

Analysis of Statically Determinate Structures

ECE479 Structural Analysis II

Text Book
Structural Analysis
by
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Lecture Outlines

- Idealized Structure
- Equations of Equilibrium
- Determinacy and Stability

Intended Learning Outcomes

By the end of today's session student's should be able to:

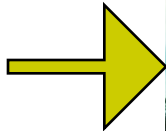
- Idealize a structure
- Determine Determinacy and Stability of structure

Why Idealize Structure?

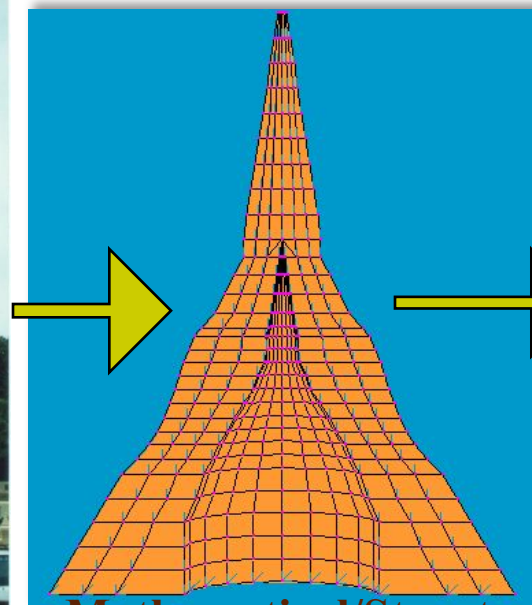
- Exact analysis --- Not possible
 - Estimate
 - Loading and its point of application
 - Strength of the Materials

EXCITATION

Loads
Vibrations
Settlements
Thermal Changes



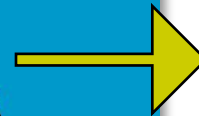
Real Structure



**Mathematical/Structural
Model**

RESPONSES

Displacements
Strains
Stress
Stress Resultants



Support Connections

Types --- Usually Three

- Pin supported connection
- Roller supported connection
- Fixed supported connection

Support Connections- Roller support



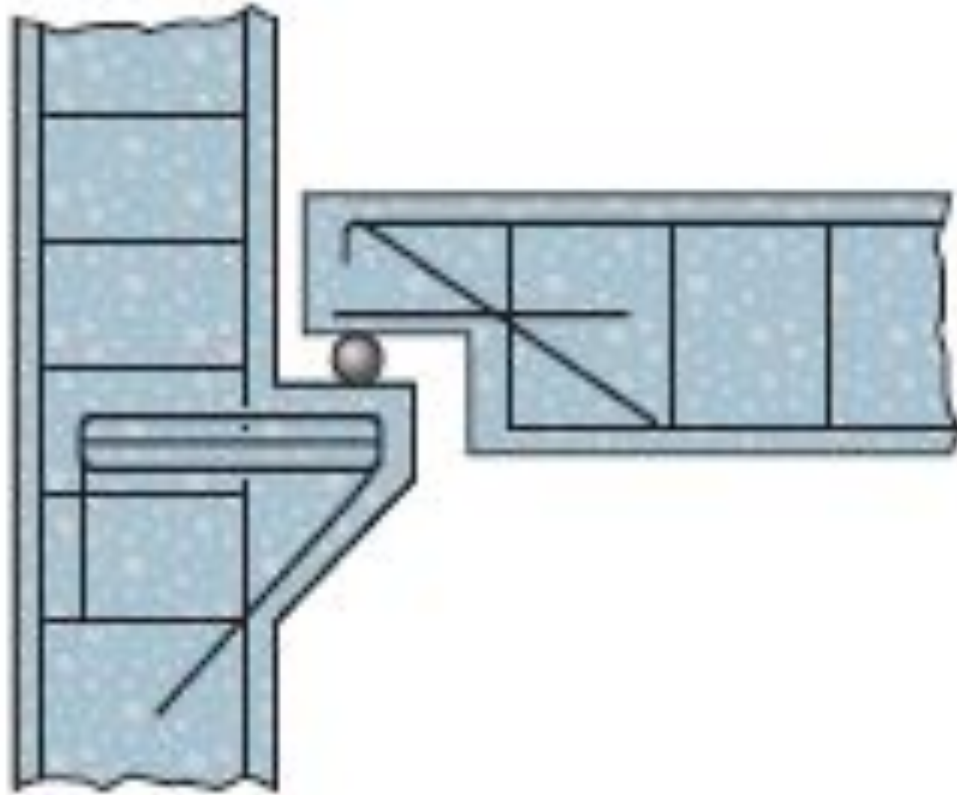
- Roller support - Deck of concrete bridge (One section considered roller supported on other section)

Support Connections- Roller support



- Roller support - Used to supports prestressed girders of a highway bridge.

