Tuzuvchi:

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- URGANCH-2021

« Impuls momenti»

Reja:

8.1 Zarrachalar момент импульси. Kuch momenti.

8.2. Impuls momenti va o’qqa nisbatan kuch momentlari.

8.3. Impuls momentining saqlanish qonuni.

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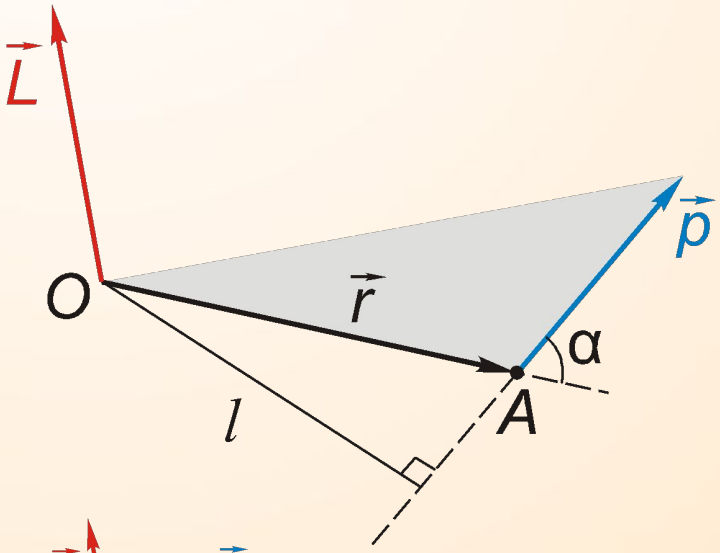
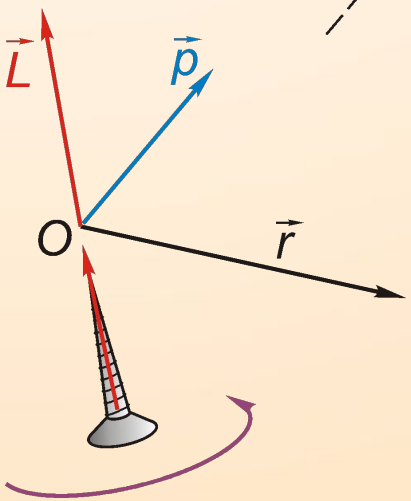
8.1 Zarrachalar момент импульси. Kuch momenti.





- Zarrachaning O nuqtasiga nisbatan impuls momenti - bu zarrachaning radiusi vektorining impulsi bo'yicha vektor ko'paytmasiga sonli teng bo'lgan fizik kattalik.

$$\vec{L} = \vec{r} \times \vec{p} \qquad (8.1)$$

$$L = r \cdot p \cdot \sin\alpha = l \cdot p \qquad (8.2)$$

Birliklar sistemasida SI:

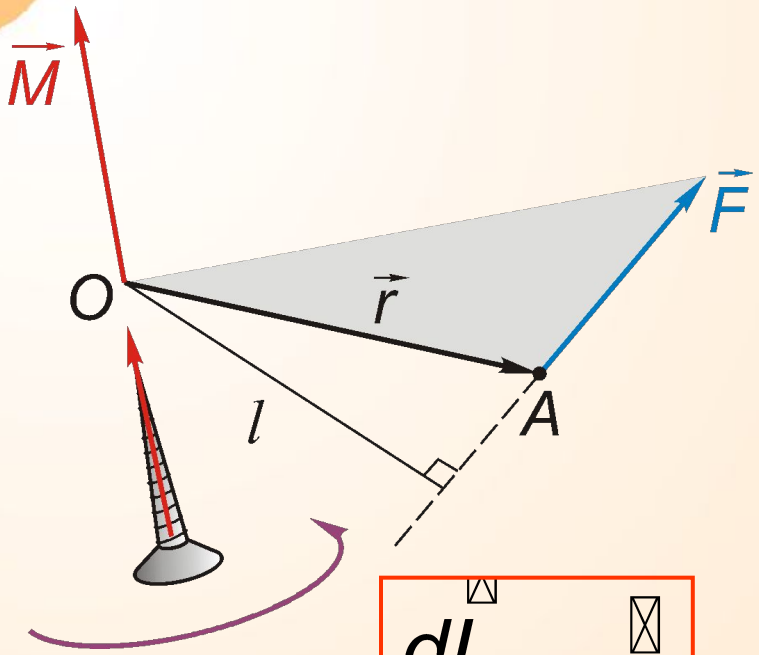
impuls momenti – 1м·1кг·м/с=1 кг·м^2/с

$$\frac{d\vec{L}}{dt} = \frac{d\vec{r}}{dt} \times \vec{p} + \vec{r} \times \frac{d\vec{p}}{dt} \qquad (8.3)$$

$$\frac{d\vec{r}}{dt} = \vec{v} \uparrow\uparrow \vec{p} \quad \Rightarrow \quad \frac{d\vec{r}}{dt} \times \vec{p} = 0$$

