The State of Techniques for Solving Large Imperfect-Information Games

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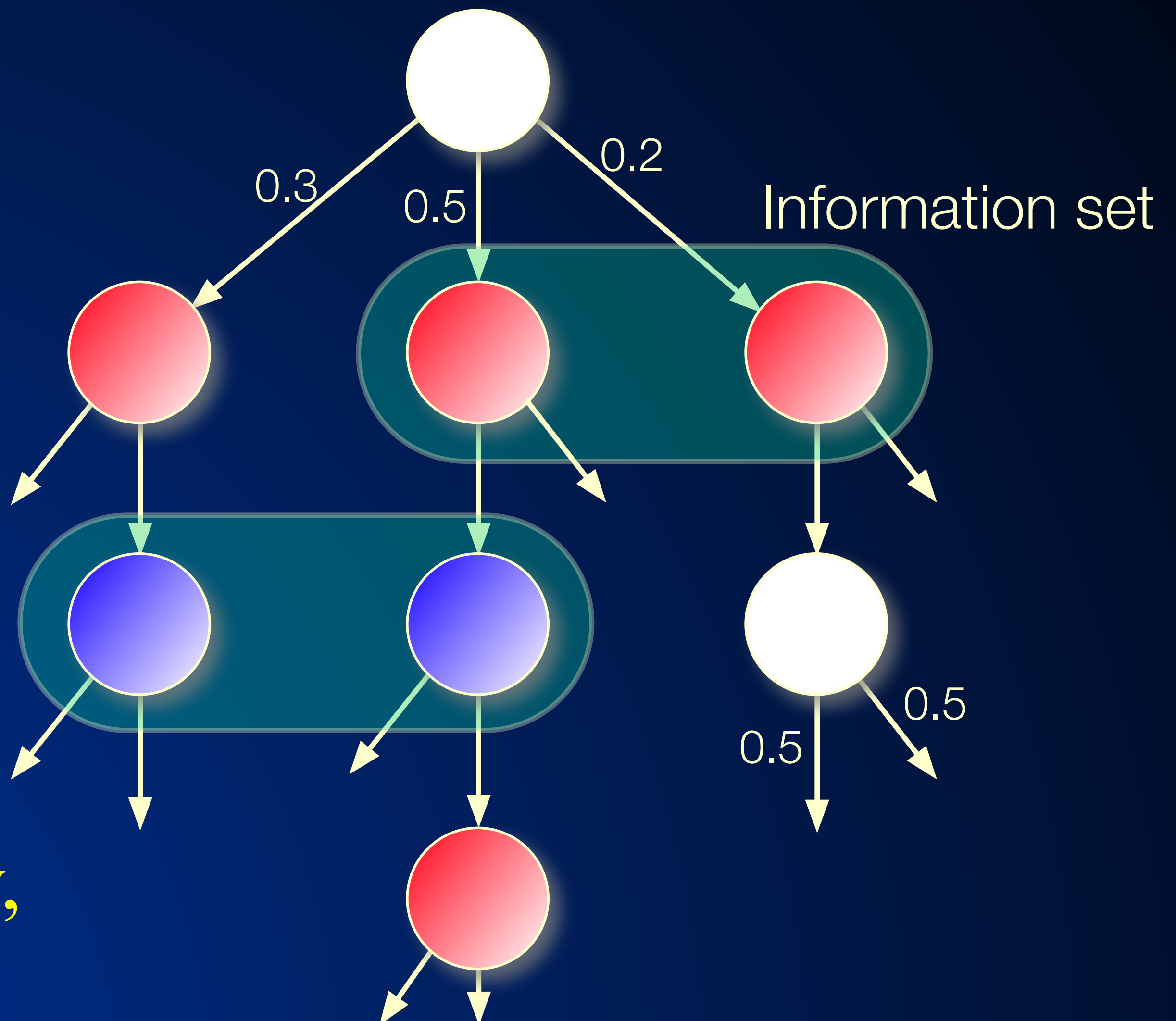
Incomplete-information game tree



Strategy,
beliefs

Tackling such games

- Domain-independent techniques
- Techniques for complete-info games don't apply
- Challenges
  - Unknown state
  - Uncertainty about what other agents and nature will do
  - Interpreting signals and avoiding signaling too much

- **Definition**. A Nash equilibrium is a *strategy* and *beliefs* for each agent such that no agent benefits from using a different strategy
  - Beliefs derived from strategies using Bayes' rule

Most real-world games are like this

- Negotiation
- Multi-stage auctions (FCC ascending, combinatorial)
- Sequential auctions of multiple items
- Political campaigns (TV spending)
- Military (allocating troops; spending on space vs ocean)
- Next-generation (cyber)security (jamming [DeBruhl et al.]; OS)
- Medical treatment [Sandholm 2012, AAAI-15 SMT Blue Skies]
- …