Support Connections- Roller support



- Roller supported Concrete connection

Support Connections – Pin support

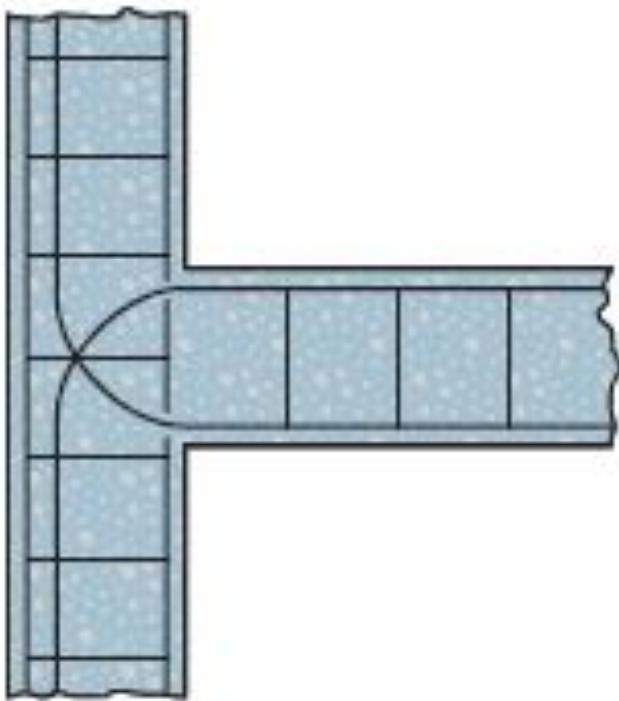


Pin support - Steel girder Railway bridge

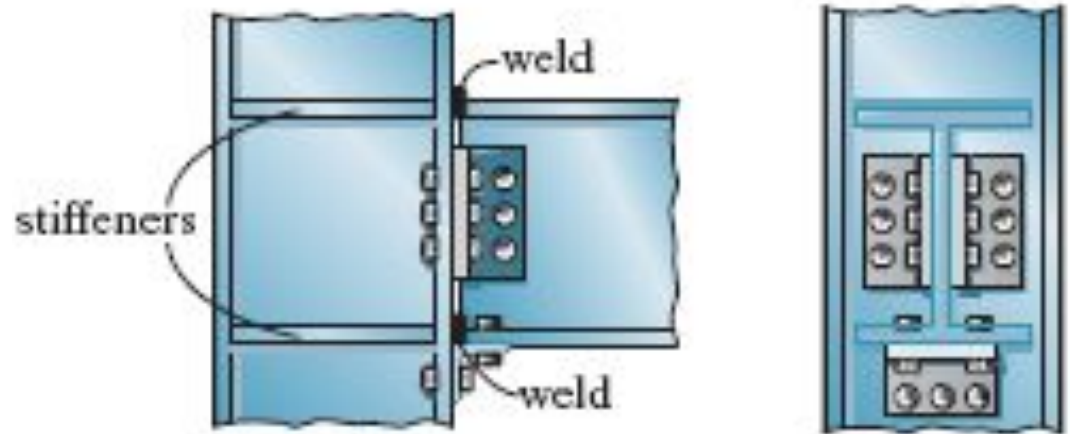


Pin supported
Metal connection

Support Connections – Fixed support



Fixed supported
Concrete connection



Fixed supported
Metal connection

**Hinge
Support**



**Roller
Support**

Equations of Equilibrium

- For complete static equilibrium in 2D, three requirements must be met:
 1. External Horizontal forces balance (translation).
 2. External Vertical forces balance (translation).
 3. External Moments balance about any point (rotational).