$$\frac{d\vec{p}}{dt} = \vec{F}$$

$\vec{M} = \vec{r} \times \vec{F}$ - F Kuch momenti O nuqtaga nisbatan



$$\vec{M} = \vec{r} \times \vec{F} \qquad (8.4)$$

$$M = r \cdot F \cdot \sin\alpha = l \cdot F \qquad (8.5)$$

Birliklar sistemasida SI :
kuch momenti – 1 Н·м

$$\frac{d\vec{L}}{dt} = \vec{M}$$

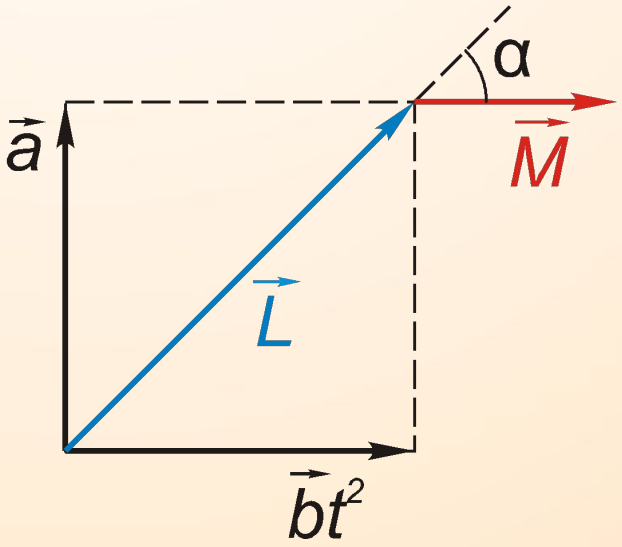
Impuls momenti tezligining o'zgarishi tashqi kuchlar momentiga teng (8.6)

$$d\vec{L} = \vec{M} \cdot dt$$ - Impuls momenti kuchi (8.7)

$$\vec{L}_2 - \vec{L}_1 = \int_0^t \vec{M}(t)dt \qquad (8.8)$$

Topshiriq 8.1

Zarrachalar impuls momenti bazi nuqtalarga nisbatan vaqt birligida t quyidagi qonun bo'yicha $\vec{L}(t) = \vec{a} + \vec{b} \cdot t^2$, bu erda $\vec{a}$ и $\vec{b}$ - bazi o'zgarmas vektorlar, bunda $\vec{a} \perp \vec{b}$. $\vec{L}$ va $\vec{M}$ vektorlar orasidagi burchak 45°ga teng bo'lsa, zarrachalarga tasir etuvchi kuch momentini aniqlash talab qilinadi.



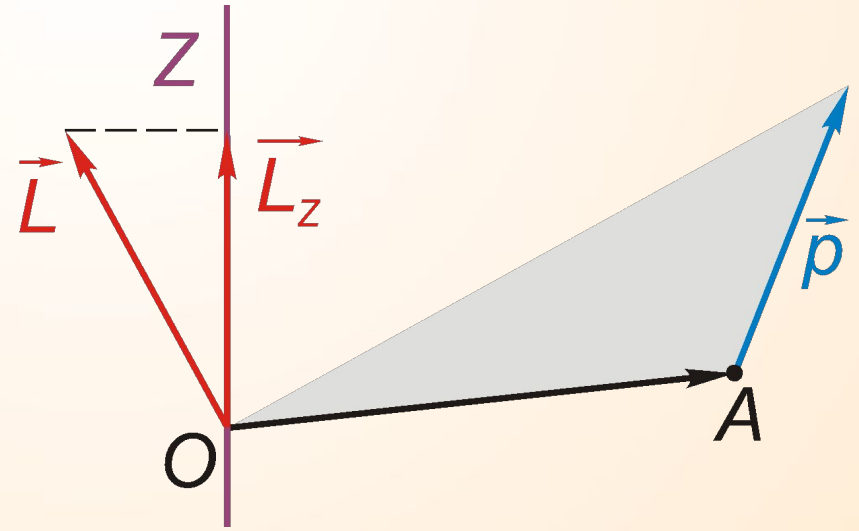
$$\vec{M} = \frac{d\vec{L}}{dt} = 2\vec{b} \cdot t$$

