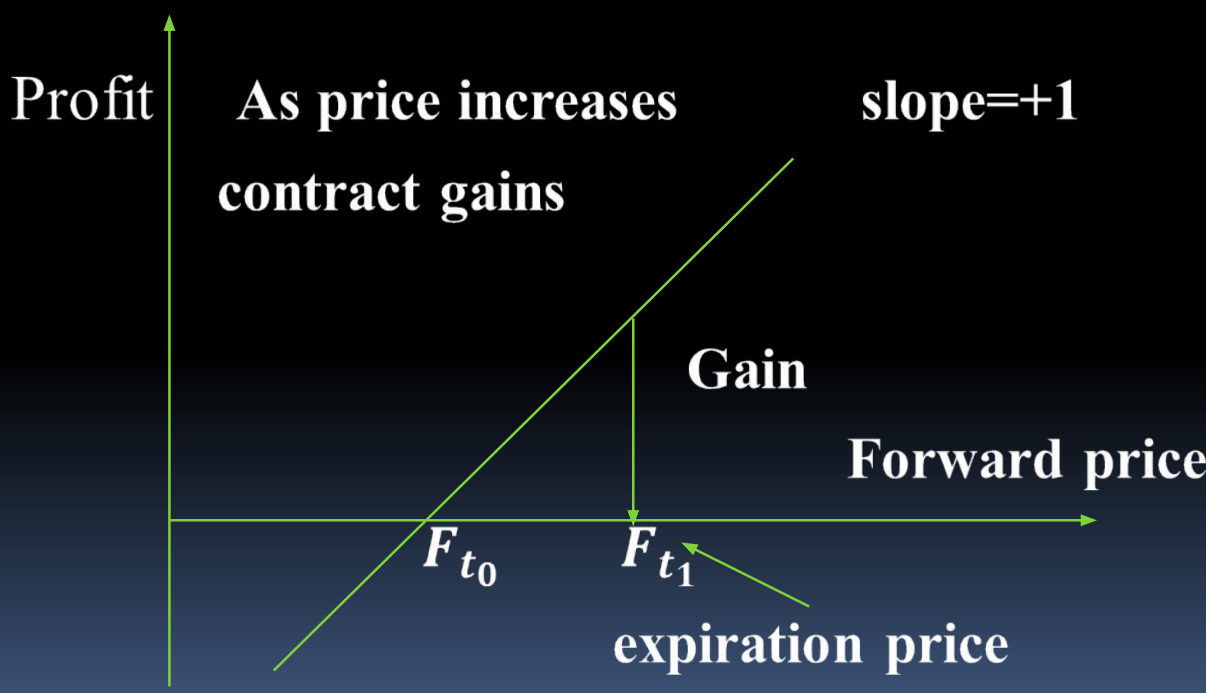
- **A forward is a contract written at time $t_0$ with a commitment to accept delivery ( or deliver) $N$ units of underlying asset at future date $t_1$ at forward price $F_{t_0}$.**

- $F_{t_0}$ is used at settlement of the contract at time $t_1$ .