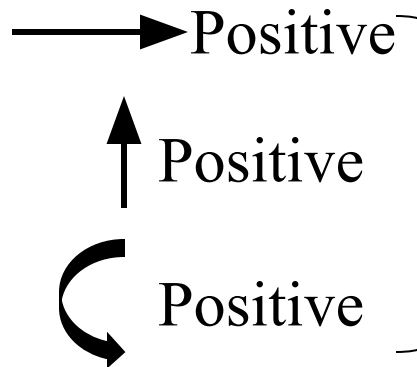
Equations of Equilibrium

- For two-dimensional system of forces and moments, the equilibrium equations are:

1. $\Sigma F_x = 0$

2. $\Sigma F_y = 0$

3. $\Sigma M_z = 0$



← *Sign Conventions*

Determinate vs Indeterminate Structure

- When all the forces in a structure can be determined from the equilibrium equations, the structure is referred to as *statically determinate*.
- When the unknown forces in a structure are more than the available equilibrium equations, that structure is known as *statically indeterminate*.

Determinacy

- For a coplanar structure, there are at most three equilibrium equations for each part.

If there is a total of n parts and r force and moment reaction components, we have

$r = 3n$ *statically determinate*

$r > 3n$ *statically indeterminate*

Determinate vs Indeterminate Structure – Examples (Beams)



(a)

$$r = 3, n = 1, 3 = 3(1)$$



Statically determinate



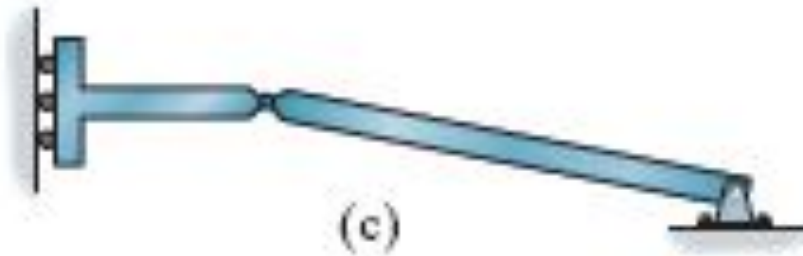
(b)

$$r = 5, n = 1, 5 > 3(1)$$



Statically indeterminate to the second degree

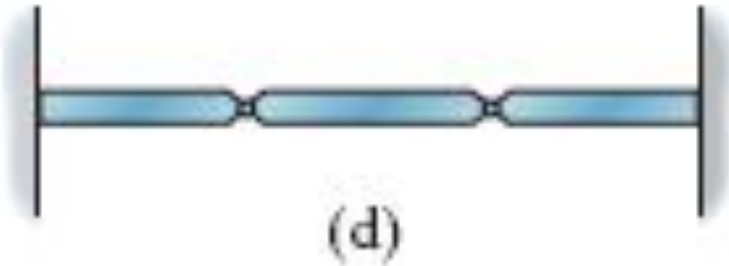
Determinate vs Indeterminate Structure – Examples (Beams)