$$a = b \cdot t_0^2$$

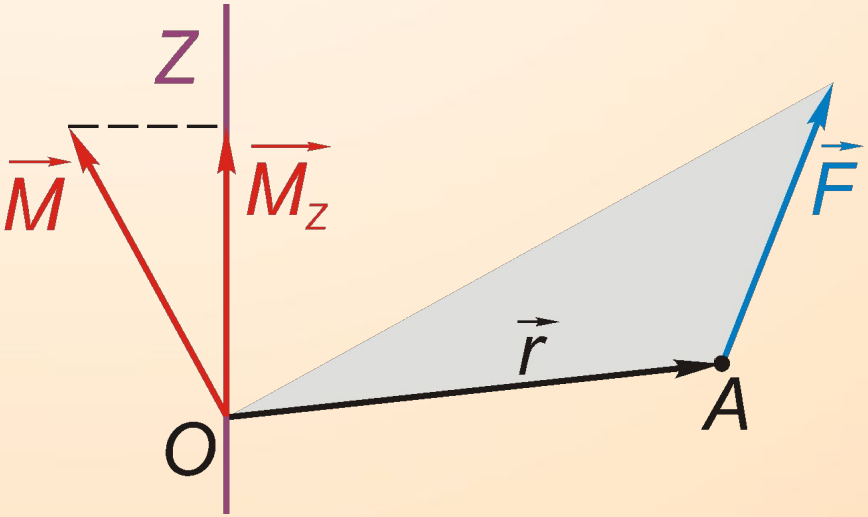
$$t_0 = \sqrt{a/b}$$

$$\vec{M} = 2\sqrt{a/b} \cdot \vec{b}$$

8.2. Impuls momenti va o’qqa nisbatan kuch momentlari.

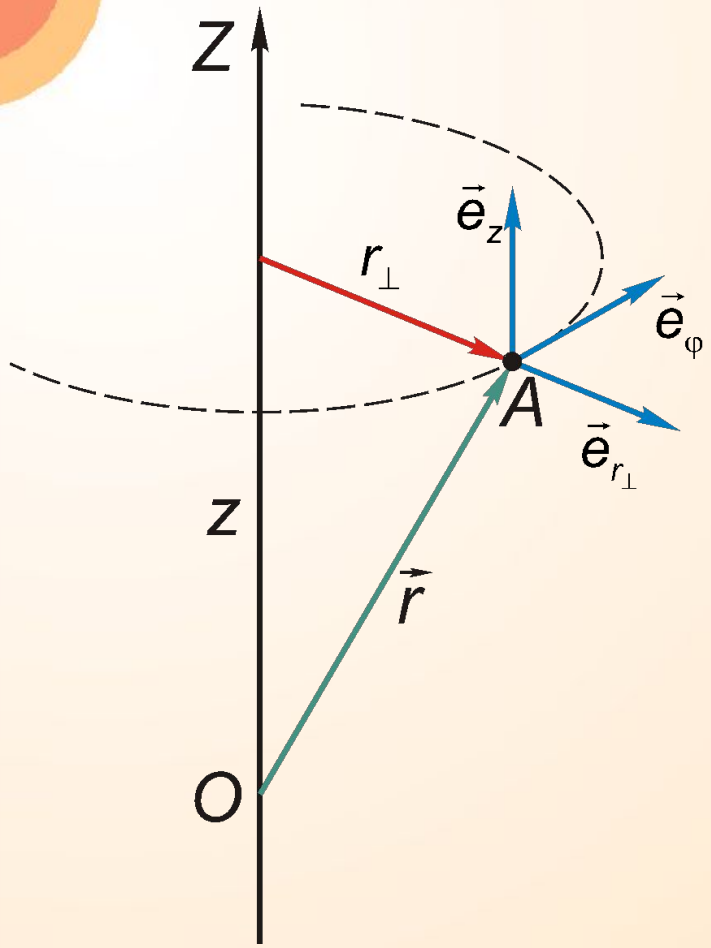


$$\frac{dL_z}{dt} = M_z \qquad (8.9)$$



$$L_z = \left(\vec{r} \times \vec{p}\right)_z$$

$$M_z = \left(\vec{r} \times \vec{F}\right)_z$$



$$L_z = mr_\perp^2 \omega_z \qquad (8.10)$$

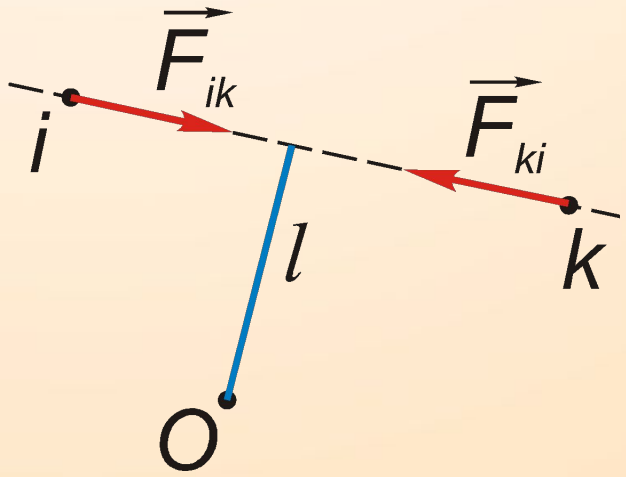
$$M_z = r_\perp F_\varphi$$

8.3. Impuls momentining saqlanish qonuni.

- Zarrachalar yoki jismlar sistemasining impuls momentlari bu sistemaga kiruvchi barcha jismlar impuls momentlarining vector y'ig'indisiga teng.

$$\vec{L} = \sum \vec{L}_i \qquad (8.11)$$

$$\frac{d\vec{L}}{dt} = \sum \frac{d\vec{L}_i}{dt} = \sum \vec{M}_i^{\text{ichki}} + \sum \vec{M}_i^{\text{tashqi}}$$



$$\vec{M}_{ik} = -\vec{M}_{ki}$$

$$\frac{d\vec{L}}{dt} = \vec{M}_{\text{tashqi}}$$

$$\vec{L}_2 - \vec{L}_1 = \int_0^t \vec{M}_{\text{tashqi}}(t)dt \qquad (8.13)$$

Agarda sistema yopiq bo'lsa, unga tashqi kuchlar tasir qilmaydi va uning momenti nolga teng bo'ladi.

Shuning uchun quyidagi qonun o'rinli bo'ladi:

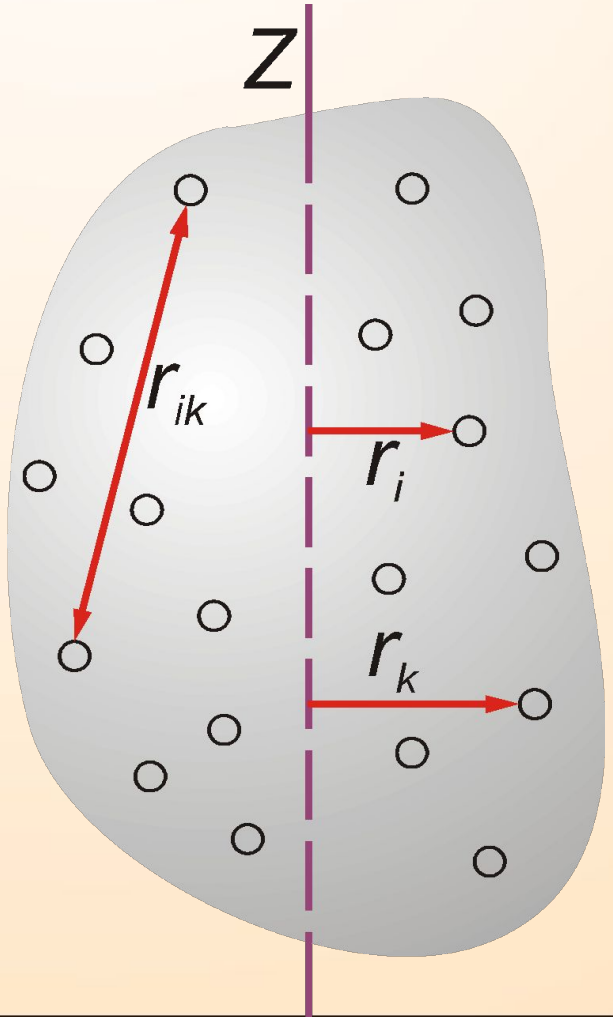
Impuls momentining saqlanish qonuni.