Poker

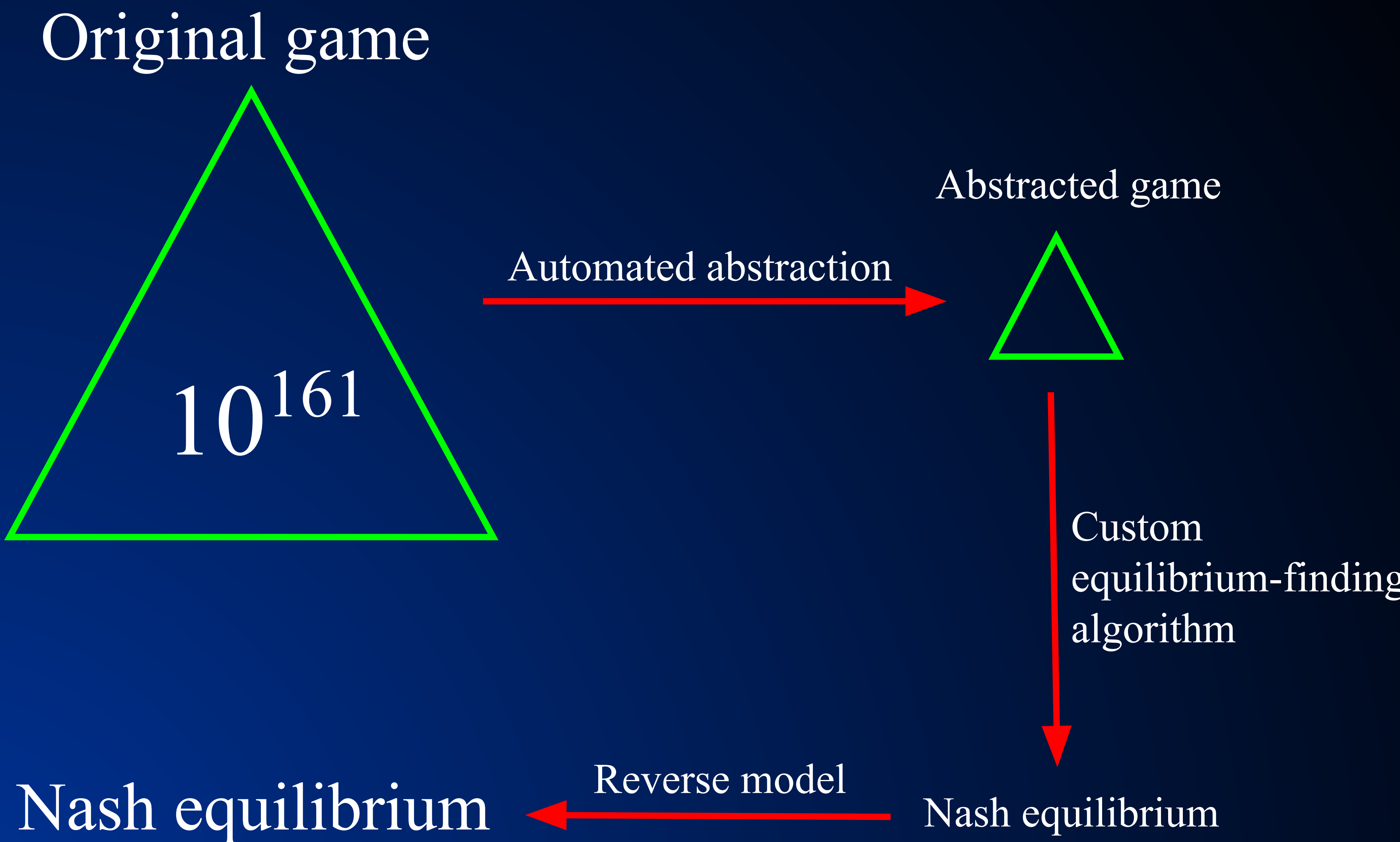
Recognized challenge problem in AI since 1992 [Billings, Schaeffer, …]

- Hidden information (other players' cards)
- Uncertainty about future events
- Deceptive strategies needed in a good player
- Very large game trees

NBC National Heads-Up Poker Championship 2013

Our approach [Gilpin & Sandholm EC-06, *J. of the ACM* 2007…]

Now used basically by all competitive Texas Hold'em programs



Foreshadowed by Shi & Littman 01, Billings et al. IJCAI-03

Lossless abstraction

[Gilpin & Sandholm EC-06, *J. of the ACM* 2007]

Information filters

- Observation: We can make games smaller by filtering the information a player receives
- Instead of observing a specific signal exactly, a player instead observes a **filtered set** of signals
  - *E.g.* receiving signal {A♠,A♣,A♥,A♦} instead of A♥

Solved Rhode Island Hold'em poker

- AI challenge problem [Shi & Littman 01]
  - 3.1 billion nodes in game tree
- Without abstraction, LP has 91,224,226 rows and columns => unsolvable
- *GameShrink* ran in one second
- After that, LP had 1,237,238 rows and columns (50,428,638 non-zeros)
- Solved the LP
  - CPLEX *barrier* method took 8 days & 25 GB RAM
- Exact Nash equilibrium
- Largest incomplete-info game solved by then by over 4 orders of magnitude

Lossy abstraction

Texas Hold'em poker

Nature deals 2 cards to each player

Round of betting

Nature deals 3 shared cards

Round of betting

Nature deals 1 shared card

Round of betting

Nature deals 1 shared card

Round of betting

- 2-player Limit has $\sim 10^{14}$ info sets
- 2-player No-Limit has $\sim 10^{161}$ info sets
- Losslessly abstracted game too big to solve
  => abstract more
  => lossy

Important ideas for practical game abstraction 2007-13

- Integer programming [Gilpin & Sandholm AAMAS-07]
- Potential-aware [Gilpin, Sandholm & Sørensen AAAI-07, Gilpin & Sandholm AAAI-08]
- Imperfect recall [Waugh et al. SARA-09, Johanson et al. AAMAS-13]

Leading practical abstraction algorithm: Potential-aware imperfect-recall abstraction with earth-mover's distance

[Ganzfried & Sandholm AAAI-14]

- Bottom-up pass of the tree, clustering using histograms over next-round clusters
  - EMD is now in multi-dimensional space
    - Ground distance assumed to be the (next-round) EMD between the corresponding cluster means

Techniques used to develop *Tartanian7*, program that won the heads-up no-limit Texas Hold'em ACPC-14
[Brown, Ganzfried, Sandholm AAMAS-15]