# Payoff diagram for forward contract; Long position

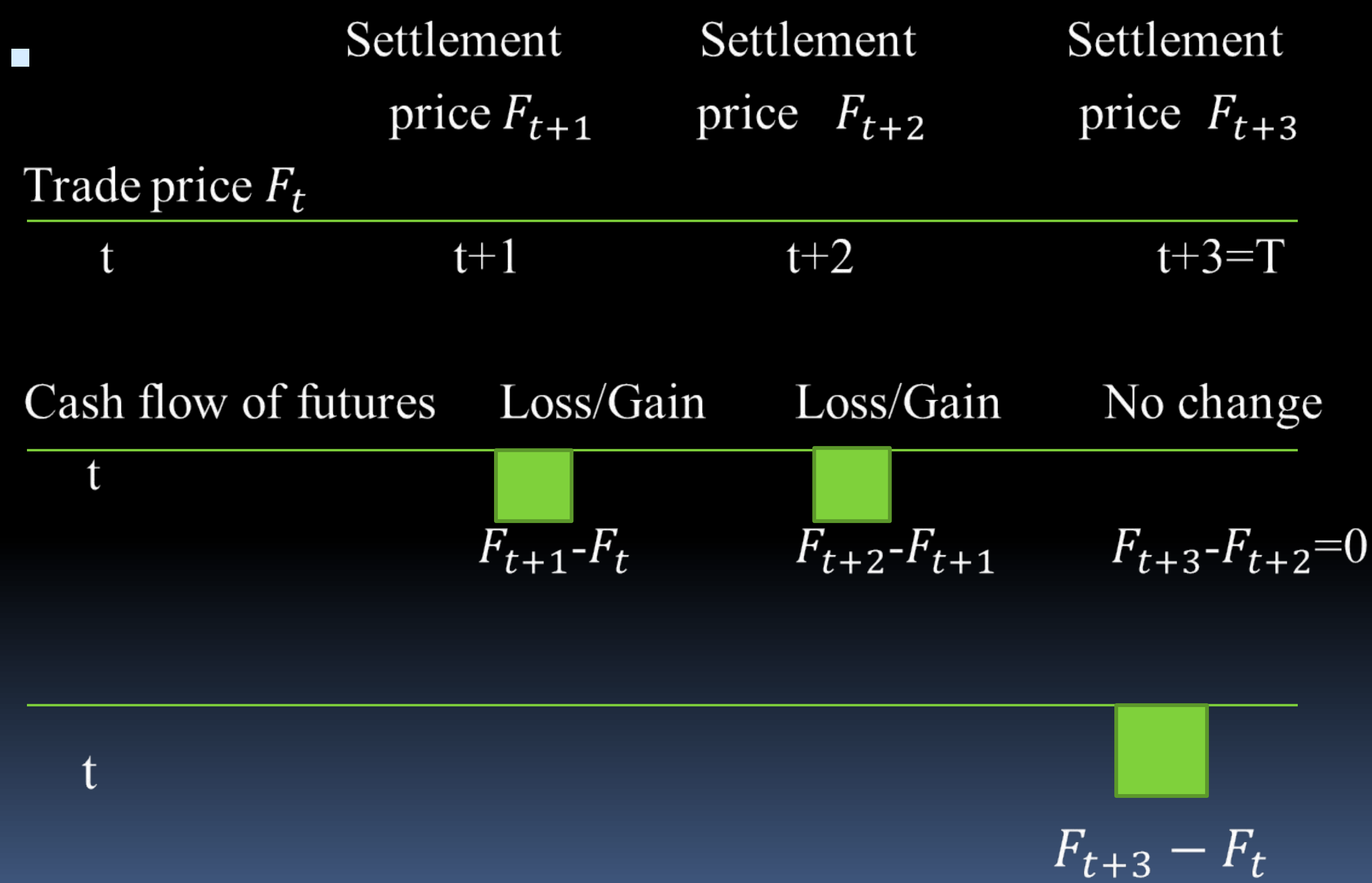
- Organized exchanges are formal entities . Traded instruments and trading procedures are standardized.



# Types of forward and future contracts

- ***Forwards on interest rates***
- ***Forwards on currencies***
- ***Futures on commodities***
- ***Futures on loans and deposits***
- ***Futures and forwards on stocks and stock indices***
- ***Futures contracts on interest rate swaps***
- ***Futures contacts  on volatility indices***

# Cash flows comparison: Futures vs forwards



# Forward and futures prices

- Forward and futures contract prices can be derived from spot market prices based on no arbitrage principles.

- Let’s consider crude oil forwards contract for instance.

- Convenience yield is defined as the amount of benefit associated with physically owning a particular good instead of owning futures contract for that good.

# Forward and futures prices 2

- Let's denote by $c$ the percentage costs associated with holding the physical amount of oil.

- $S_0$ stands for spot price , $F$ is the futures contract price and $r_F$ is the risk-free rate.

- Consider two alternative :

*a) Buy underlying asset and store until T (maturity of futures)*

*b) Buy underlying asset and hold it until T*

# Forward and futures prices 3

- No arbitrage implies that those two alternative should have the same value. Thus:

$$F = S_0(1+r_F-(\text{y-c}))^T$$

- Now let's consider the forwards contract which has stock as underlying asset . Let's denote dividends by *d*.

