

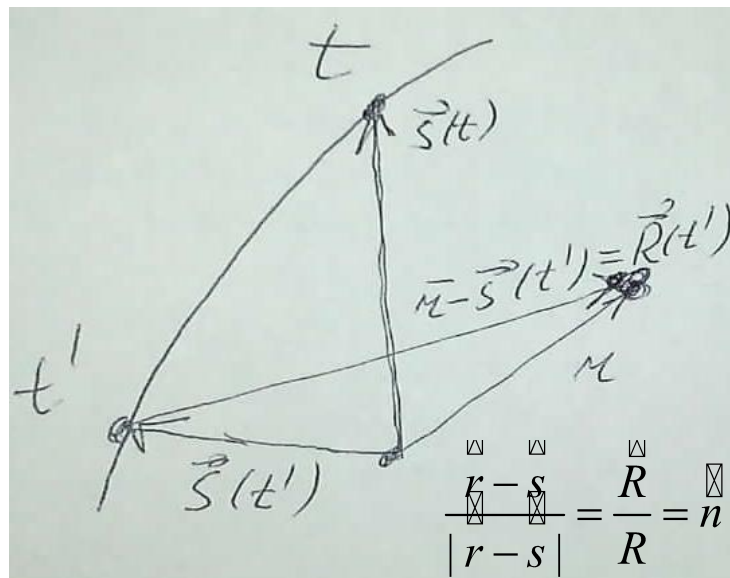


6. Потенциалы Лиенара-Вихерта. Поле точечного заряда.

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6.1 Потенциалы Лиенара-Вихерта



$s(t)$ — положение

$$\rho(r, t) = e \delta(r - s(t))$$

$$j(r, t) = eV(t) \delta(r - s(t))$$

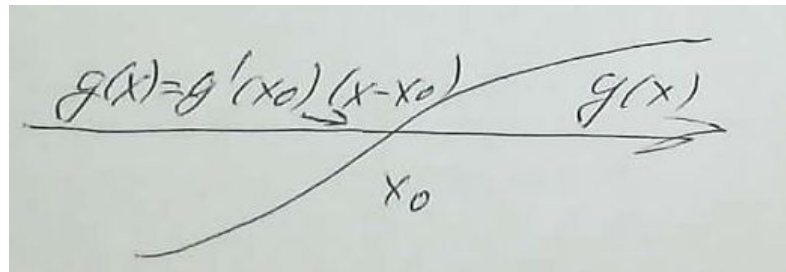
$$V = \frac{ds}{dt}$$

$$\varphi(r, t) = \int \frac{\delta(t - t' - \frac{|r - r'|}{c})}{|r - r'|} \delta(r' - s(t')) dr' dt'$$

$$\begin{aligned} A(r, t) &= \frac{e}{c} \int \frac{\delta(t - t' - \frac{|r - r'|}{c})}{|r - r'|} V(t') \delta(r' - s(t')) dr' dt' = \\ &= \frac{e}{c} \int_{-\infty}^{+\infty} \frac{V(t')}{|r' - s(t')|} \delta(t' - (t - \frac{|r' - s(t')|}{c})) dt' \end{aligned}$$

$$\int \delta(g(x)) f(x) dx = \int \delta(g'(x_0)(x - x_0)) f(x) dx = \int \frac{1}{|g'(x_0)|} \delta(x - x_0) f(x) dx = \frac{1}{|g'(x_0)|} f(x_0)$$

$$g(x_0) = 0$$



6.1(+) Потенциалы Лиенара-Вихерта

$$\delta(g(x)) = \frac{1}{|g'(x_0)|} \delta(x - x_0) \quad g(t') = t' - \left(t - \frac{|r - s(t')|}{c}\right) = 0$$

$$(t - t') = \frac{|r - s(t')|}{c} \Rightarrow t' = t'_k(r, t)$$

$$\frac{dg(t')}{dt'} = 1 + \frac{1}{c} \frac{\partial |r - s(t')|}{\partial (r - s)} * \frac{\partial (r - s)}{\partial t'} = 1 + \frac{1}{c} \frac{x_i - s_j}{|r - s|} (-V_j) = 1 - \frac{1}{c} \frac{r - s}{|r - s|} V = 1 - \frac{nV}{c}$$

$$A(r, t) = \frac{e}{c} \frac{V(t')}{|r - s(t')| \left(1 - \frac{nV}{c}\right)}, [t' = t'_k(r, t)]$$