- Enables massive distribution or leveraging ccNUMA
- Abstraction:
  - Top of game abstracted with any algorithm
  - Rest of game split into equal-sized disjoint pieces based on public signals
    - This (5-card) abstraction determined based on transitions to a *base abstraction*
  - At each later stage, abstraction done within each piece separately
- Equilibrium finding (see also [Jackson, 2013; Johanson, 2007])
  - "Head" blade handles top in each iteration of External-Sampling MCCFR
  - Whenever the rest is reached, sample (a flop) from each public cluster
  - Continue the iteration on a separate blade for each public cluster. Return results to head node
  - Details:
    - Must weigh each cluster by probability it would've been sampled randomly
    - Can sample multiple flops from a cluster to reduce communication overhead

Lossy Game Abstraction with Bounds

Lossy game abstraction with bounds

- Tricky due to abstraction pathology [Waugh et al. AAMAS-09]

- Prior lossy abstraction algorithms had no bounds
  - First exception was for stochastic games only [S. & Singh EC-12]

- We do this for general extensive-form games [Kroer & S. EC-14]
  - Many new techniques required
  - For both action and state abstraction
  - More general abstraction operations by also allowing one-to-many mapping of nodes

Bounding abstraction quality

Main theorem:

For any Nash equilibrium σ' in M', any undivided lifted strategy σ is an ϵ-Nash equilibrium in M

where $\varepsilon = \max_{i \in \text{Players}} \varepsilon_i$

Set of heights for player i

Set of heights for nature

Nature distribution error at height j

$$\epsilon_i = 2\epsilon_i^R + \sum_{j \in \mathcal{H}_i} \epsilon_j^0 \overline{W} + \sum_{j \in \mathcal{H}_0} 2\epsilon_j^0 \overline{W}$$

Reward error

Nature distribution error at height j

Maximum utility in abstract game

Hardness results

- Determining whether two subtrees are "extensive-form game-tree isomorphic" is graph isomorphism complete
- Computing the minimum-size abstraction given a bound is NP-complete
  - Holds also for minimizing a bound given a maximum size
- Doesn't mean abstraction with bounds is undoable or not worth it computationally

Extension to imperfect recall

- Merge information sets
- Allows payoff error
- Allows chance error

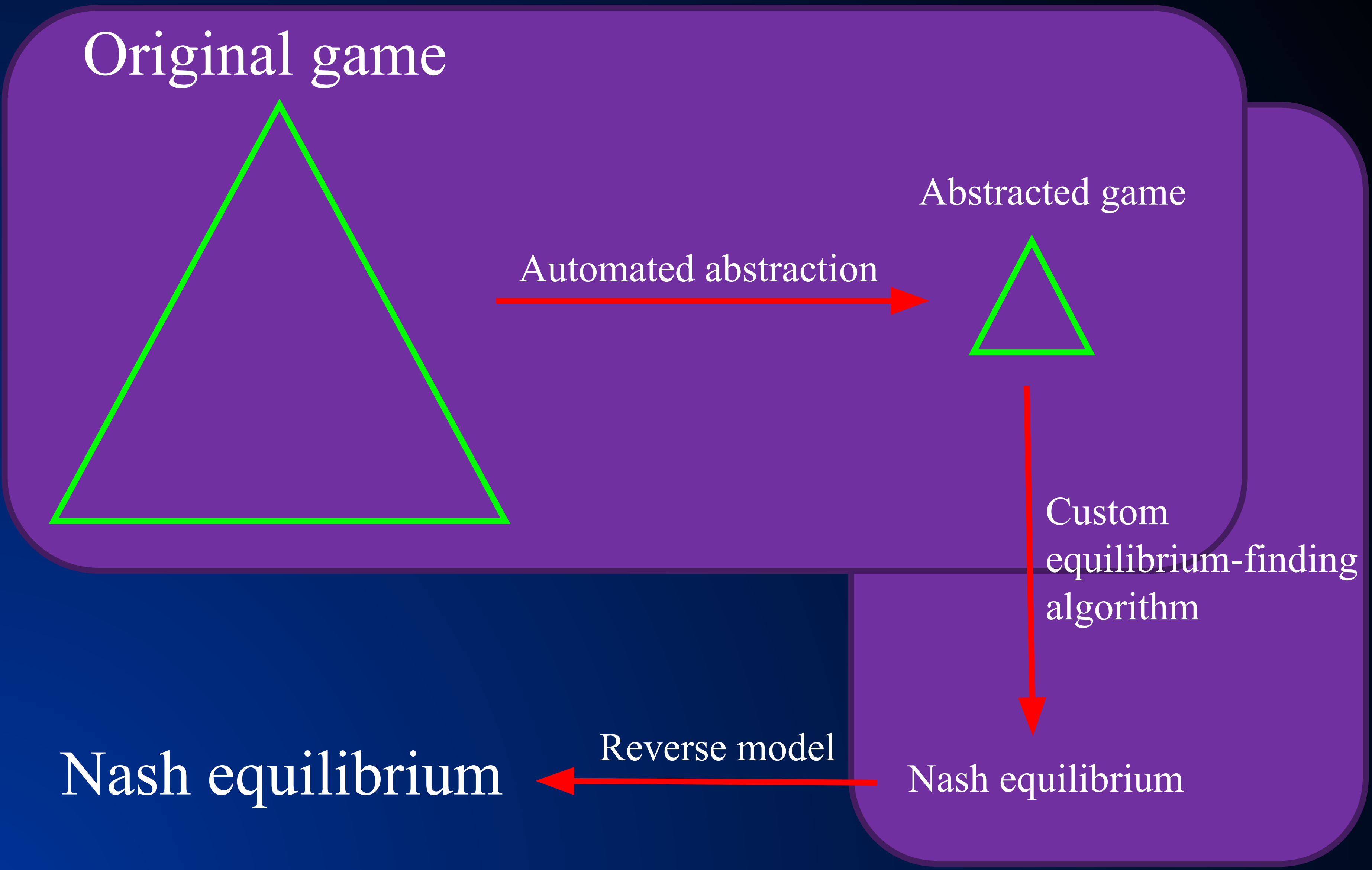
- Going to imperfect-recall setting costs an error increase that is linear in game-tree height