- Using the same no arbitrage condition we can write:

$$F = S_0(1+r_F-d))^T$$

# Forward and futures prices 4

- Finally consider the forwards contact which has treasury bond as underlying asset.
- Denote by $r$ the spot interest rate and by $y$ the coupon yield.
- Using no arbitrage we determine the price of this forwards contract as :

$$F = S_0(1+\mathrm{r}-\mathrm{y})^T$$

# Synthetic instrument concept

- Financial instruments can be visualized as bundles of cash flows, which allows to trade cash flows that with different characteristics and different risks.
- Using financial engineering methods we analyze cash flows generated by an instrument during the lifetime of its contract.
- Then , using other more liquid financial instruments, the portfolio that replicates these cash flows exactly is formed.

This portfolio is called replacing portfolio or synthetic.

# Forward loan

- Forward loan is engineered like a usual forward contract with loan being an underlying asset.
- At settlement date $t_1$ the traders receives(delivers) a loan which matures at $t_2$.
- The contract also specifies interest rate applied to this loan. Denote this rate by $F(t_0,t_1, t_2)$.

# Forward loan importance

- Forward loan is successfully used in the following cases:

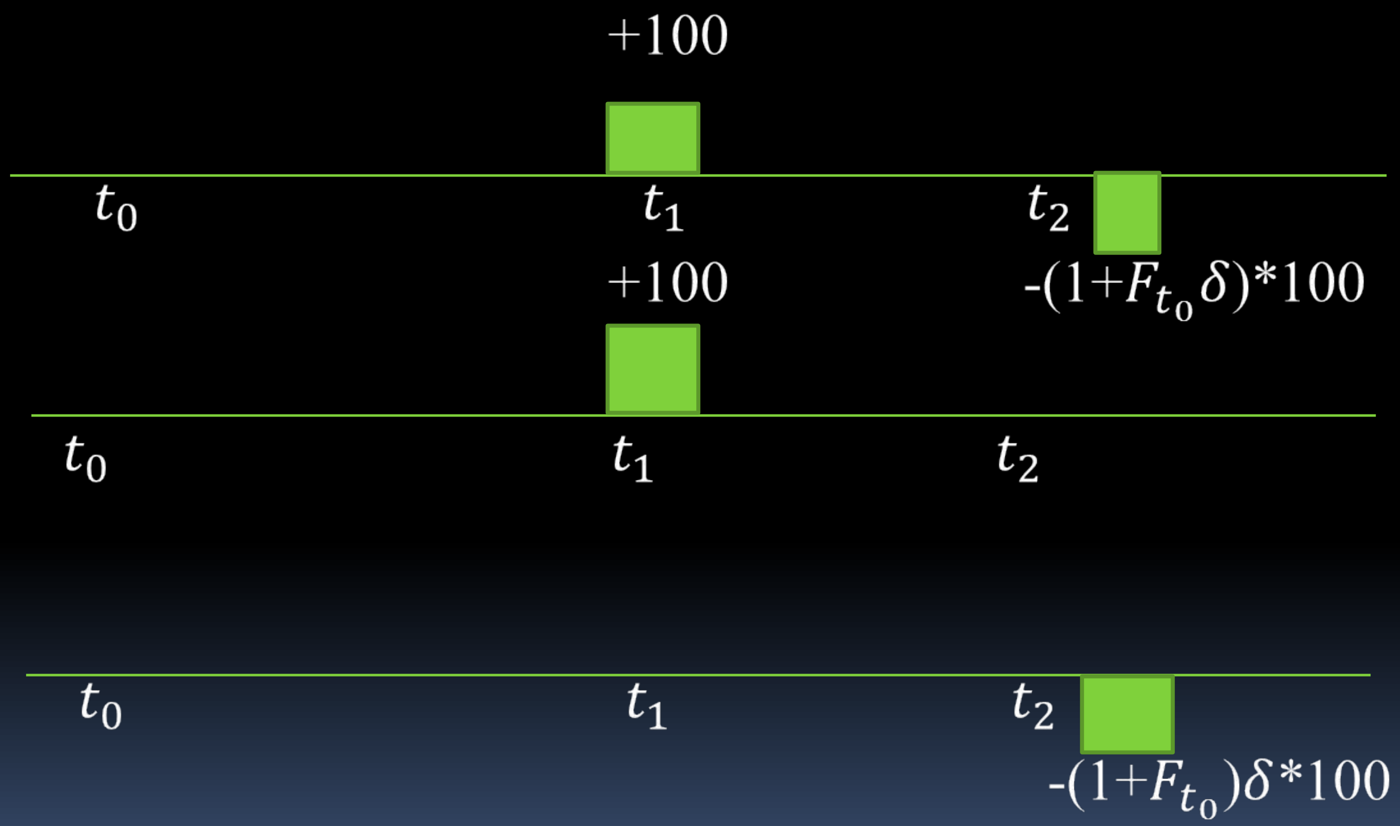
a) Business wants to lock the current low borrowing rate .