Yopiq sistemadagi jismlarning impuls momenti vaqt o'tishi bilan o'zgarmaydi.

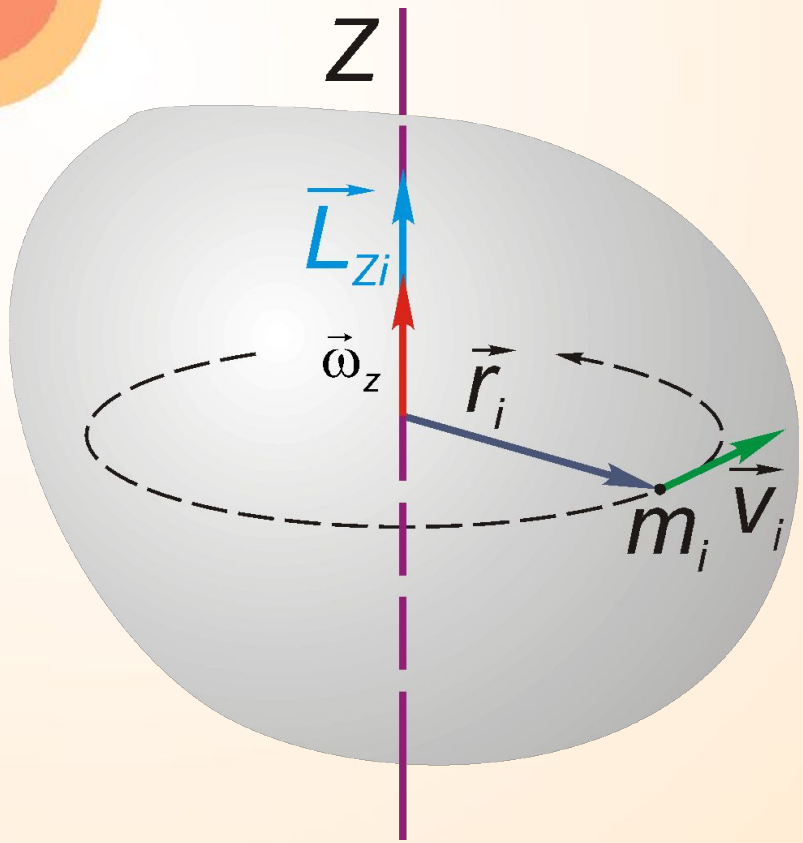
$$\vec{L} = \sum \vec{L}_i(t) = const$$

8.4. Jismning qo’zg’almas o’q atrofida aylanishi.

- Absalyut qattiq jism- bu jismning xoxlagan ikkita nuqtasi orasidagi masofa tashqi tasir natijasida o’zgarmasdan qolishiga aytiladi.



$$\frac{dL_Z}{dt} = M_{\text{tashqiZ}} \qquad (8.14)$$



$$L_z = \sum L_{iz} = \left(\sum m_i r_i^2\right) \cdot \omega_z$$

Jismning Z o'qiga nisbatan inersiya momenti –

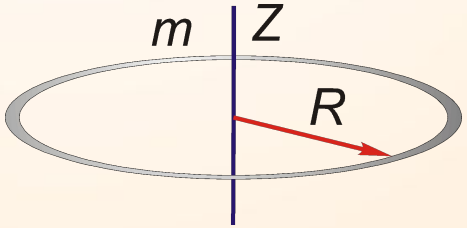
$$I_z = \sum m_i r_i^2$$

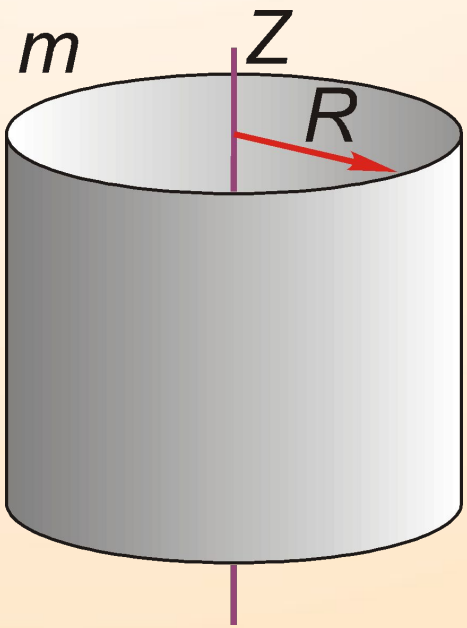
$$L_z = I_z \cdot \omega_z \qquad (8.15)$$

8.4.1. Bazi jismlarning inersiya momentlari.

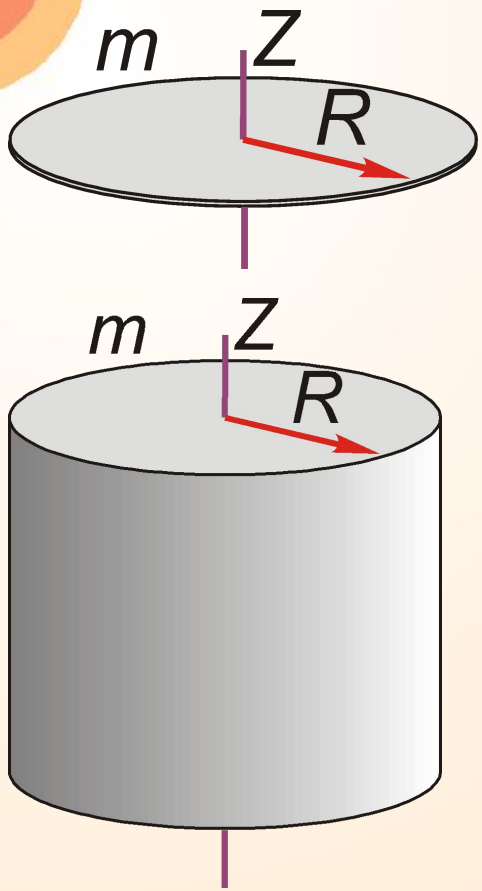
$$I_z = \sum m_i r_i^2 = \int r^2 dm = \int \rho r^2 dv$$