- Exponentially stronger bounds and broader class (abstraction can introduce nature error) than [Lanctot et al. ICML-12], which was also just for CFR

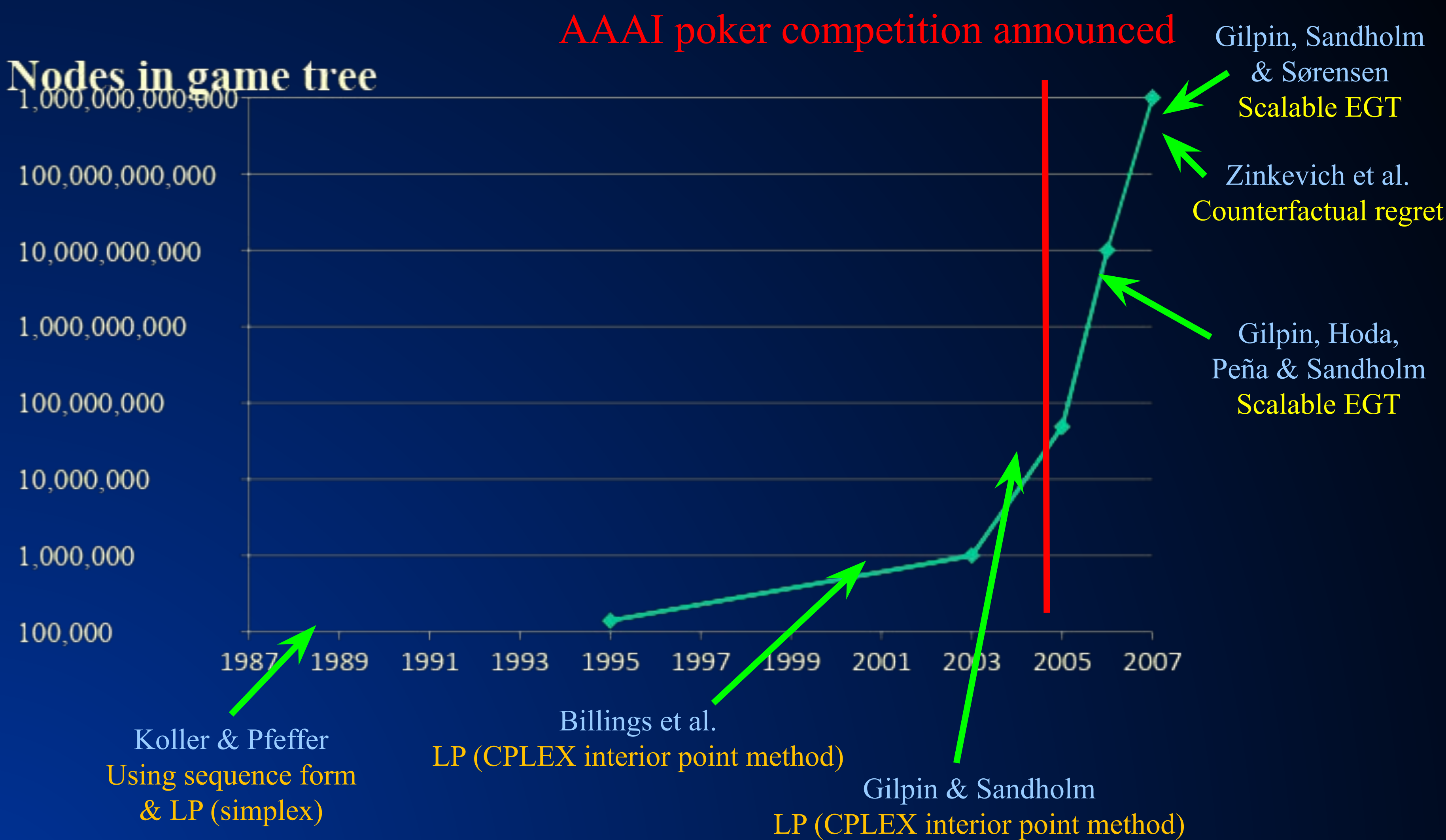
[Kroer and Sandholm IJCAI-15 workshop]

Role in modeling

- All modeling is abstraction
- These are the first results that tie game modeling choices to solution quality in the actual world!