b) Banks want to lock the current “high” lending rate.

c) A business will face a liability depending on floating rate at a future date and wants to hedge against the risk by a future loan with known cost.

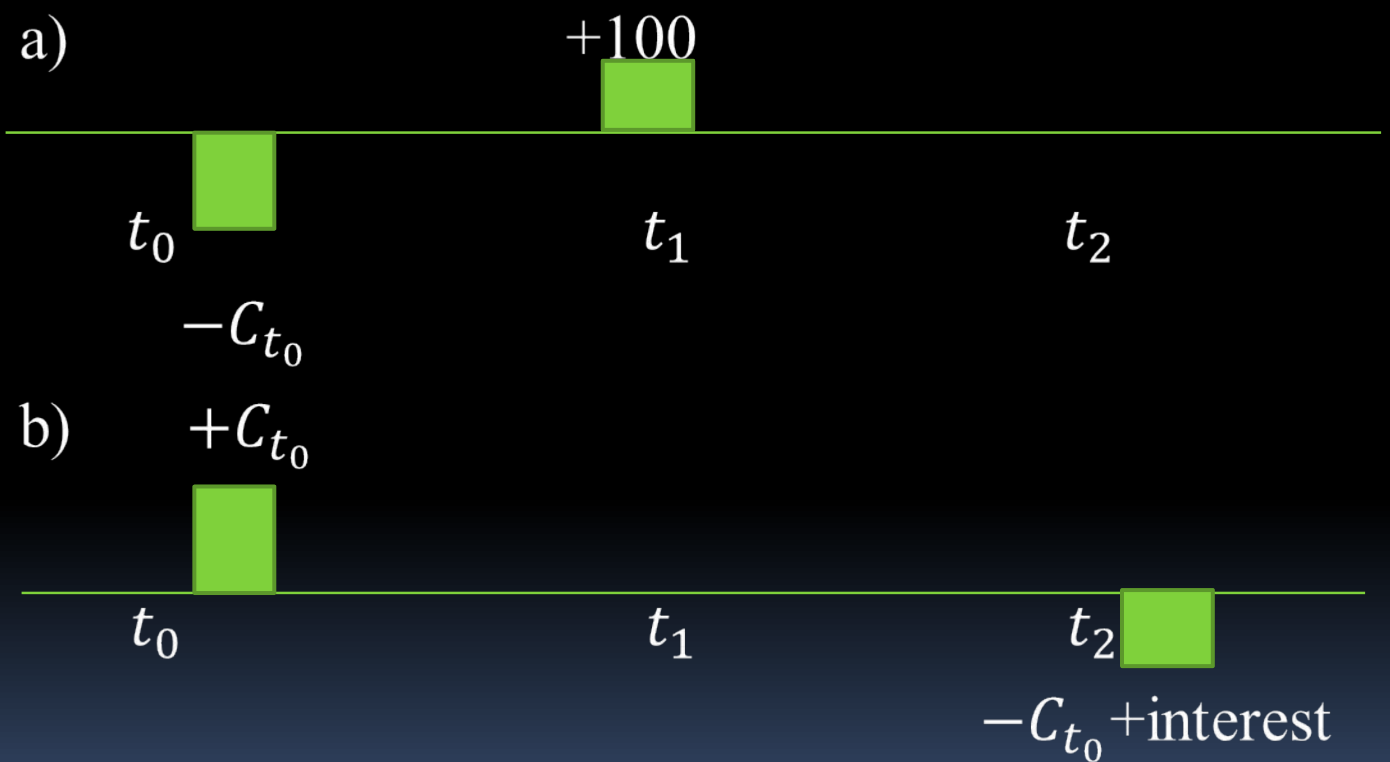
# Cash flows diagram

- To construct the synthetic for forward loan first decompose cash flows.