Birliklar sistemasida SI :
inersiya momenti – 1 кг·м^2





$$I_z = \int r^2 dm = R^2 \int dm = mR^2$$



$$I_z = \int \rho r^2 dV$$

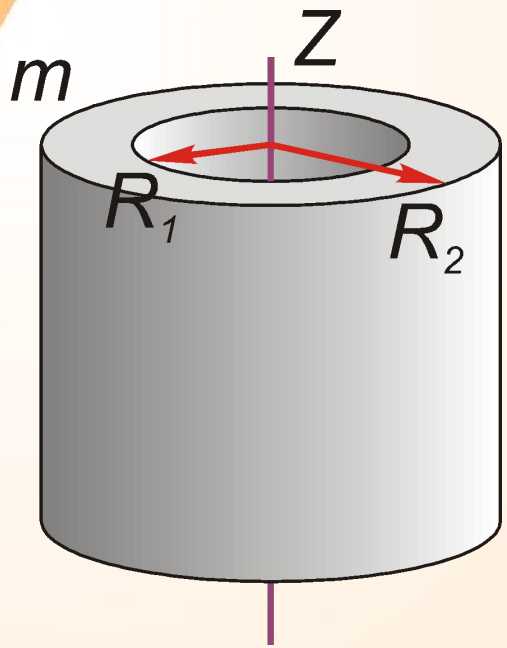
$$dV = zr \cdot dr \cdot d\varphi \cdot dz$$

$$I_Z = \iiint \rho r^3 z \cdot dr \cdot d\varphi \cdot dz = \rho \int_0^h dz \cdot \int_0^R r^3 dr \cdot \int_0^{2\pi} d\varphi =$$

$$= \rho \cdot h \cdot \frac{R^4}{4} \cdot 2\pi = \rho\left(\pi R^2 h\right)\frac{R^2}{2} = \rho V \frac{R^2}{2}$$

$$\rho V = m$$

$$\boxed{I_z = \frac{mR^2}{2}}$$



$$I_{z2} = I_{z1} + I_z$$

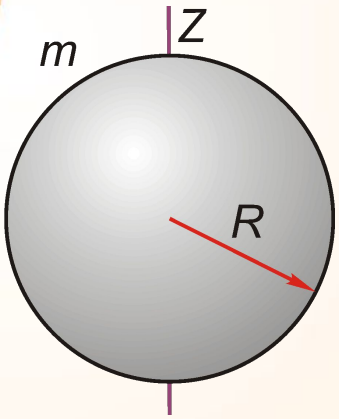
$$I_{z1} = \frac{1}{2} m_1 R_1^2 = \frac{1}{2} \rho \pi R_1^2 h R_1^2$$

$$I_{z2} = \frac{1}{2} m_2 R_2^2 = \frac{1}{2} \rho \pi R_2^2 h R_2^2$$

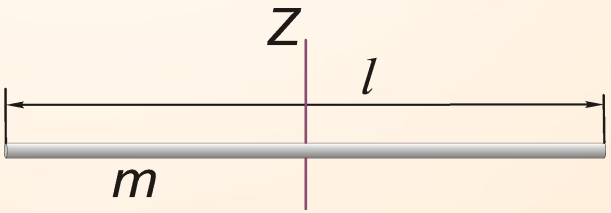
$$I_z = I_{z2} - I_{z1} = \frac{1}{2} \rho \pi h \left(R_2^4 - R_1^4 \right) = \frac{1}{2} \rho \pi h \left(R_2^2 - R_1^2 \right) \left(R_2^2 + R_1^2 \right)$$

$$\rho \pi h \left(R_2^2 - R_1^2 \right) = m$$

$$I_z = \frac{1}{2} m \left(R_2^2 + R_1^2 \right)$$



$$I_z = \frac{2}{5} mR^2$$



$$I_z = \int \rho r^2 dV$$

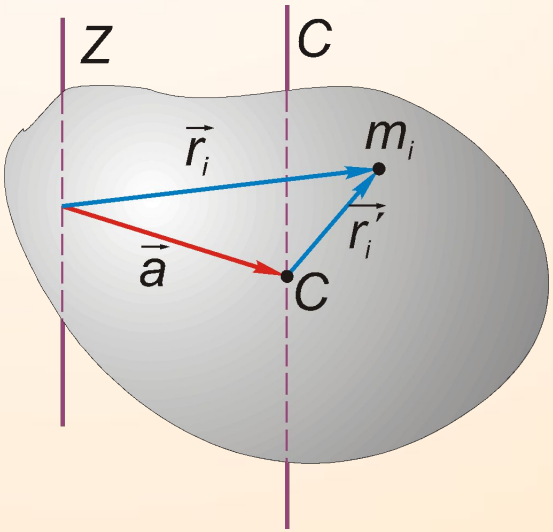
$$I_z = \int_{-\frac{l}{2}}^{\frac{l}{2}} \rho r^2 \cdot S dr = \rho S \left(\frac{1}{3} \left(\frac{l}{2} \right)^3 - \frac{1}{3} \left(-\frac{l}{2} \right)^3 \right)$$

$$= (\rho S l) \frac{l^2}{12}$$

$$I_z = \frac{1}{12} ml^2$$

Shtayner teoremasi:

Ixtiyoriy Z o'qiga nisbatan inersiya momenti I_z C o'qiga nisbatan inersiya momenti I_c ga teng bo'lib, jismning og'irlik markazidan o'tadi va jism massasi m, o'qlaroro masofa a ning kvadratining yig'indisiga teng.



$$I_Z = I_C + ma^2 \qquad (8.16)$$

$$\vec{r}_i = \vec{r}_i' + \vec{a}$$

$$I_Z = \sum m_i r_i^2 = \sum m_i \left(\vec{r}_i' + \vec{a}\right)^2$$

$$I_Z = \sum m_i r_i'^2 + 2\vec{a}\sum m_i \vec{r}_i' + \sum m_i a^2 \qquad (8.17)$$

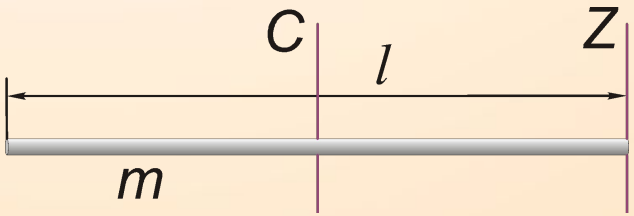
$$I_z = \sum m_i r_i'^2 + 2\vec{a}\sum m_i \vec{r}_i' + \sum m_i a^2$$

$$\sum m_i r_i'^2 = I_C$$

$$\sum m_i \vec{r}_i' = m\vec{r}_C' = 0$$

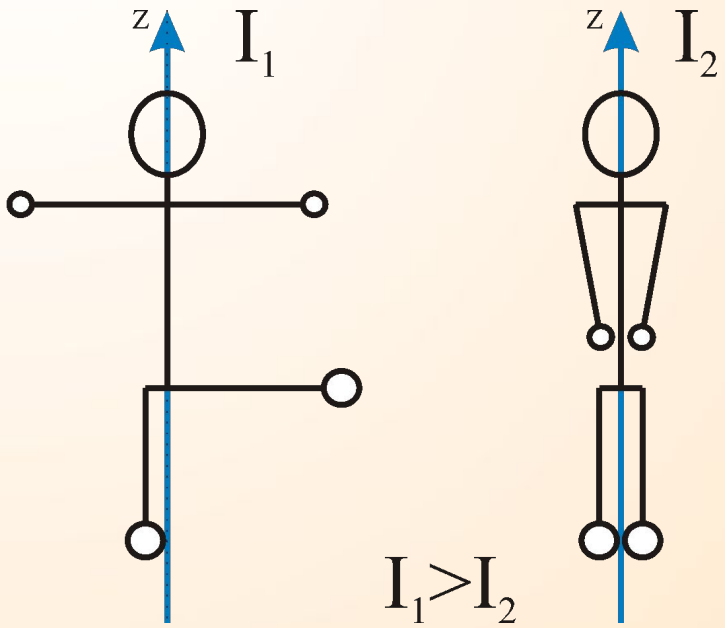
$$\sum m_i a^2 = ma^2$$

$$\boxed{I_z = I_C + ma^2}$$



$$I_z = \frac{1}{12}ml^2 + m\left(\frac{l}{2}\right)^2 = \frac{1}{3}ml^2$$

$$I_z \cdot \omega_z = const \qquad (8.18)$$



$$I_{z1} \cdot \omega_1 = I_{z2} \cdot \omega_2$$

$$\omega_1 < \omega_2$$

Topshiriq 8.2

Sterjen uzunligi 1,2 m va o'g'irligi 1 kg vertical o'qqa maxkamlangan, uning uzunligiga perpendikulyar joylashgan bo'lib, markazidan kesib o'tadi. Agar sterjen uchiga 8 g og'irlikdagi o'q 100 m/s tezlik bilan gorizontal tekislik bo'yicha tegsa, sterjen qaysi burchak tezlik bilan aylanadi?

Berilganlar:

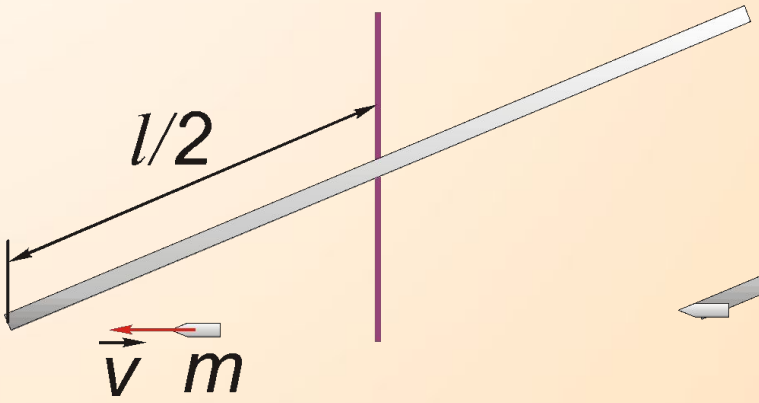
$l = 1{,}2$ m

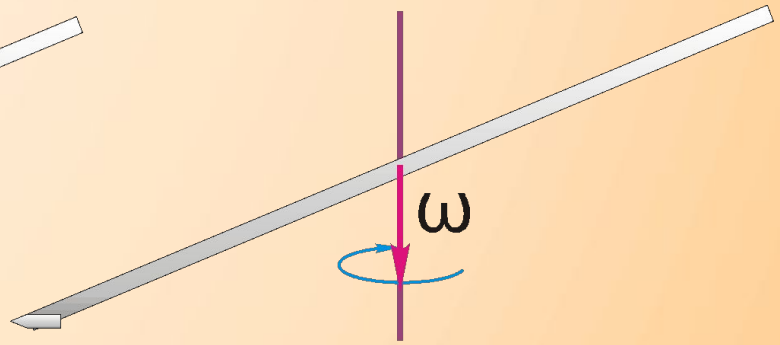
$M = 1$ kg

$m = 8$ g $= 8 \cdot 10^{-3}$ kg

$v = 100$ m/s

ω - ?





