TURING MACHINE

LEARNING OBJECTIVES

- **state the function of a Turing Machine and a Universal Turing Machine**

- **explain the algorithm of the Turing machine at an elementary level, using a table diagram of the process**

WHO?

Alan Mathison Turing - English mathematician, logician, cryptographer.

In 1937, the proposed refinement of the concept of the algorithm as a process that can be accomplished with a special machine called a Turing machine in the future.

The concept of "Turing machine" was formulated for 9 years before the first computer.

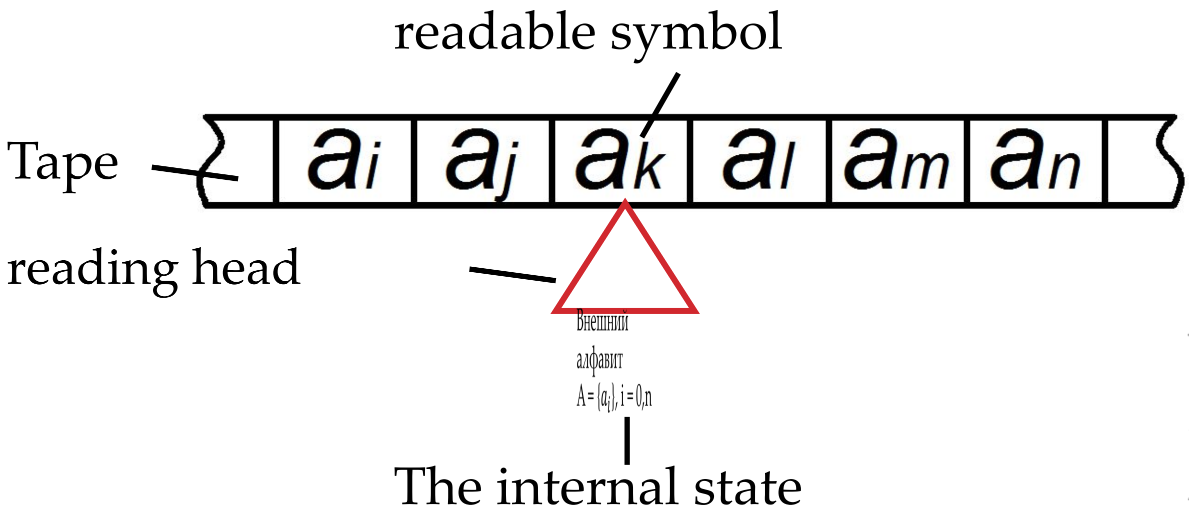
What?

Turing machine - a mathematical (imaginary) vehicle, not a machine physical. It is a mathematical object as a function, derivative, integral, etc.

Turing machines provide a general or formal model of computation and can be used to determine whether or not a task is computable.

A universal Turing machine (UTM) is a Turing machine that can execute other Turing machines by simulating the behaviour of any Turing machine.

THE DEVICE TURING MACHINE



Внешний алфавит
$A = \{a_i\}$, $i = 0{,}n$

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Tape:

- Potentially infinite;
- In one cell - one character;
- The empty cell is filled with the symbol a0.

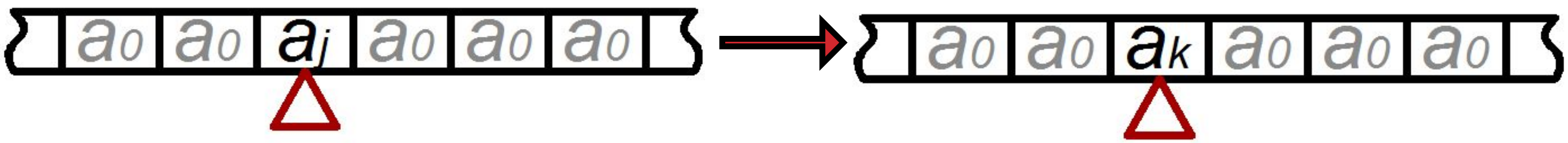
Head:

- At any given time there is only one internal state;
- Initial state - q1;
- The final state - q0.

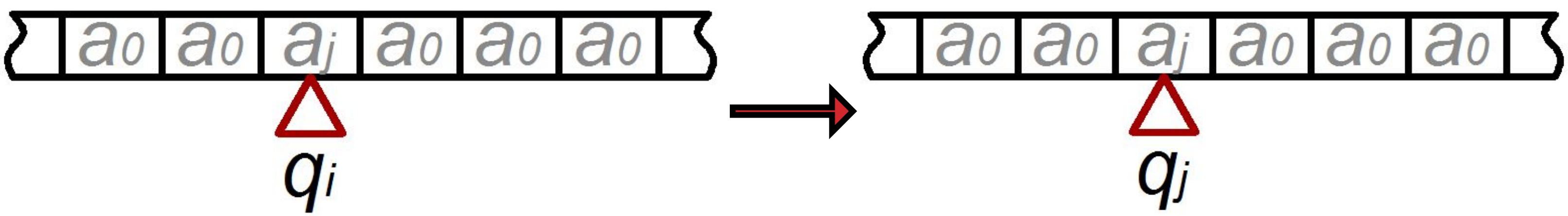
Actions Turing machine

In a single stroke of his work Turing machine can:

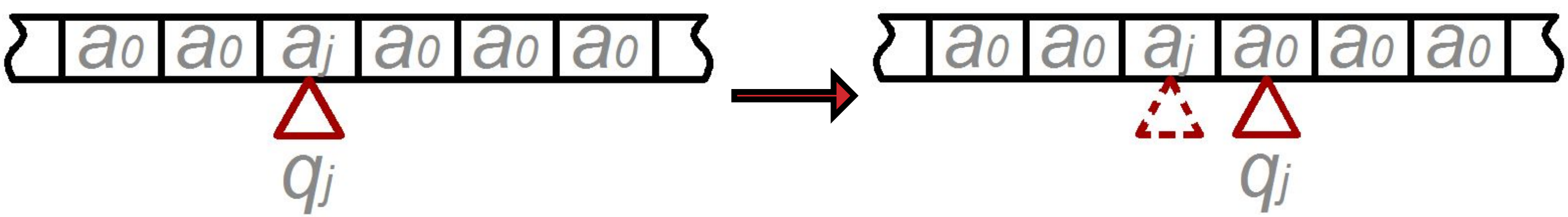
1) Change / do not change the character recorded on a tape



2) Change / do not change their internal state



3) Move the head on the tape left / right / not move the head



Внешний алфавит
$A = \{a_i\}, i = 0,n$

Program - a set of machine instructions.

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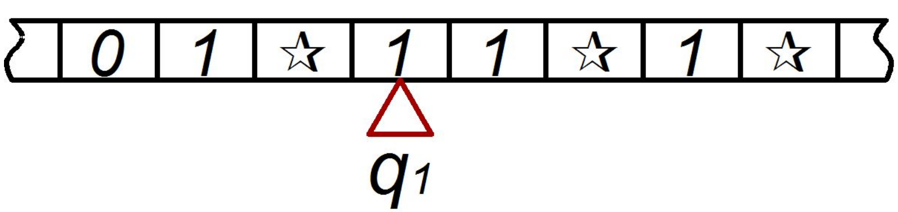
Machine



Configuration:

Внешний алфавит
$A = \{a_i\}, i = 0,n$

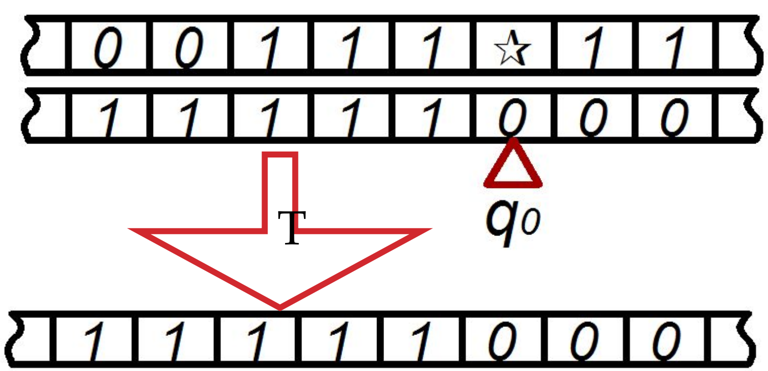
THE PROGRAM FOR A TURING MACHINE

Programs for Turing machines are recorded in the form of a table where the first column and row contains the letters of the alphabet and external possible internal state machine (the internal alphabet). The contents of the table is a command to Turing machines.

An example of a Turing machine

Consider the work of the Turing machine, which has the following program:

Q / A	q_1	q_2	q_3
0	$q_1$0L	$q_3$1R	$q_1$0L
1	$q_2$0L	$q_2$1L	$q_3$1R
☆	$q_0$0	q_2☆L	q_3☆R



$f(a,b) = a + b$

111☆1$q_1$1	1111☆$q_1$10
111☆$q_2$10	1111q_2☆00
111q_2☆10	111$q_2$1☆00
11$q_2$1☆10	…
1$q_2$11☆10	$q_2$1111☆00
$q_2$111☆10	$q_2$01111☆00
$q_2$0111☆10	1$q_3$1111☆00
1$q_3$111☆10	…
…	1111$q_3$1☆00
1111q_3☆10	11111q_3☆00
1111☆$q_3$10	11111☆$q_3$00
1111☆1$q_3$0	11111q_1☆00
	11111$q_0$000

Task.

On the tape recorded an integer. Wanted to get on tape number that is greater than 1. For example, if we are given a number of 53, the result should be 54.

Decision.

To solve this problem we suggest the following steps:

1. Machine distilled under the last digit of the number.

2. If this is a number from 0 to 8, then replace it with the numeral 1 and to stay longer; eg:

1 9 5 7 → 1 9 5 7→1 9 5 7 → 1 9 5 7→1 9 5 8

↑ ↑ ↑ ↑ ↑

3. If this figure is 9, then change it to 0 and shift to automatic
previous digit, then increase in the same manner on the penultimate figure 1; eg:

6 4 9 → 6 4 9→6 4 9 → 6 4 0 → 6 5 0

↑ ↑ ↑ ↑ ↑

4. Special case: only nine in number (e.g., 99). Then the machine will move to the left, replacing nine to zero, and in the end will be at the empty cage. In this empty cell to be written and 1 stop (the answer is 100):

9 9 → 9 9 → 9 0 → 0 0 → 1 0 0

↑ ↑ ↑ ↑ ↑

As an MT for the program steps are described as follows:

AQ	0	1	2	3	4	5	6	7	8	9	a_0
q_1	0,R,q1	1,R,q1	2,R,q1	3,R,q1	4,R,q1	5,R,q1	6,R,q1	7,R,q1	8,R,q1	9,R,q1	a_0,L,q2
q_2	1,S,!	2, S,!	3, S,!	4, S,!	5, S,!	6, S,!	7, S,!	8, S,!	9, S,!	0,L,q2	1,S,!

Conclusions:

- Turing machine - rigorous mathematical analog of the notion of "algorithm".

- The principle of operation of a Turing machine is the basis of all modern computers.

http://www.inf1.info/Turing