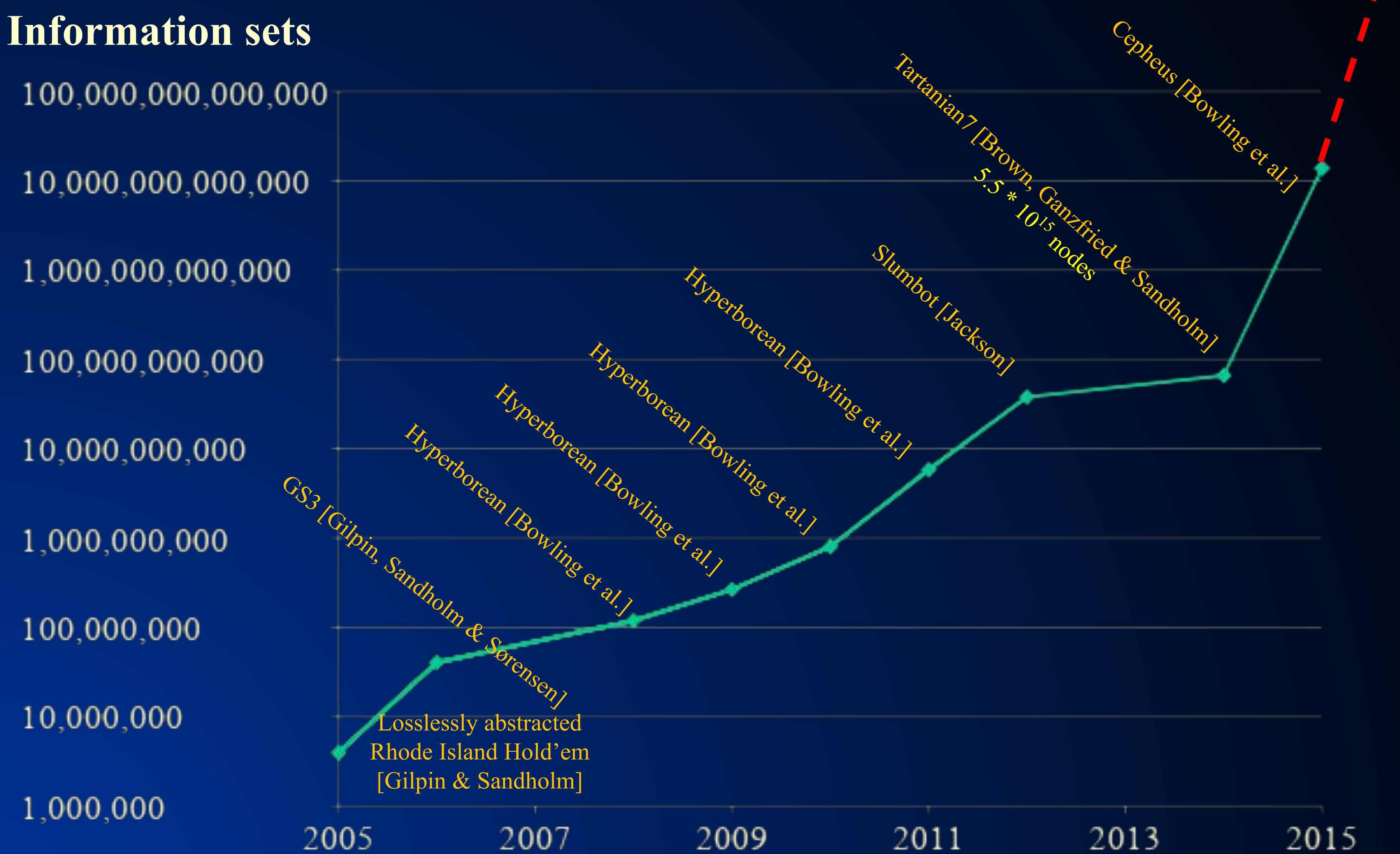

Original game
Abstracted game
Automated abstraction
Custom
equilibrium-finding
algorithm
Reverse model
Nash equilibrium
Nash equilibrium

Scalability of (near-)equilibrium finding in 2-player 0-sum games



Regret-based pruning [Brown & Sandholm NIPS-15]

Scalability of (near-)equilibrium finding in 2-player 0-sum games…



Leading equilibrium-finding algorithms for 2-player 0-sum games

Counterfactual regret (CFR)

- Based on no-regret learning
- Most powerful innovations:
  - Each information set has a separate no-regret learner [Zinkevich et al. NIPS-07]
  - Sampling [Lanctot et al. NIPS-09, …]
- $O(1/\varepsilon^2)$ iterations
  - Each iteration is fast
- Parallelizes
- Selective superiority
- Can be run on imperfect-recall games and with >2 players (without guarantee of converging to equilibrium)