$$L_1 = mv\frac{l}{2}$$

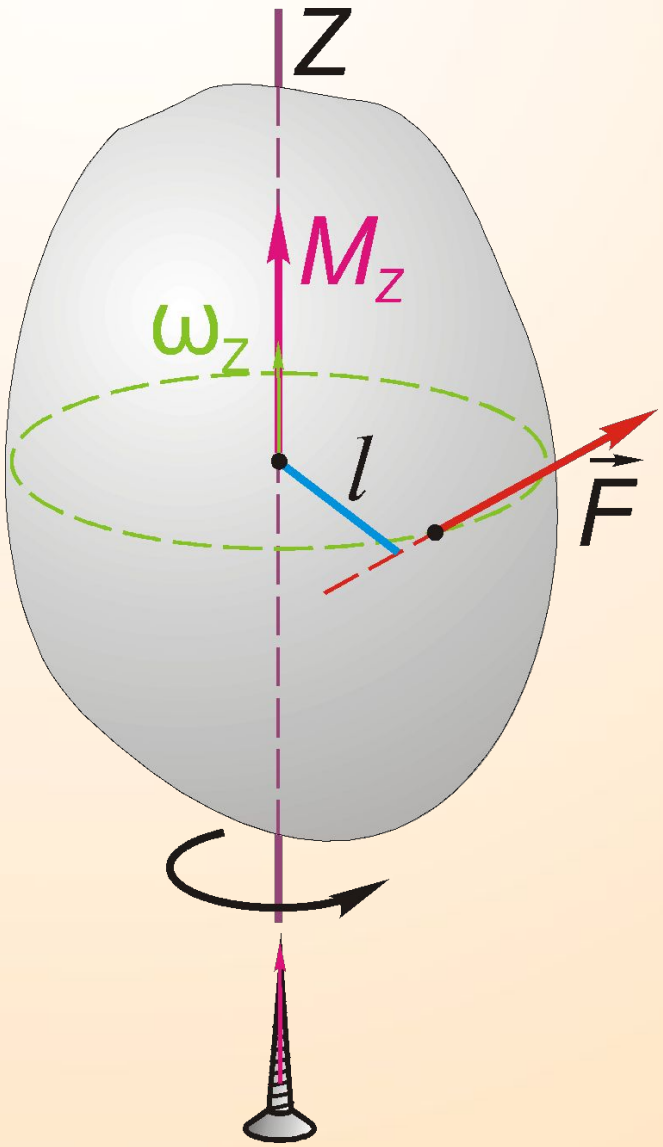
$$L_2 = (I_{ст} + I_п) \cdot \omega$$

$$L_2 = \left(\frac{1}{12} M l^2 + m \left(\frac{l}{2} \right)^2 \right) \cdot \omega = \frac{l^2 \omega}{12} (M + 3m)$$

$$L_1 = L_2 \qquad mv \frac{l}{2} = \frac{l^2 \omega}{12} (M + 3m)$$

$$\omega = \frac{6mv}{l(M + 3m)} = \frac{6 \cdot 8 \cdot 10^{-3}\ \text{кг} \cdot 100\ \text{м/с}}{1{,}2\ \text{м} \cdot (1\text{кг} + 3 \cdot 8 \cdot 10^{-3}\ \text{кг})} = 3{,}9\ \text{рад/с}$$