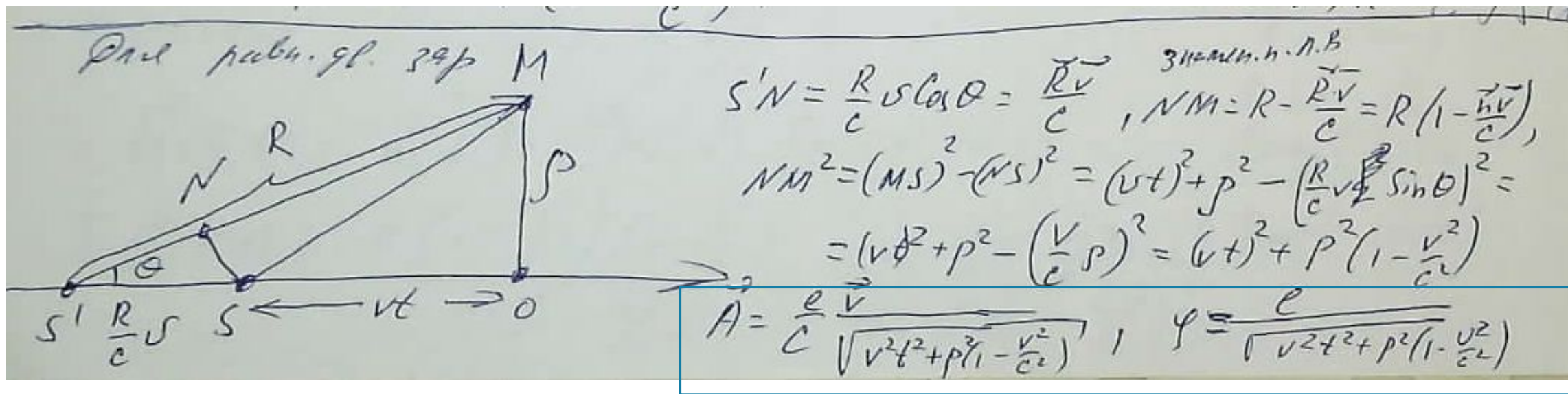
$$\varphi(r, t) = \frac{e}{|r - s(t')| \left(1 - \frac{nV}{c}\right)}, [t' = t'_k(r, t)]$$

$$\frac{r - s}{|r - s|} = \frac{R}{R} = n$$

6.1(++) Потенциалы Лиенара-Вихерта (равн движ заряд)

$$(t - t') = \frac{|r - s(t')|}{c} \Rightarrow t' = t'_k(r, t)$$

$$A(r, t) = \frac{e}{c} \frac{V(t')}{|r - s(t')| (1 - \frac{nV}{c})}, [t' = t'_k(r, t)]$$



6.2 Поле точечного заряда (H=rotA ...)

$$\frac{\partial}{\partial x_j} \frac{1}{R} = \frac{\partial}{\partial R} \frac{1}{R} \frac{\partial R}{\partial x_j} = -\frac{1}{R^2} \frac{R_j}{R}$$

$$\vec{H} = \frac{e}{c} \nabla \times \int \frac{\vec{v}(t')}{R} \delta(t-t'-\frac{R}{c}) dt' = \frac{e}{c} \varepsilon_{ijk} \frac{\partial}{\partial x_j} \int \frac{v_k(t')}{R} \delta(t-t'-\frac{R}{c}) dt'$$

$$H = \frac{e}{c} \int [\varepsilon_{ijk} (-\frac{R_j}{R}) v_k \delta(t-t'-\frac{R}{c}) + \varepsilon_{ijk} \frac{1}{R} v_k \delta'(t-t'-\frac{R}{c}) (-\frac{1}{c}) \frac{R_j}{R}] dt'$$

$$H = \frac{e}{c} \frac{(\vec{n} \times \vec{v})(-1)}{R^2 (1-\frac{nv}{c})} + \dots = \frac{e}{c R^2} \frac{(\vec{n} \times \vec{v})}{(1-\frac{nv}{c})} + \dots; t' = t'_k$$

$$\delta(t-t'-\frac{R}{c}) = \frac{\delta'(t-\frac{R}{c}-t')}{1-\frac{nv}{c}}$$

$$\int_{-\infty}^{\infty} f(x) \frac{\partial \delta(x-a)}{\partial x} dx = f(x) \delta(x-a) |_{(-\infty; \infty)} - \int_{-\infty}^{\infty} \delta(x-a) f'(x) dx = -\frac{\partial f(a)}{\partial x}$$

$$\frac{e}{c} \int \frac{(\vec{n} \times \vec{v})}{cR} \frac{d\delta(t-t'-\frac{R}{c})}{dt'} \frac{1}{\frac{d(t-t'-\frac{R}{c})}{dt'}} dt' = \frac{e}{c} \int \frac{(\vec{n} \times \vec{v})}{(-1)(1-\frac{nv}{c})R} d\delta(t-t'-\frac{R}{c}) =$$

$$= (-1)(-1) \frac{e}{c} \int_{-\infty}^{\infty} \frac{\delta(t'-t'_k)}{(1-\frac{nv}{c})R} \frac{d}{dt'} \frac{(\vec{n} \times \vec{v})}{(1-\frac{nv}{c})} dt'$$

$$\frac{dR}{dt'} = -nv; \frac{dn}{dt'} = \frac{d}{dt'} \frac{R}{R} = -\frac{v}{R} + (-\frac{1}{R^2}) (\frac{R}{R} * (-v)) R = -\frac{v}{R} + \frac{n(nv)}{R} = \frac{n \times (n \times v)}{R}$$

$$H = \frac{e(1-\frac{v^2}{c^2})(\vec{n} \times \vec{v})}{cR^2(1-\frac{nv}{c})^3} + \frac{e(\frac{d\vec{v}}{dt} \times \vec{n} + \vec{n} \times [\vec{v} \times \frac{d\vec{v}}{dt}] \times \vec{n})}{c^3 R(1-\frac{nv}{c})^3}$$