Scalable EGT

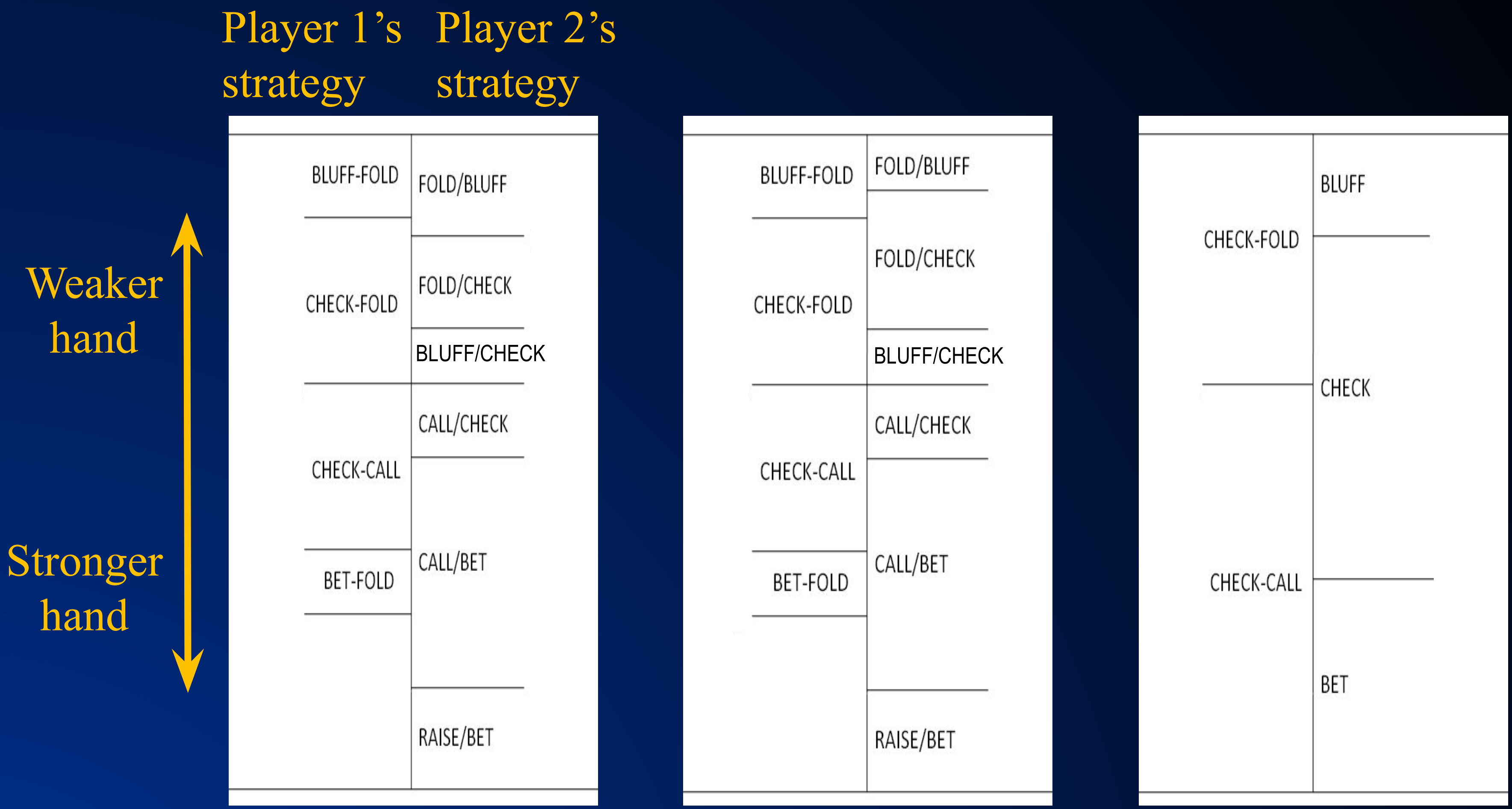
- Based on Nesterov's Excessive Gap Technique
- Most powerful innovations: [Hoda, Gilpin, Peña & Sandholm WINE-07, *Mathematics of Operations Research* 2011]
  - Smoothing fns for sequential games
  - Aggressive decrease of smoothing
  - Balanced smoothing
  - Available actions don't depend on chance => memory scalability
- $O(1/\varepsilon)$ iterations
  - Each iteration is slow
- Parallelizes
- New $O(\log(1/\varepsilon))$ algorithm [Gilpin, Peña & Sandholm AAAI-08, *Mathematical Programming* 2012]

Better first-order methods

[Kroer, Waugh, Kılınç-Karzan & Sandholm EC-15]

- New prox function for first-order methods such as EGT and Mirror Prox
  - Gives first explicit convergence-rate bounds for general zero-sum extensive-form games (prior explicit bounds were for very restricted class)
  - In addition to generalizing, bound improvement leads to a linear (in the worst case, quadratic for most games) improvement in the dependence on game specific constants
- Introduces gradient sampling scheme
  - Enables the first stochastic first-order approach with convergence guarantees for extensive-form games
  - As in CFR, can now represent game as tree that can be sampled
- Introduces first first-order method for imperfect-recall abstractions
  - As with other imperfect-recall approaches, not guaranteed to converge

Computing equilibria by leveraging qualitative models



- **Theorem.** Given F_1, F_2, and a qualitative model, we have a complete mixed-integer linear feasibility program for finding an equilibrium
- Qualitative models can enable proving existence of equilibrium & solve games for which algorithms didn't exist

[Ganzfried & Sandholm AAMAS-10 & newer draft]

Simultaneous Abstraction and Equilibrium Finding in Games

[Brown & Sandholm IJCAI-15 & new manuscript]

Problems solved

- Cannot solve without abstracting, and cannot principally abstract without solving
  - SAEF abstracts and solves simultaneously
- Must restart equilibrium finding when abstraction changes
  - SAEF does not need to restart (uses discounting)
- Abstraction size must be tuned to available runtime
  - In SAEF, abstraction increases in size over time
- Larger abstractions may not lead to better strategies
  - SAEF guarantees convergence to a full-game equilibrium

OPPONENT EXPLOITATION

Traditionally two approaches

- Game theory approach (abstraction+equilibrium finding)
  - Safe in 2-person 0-sum games
  - Doesn't maximally exploit weaknesses in opponent(s)

- Opponent modeling
  - Needs prohibitively many repetitions to learn in large games (loses too much during learning)
    - Crushed by game theory approach in Texas Hold'em
    - Same would be true of no-regret learning algorithms
  - *Get-taught-and-exploited problem* [Sandholm AIJ-07]

Let’s hybridize the two approaches

- Start playing based on pre-computed (near-)equilibrium
- As we learn opponent(s) deviate from equilibrium, adjust our strategy to exploit their weaknesses
  - Adjust more in points of game where more data now available
  - Requires no prior knowledge about opponent
- Significantly outperforms game-theory-based base strategy in 2-player limit Texas Hold’em against
  - trivial opponents
  - weak opponents from AAAI computer poker competitions
- Don’t have to turn this on against strong opponents

[Ganzfried & Sandholm AAMAS-11]

Other modern approaches to opponent exploitation

- ε-safe best response
  [Johanson, Zinkevich & Bowling NIPS-07, Johanson & Bowling AISTATS-09]

- Precompute a small number of strong strategies. Use no-regret learning to choose among them
  [Bard, Johanson, Burch & Bowling AAMAS-13]

Safe opponent exploitation

- Definition. *Safe* strategy achieves at least the value of the (repeated) game in expectation

- Is safe exploitation possible (beyond selecting among equilibrium strategies)?