8.5. Dinamikaning aylanma harakat asosiy tenglamalari.



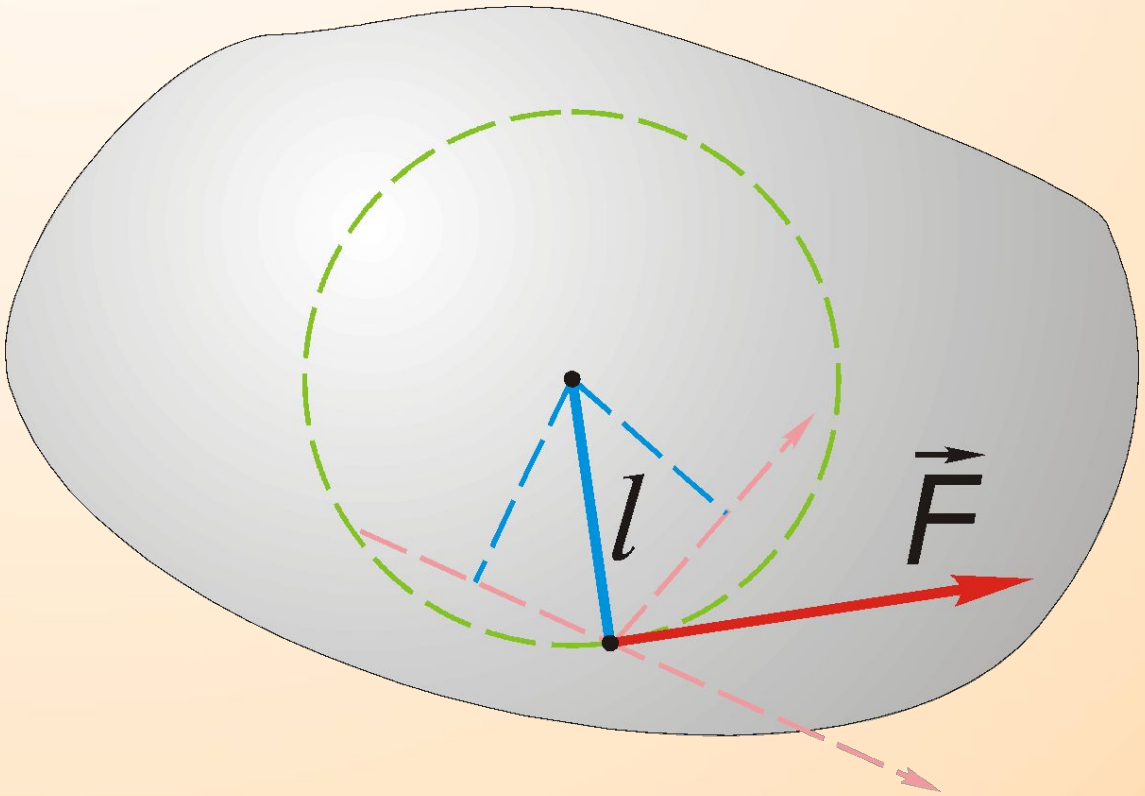
$$\frac{dL_z}{dt} = M_z$$

$$L_z = I_z \cdot \omega_z$$

$$M_z = \frac{d(I_z \cdot \omega_z)}{dt} = I_z \frac{d\omega_z}{dt}$$

$$\frac{d\omega_z}{dt} = \beta_z$$

$$\boxed{M_z = I_z \cdot \beta_z} \qquad (8.22)$$



$$M_z = F \cdot l$$

Topshiriq 8.3

Bir jinsli disk massasi 5 kg va radiusi 0,2 m markazdan o'tgan o'q atrofida aylanmoqda. Diskning burchak tezligini vaqtga bog'liqlik tenglamasi $\omega = A+Bt$, bu erda $B = 8$ рад/с^2 berilgan. Disk obodasiga qo'yilgan urinma kuch qiymati aniqlansin. Ishqalanish hisobga olinmasin.

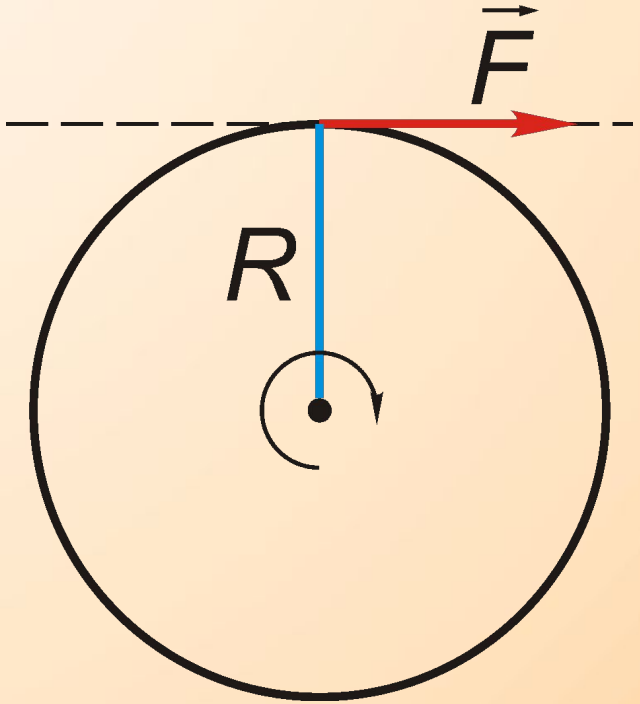
Berilgan:

$m = 5$ kg

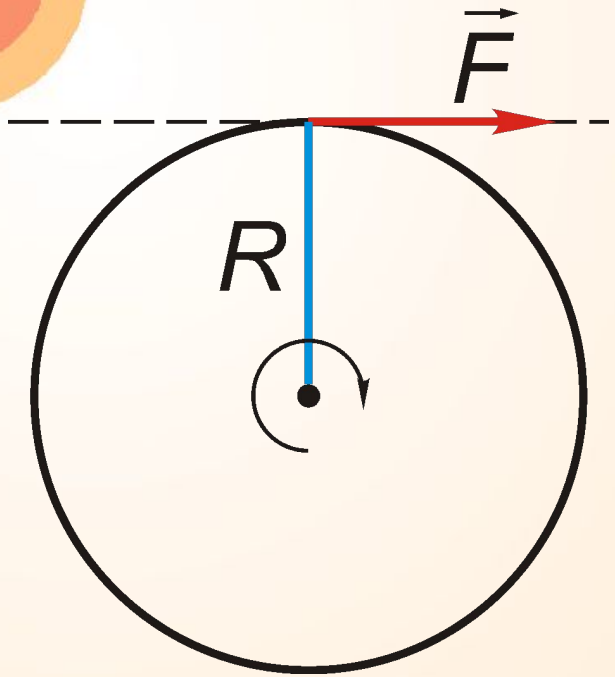
$R = 0{,}2$ m

$B = 8$ rad/s^2

F - ?



$$M_z = I_z \cdot \beta_z$$



$$M_z = F \cdot l = F \cdot R$$

$$I_z = \frac{mR^2}{2}$$

$$\beta_z = \frac{d\omega_z}{dt} = \frac{d(A + Bt)}{dt} = B$$

$$F \cdot R = \frac{mR^2 B}{2}$$

$$F = \frac{mRB}{2} = \frac{5\,\text{kg} \cdot 0{,}2\,\text{m} \cdot 8\,\text{rad/s}^2}{2} = 4\,\frac{\text{kg} \cdot \text{m}}{\text{s}^2} = 4\,\text{N}$$