

ТЕОРИЯ КРИВЫХ

Формулы Серре-Френе

Формулы Сере-Френе

$$\frac{d\bar{\tau}}{ds} = \bar{\tau}' = (\bar{r}')' = \bar{r}'' \stackrel{(18)}{=} k \cdot \bar{v}$$

$$\frac{d\bar{\beta}}{ds} = \bar{\beta}' \stackrel{(19)}{=} [\bar{\tau}; \bar{v}]' = [\bar{\tau}'; \bar{v}] + [\bar{\tau}; \bar{v}'] = [k\bar{v}; \bar{v}] + [\bar{\tau}; \bar{v}'] = [\bar{\tau}; \bar{v}']$$

$$\left. \begin{array}{l} \frac{d\bar{\beta}}{ds} \perp \bar{\beta}, \text{ в силу } \textcolor{red}{\text{утверждения 1}} \text{ раздела «Векторный анализ»} \\ \frac{d\bar{\beta}}{ds} \perp \bar{\tau} \end{array} \right\} \Rightarrow \frac{d\bar{\beta}}{ds} \parallel \bar{v}$$

$$\Rightarrow \frac{d\bar{\beta}}{ds} = -\chi \bar{v}$$

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$$\begin{aligned}\bar{v}' &= \frac{d\bar{v}}{ds} \stackrel{(22)}{=} [\bar{\beta}; \bar{\tau}]' = [\bar{\beta}'; \bar{\tau}] + [\bar{\beta}; \bar{\tau}'] = [-\chi \bar{v}; \bar{\tau}] + [\bar{\beta}; k\bar{v}] = \\ &= -\chi[\bar{v}; \bar{\tau}] + k[\bar{\beta}; \bar{v}] \stackrel{(22)}{=} \chi \bar{\beta} - k\bar{\tau} \\ &\quad \left\{ \begin{array}{l} \frac{d\bar{\tau}}{ds} = k\bar{v} \\ \frac{d\bar{v}}{ds} = -k\bar{\tau} + \chi \bar{\beta} \\ \frac{d\bar{\beta}}{ds} = -\chi \bar{v} \end{array} \right. \quad (23)\end{aligned}$$

(23) – формулы Сере-Френе.

ВЫХО

Д

Утверждение 1:

Для того чтобы $\bar{u}' \perp \bar{u}$ необходимо и достаточно, чтобы $|\bar{u}| = \text{const}$.



$$\frac{\bar{r}''}{k} = \frac{\bar{r}''}{|\bar{r}''|} = \bar{v} \quad (18)$$



$$\bar{\beta} \equiv [\bar{\tau}; \bar{\nu}] \quad (19)$$



$$\left\{ \begin{array}{lll} [\bar{\tau}; \bar{\tau}] = \bar{0} & [\bar{\nu}; \bar{\tau}] = -\bar{\beta} & [\bar{\beta}; \bar{\nu}] = -\bar{\tau} \\ [\bar{\tau}; \bar{\nu}] = \bar{\beta} & [\bar{\nu}; \bar{\nu}] = \bar{0} & [\bar{\beta}; \bar{\tau}] = \bar{\nu} \\ [\bar{\tau}; \bar{\beta}] = -\bar{\nu} & [\bar{\nu}; \bar{\beta}] = \bar{\tau} & [\bar{\beta}; \bar{\beta}] = \bar{0} \end{array} \right. \quad (22)$$