[Ganzfried & Sandholm EC-12, TEAC 2015]

Exploitation algorithms

1. Risk what you've won so far
2. Risk what you've won so far in expectation (over nature's & own randomization), i.e., risk the gifts received
   – Assuming the opponent plays a nemesis in states where we don't know

…

- **Theorem.** A strategy for a 2-player 0-sum game is safe iff it never risks more than the gifts received according to #2
- Can be used to make any opponent model / exploitation algorithm safe
- No prior (non-eq) opponent exploitation algorithms are safe
- #2 experimentally better than more conservative safe exploitation algs
- Suffices to lower bound opponent's mistakes

STATE OF TOP POKER PROGRAMS

Rhode Island Hold'em

- Bots play optimally
  [Gilpin & Sandholm EC-06, J. of the ACM 2007]

Heads-Up Limit Texas Hold’em

- Bots surpassed pros in 2008
  [U. Alberta Poker Research Group]

- “Essentially solved” in 2015 [Bowling et al.]

Heads-Up No-Limit Texas Hold’em

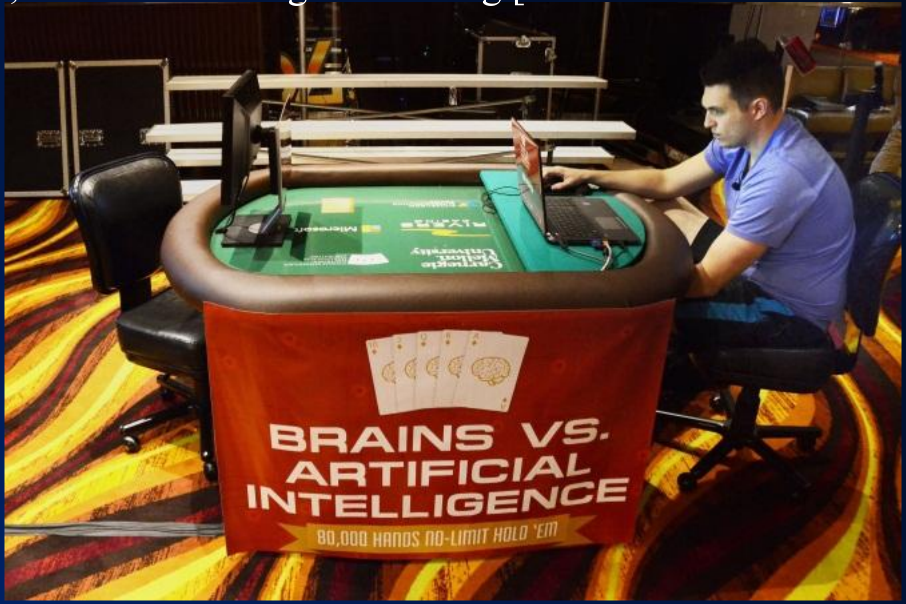
Annual Computer Poker Competition

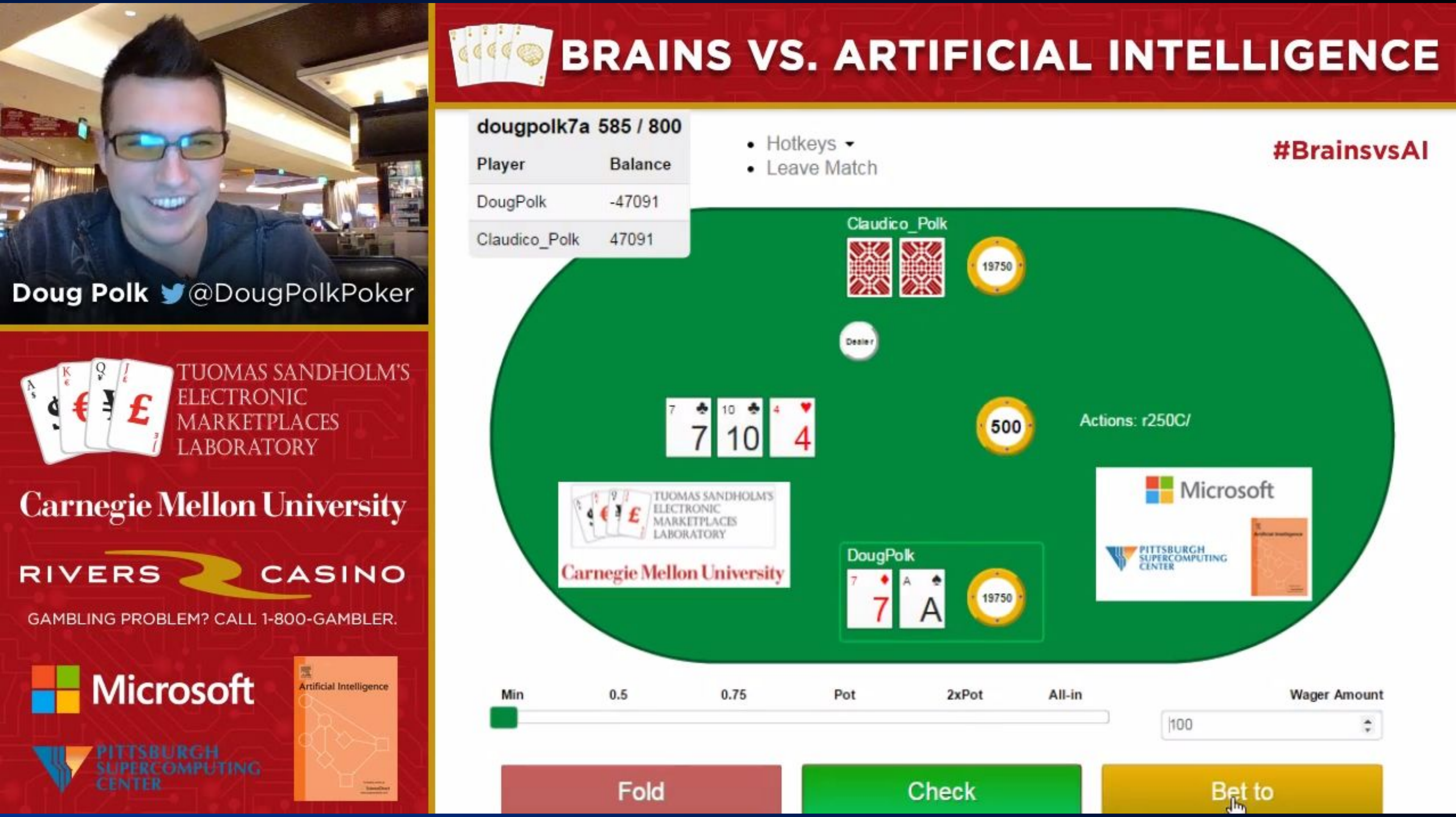
--> Claudico

- Statistical significance win against every bot
- Smallest margin in IRO: 19.76 ± 15.78
- Average in Bankroll: 342.49
  (next highest: 308.92)

“BRAINS VS AI” EVENT

- Claudico against each of 4 of the top-10 pros in this game
- 4 * 20,000 hands over 2 weeks
- Strategy was precomputed, but we used endgame solving [Ganzfried & Sandholm AAMAS-15] in some sessions




BRAINS VS. ARTIFICIAL INTELLIGENCE
Doug Polk @DougPolkPoker
TUOMAS SANDHOLM'S ELECTRONIC MARKETPLACES LABORATORY
Carnegie Mellon University
RIVERS CASINO
GAMBLING PROBLEM? CALL 1-800-GAMBLER.
Microsoft
PITTSBURGH SUPERCOMPUTING CENTER
Artificial Intelligence
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Player Balance
DougPolk -47091
Claudico_Polk 47091
Hotkeys
Leave Match
#BrainsvsAI
Claudico_Polk
19750
Dealer
500
Actions: r250C/
DougPolk
19750
Min
0.5
0.75
Pot
2xPot
All-in
Wager Amount
100
Fold
Check
Bet to

Humans' $100,000 participation fee distributed based on performance

Overall performance

- Pros won by 91 mbb/hand
  - Not statistically significant (at 95% confidence)
  - Perspective:
    - Dong Kim won a challenge against Nick Frame by 139 mbb/hand
    - Doug Polk won a challenge against Ben Sulsky 247 mbb/hand
- 3 pros beat Claudico, one lost to it
- Pro team won 9 days, Claudico won 4

Observations about Claudico's play