# Cash flows diagram 2

- Now add and subtract the same amount from two cash flows at initial date $t_0$



# Synthetic or replacing portfolio using bonds

- Cash flows in a) of the previous diagram are equivalent to long position in $t_1$ maturity discount bond.
- Cash flows in b) of the previous diagram are equivalent to short position in $t_2$ maturity discount bond.
- The price of pure discount bond with maturity $T$ and amount \$100 is determined by the following formula :

$$B(t,T) = \frac{100}{(1+R)^{\left(\frac{T-t}{365}\right)}}$$

# Synthetic or replacing portfolio using bonds 2

- It's clear that price of discount bond is inversely related to its maturity.
- Thus $B(t_0, t_2) < B(t_0, t_1)$
- It's intuitive that the value of initial cash flow $C_{t_0}$ is given by :

$$B(t_0, t_1) = C_{t_0}$$

To create correct synthetic we need to short more than one unit of longer maturity discount bond as it has lower price.

# Synthetic or replacing portfolio using bonds 3

- The number $\lambda$ of units of $t_2$ maturity discount bond which is needed to be used , can be determined as follows:

$$\lambda B(t_0, t_2) = C_{t_0}$$

**First possible synthetic for forward loan**

$\{Buy\ one\ t_1\ maturity\ discount\ bond, short\ \lambda\ units\ of\ t_2\ maturity\ discount\ bond\}$

- No arbitrage condition requires the following to hold:

$$1 + F_{t_0}\delta = \frac{B(t_0, t_1)}{B(t_0, t_2)}$$

# Synthetic using money market instrument

- $C_{t_0}$ borrowed on money market at interbank rate $L_{t_0}^2$.
- The discounted present value of $t_2$ cash flows is given by:

$$C_{t_0} = \frac{100(1 + F_{t_0}\delta)}{(1 + L_{t_0}^2\delta^2)}$$

- Next redeposit $C_{t_0}$ at rate $L_{t_0}^1$ at a shorter maturity. This will generate the following cash flow

$$C_{t_0}(1+L_{t_0}^1\delta^1)=100$$

# Contractual equations

- Bond synthetic contractual equation

| | | | | |
|---|---|---|---|---|
| Forward loan that begins in $t_1$ and ends in $t_2$ | = | Short B($t_o, t_1$)/ B($t_o, t_2$) of $t_2$ maturity bond | + | Long $t_1$ maturity bond |

- Money market synthetic contractual equation

| | | | | |
|---|---|---|---|---|
| Forward loan that begins in $t_1$ and ends in $t_2$ | = | Loan with maturity $t_2$ | + | Deposit with maturity $t_1$ |

# Forward rate agreement

- *Paid-in-arrears forward rate agreement(FRA)* specifies amount N , dates $t_1$ *and* $t_2$ and price $F_{t_0}$ .
- The buyer of this agreement accepts the receipt of $N\delta(L_{t_1} - F_{t_0})$ at expiration if $L_{t_1} > F_{t_0}$.
- He pays $N\delta(F_{t_0} - L_{t_1})$ if $L_{t_1} < F_{t_0}$ at expiration.
- For market traded FRAs the settlement occurs in $t_1$ and cash flows are discounted by variable LIBOR rate.

# Contractual equations

- The amount $N\delta F_{t_0}$ can be interpreted as time 0 "market value" of random cash flow $N\delta L_{t_1}$

Buying a FRA $=$ Fixed rate loan starting $t_1$ ending $t_2$ $+$ Floating rate deposit starting $t_1$ ending $t_2$

Buying a FRA $=$ Loan with maturity $t_2$ $+$ Deposit with maturity $t_1$ $+$

$+$ Floating rate deposit starting $t_1$ ending $t_2$