аналогично

$$\vec{E} = -\nabla \varphi - \frac{1}{c} \frac{\partial \vec{A}}{\partial t}$$

$$\vec{H}(\vec{r}, t) = \vec{n}(t') \times \vec{E}(\vec{r}, t'); t' = t'_k$$

$$\vec{E}(\vec{r}, t) = \frac{e(1-\frac{v^2}{c^2})(\vec{n} - \frac{\vec{v}}{c})}{R^2(1-\frac{nv}{c})^3} + \frac{en \times [(\vec{n} - \frac{\vec{v}}{c}) \times \frac{d\vec{v}}{dt}]}{c^2 R(1-\frac{nv}{c})^3}$$

6.3 Поток энергии. Формула Лармора.

$$\vec{E}(r, t) = \frac{e(1 - \frac{v^2}{c^2})(\vec{n} - \frac{\vec{v}}{c})}{R^2(1 - \frac{nv}{c})^3} + \frac{en \times [(\vec{n} - \frac{\vec{v}}{c}) \times \frac{d\vec{v}}{dt}]}{c^2 R(1 - \frac{nv}{c})^3}$$

Кулон

Излучение

поток энергии в телесный угол $d\Omega$ ($\vec{H} = \vec{n} \times \vec{E}$)

$$dI = \vec{S} \cdot \vec{n} R^2 d\Omega = \frac{c}{4\pi} (\vec{E} \times (\vec{n} \times \vec{E})) \cdot \vec{n} R^2 d\Omega =$$

$$= (\text{только второе слагаемое}, \vec{E} \perp \vec{n}) = \frac{c}{4\pi} E^2 R^2 d\Omega$$

$$\frac{dI}{d\Omega} = \frac{e^2 \left(\vec{n} \times [(\vec{n} - \frac{\vec{v}}{c}) \times \frac{d\vec{v}}{dt}] \right)^2}{4\pi c^3 (1 - \frac{nv}{c})^6}$$

$$\frac{v}{c} \ll 1$$

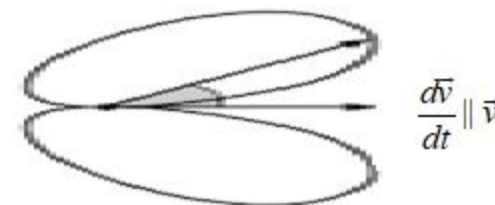
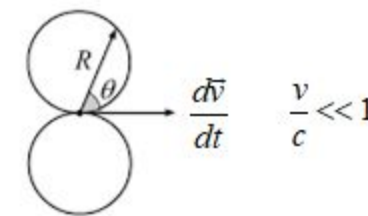
$$\frac{dI}{d\Omega} = \frac{e^2}{4\pi c^3} [\vec{n} \times (\vec{n} \times \frac{d\vec{v}}{dt})]^2 = [\vec{n} * (\vec{n} * \vec{v}) - \frac{d\vec{v}}{dt}]^2 = \sin^2 \theta (\frac{d\vec{v}}{dt})^2 * \frac{e^2}{4\pi c^3}$$

Формула Лармора для нерелятивистского заряда

$$P = \int \frac{dI}{d\Omega} d\Omega = 2\pi \frac{e^2 (\frac{d\vec{v}}{dt})^2}{4\pi c^3} \int_{-1}^1 d\cos\theta (1 - \cos^2 \theta) =$$

$$\frac{e^2 (\frac{d\vec{v}}{dt})^2}{2c^3} (z - \frac{z^3}{3}) \Big|_{-1;1} = \frac{e^2 (\frac{d\vec{v}}{dt})^2}{2c^3} \frac{4}{3} = \frac{2}{3} \frac{e^2 (\frac{d\vec{v}}{dt})^2}{c^3} = \frac{2}{3} \frac{e^2 \dot{v}^2}{c^3}$$

$$\frac{v}{c} \ll 1, \frac{d\vec{v}}{dt} \parallel \vec{v} \quad \frac{dI}{d\Omega} = \sin^2 \theta (\frac{d\vec{v}}{dt})^2 * \frac{e^2 \dot{v}^2 \sin^2 \theta}{4\pi c^3 (1 - \frac{v}{c} \cos \theta)^6}$$



На этом обсуждение темы
«Потенциалы Лиенара-Вихерта.
Поле точечного заряда.»
в курсе онлайн-лекций завершено

До новых встреч!