- Strengths (beyond what pros typically do):
  - Small bets & huge all-ins
  - Perfect balance
  - Randomization: not "range-based"
  - "Limping" & "donk betting"
- Weaknesses:
  - Coarse handling of "card removal" in endgame solver
    - Because endgame solver only had 20 seconds
  - Action mapping approach
  - No opponent exploitation

Multiplayer poker

- Bots aren't very strong (at least not yet)
  - Exception: programs are very close to optimal in jam/fold games [Ganzfried & Sandholm AAMAS-08, IJCAI-09]

Conclusions

- Domain-independent techniques
- Abstraction
  - Automated lossless abstraction—exactly solves games with billions of nodes
  - Best practical lossy abstraction: potential-aware, imperfect recall, EMD
  - Lossy abstraction with bounds
    - For action and state abstraction
    - Also for modeling
  - Simultaneous abstraction and equilibrium finding
  - (Reverse mapping [Ganzfried & S. IJCAI-13])
  - (Endgame solving [Ganzfried & S. AAMAS-15])
- Equilibrium-finding
  - Can solve 2-person 0-sum games with 10^{14} information sets to small ε
    - $O(1/\varepsilon^2)$ -> $O(1/\varepsilon)$ -> $O(\log(1/\varepsilon))$
  - New framework for fast gradient-based algorithms
    - Works with gradient sampling and can be run on imperfect-recall abstractions
  - Regret-based pruning for CFR
  - Using qualitative knowledge/guesswork
- Pseudoharmonic reverse mapping
- Opponent exploitation
  - Practical opponent exploitation that starts from equilibrium
  - Safe opponent exploitation

Current & future research

- Lossy abstraction with bounds
  - Scalable algorithms
  - With structure
  - With generated abstract states and actions
- Equilibrium-finding algorithms for 2-person 0-sum games
  - Even better gradient-based algorithms
  - Parallel implementations of our O(log(1/ε)) algorithm and understanding how #iterations depends on matrix condition number
  - Making interior-point methods usable in terms of memory
  - Additional improvements to CFR
- Endgame and "midgame" solving with guarantees
- Equilibrium-finding algorithms for >2 players
- Theory of thresholding, purification [Ganzfried, S. & Waugh AAMAS-12], and other strategy restrictions
- Other solution concepts: sequential equilibrium, coalitional deviations, …
- Understanding exploration vs exploitation vs safety
- Application to other games (medicine, cybersecurity, etc.)

Thank you!

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