$$r = 6, n = 2, 6 = 3(2)$$



Statically determinate

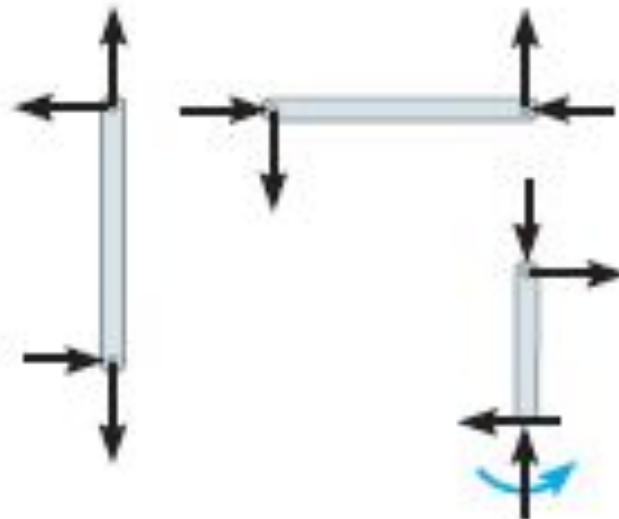
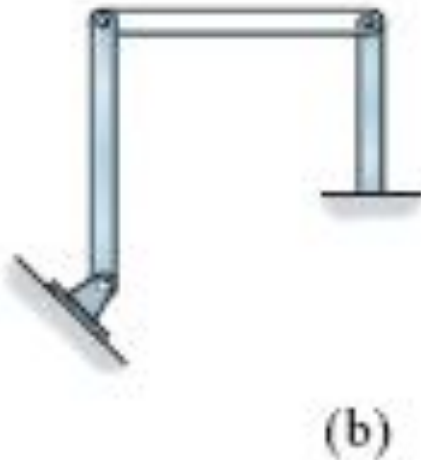
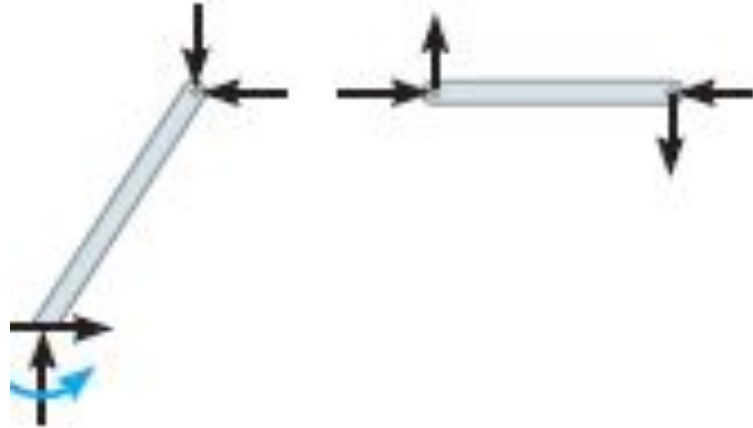
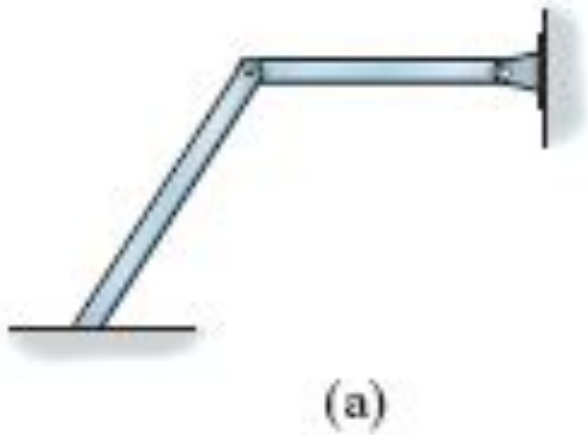


$$r = 10, n = 3, 10 > 3(3)$$

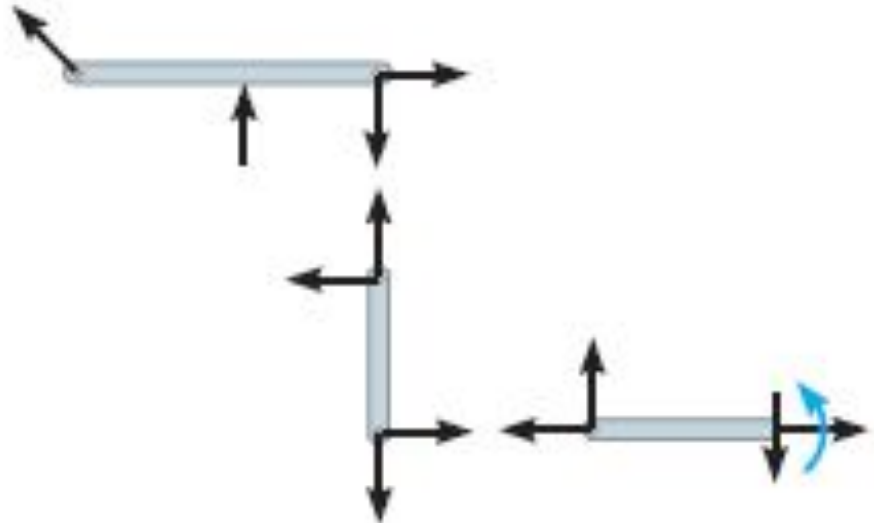
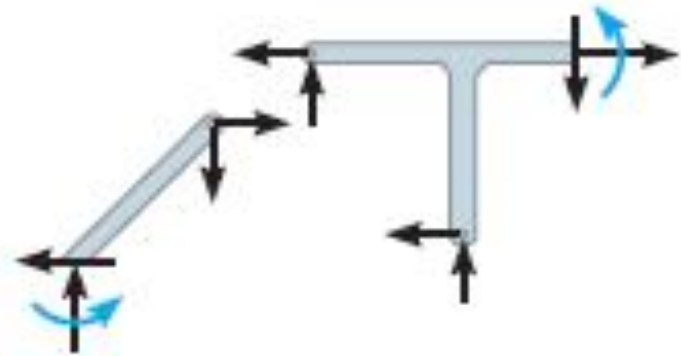
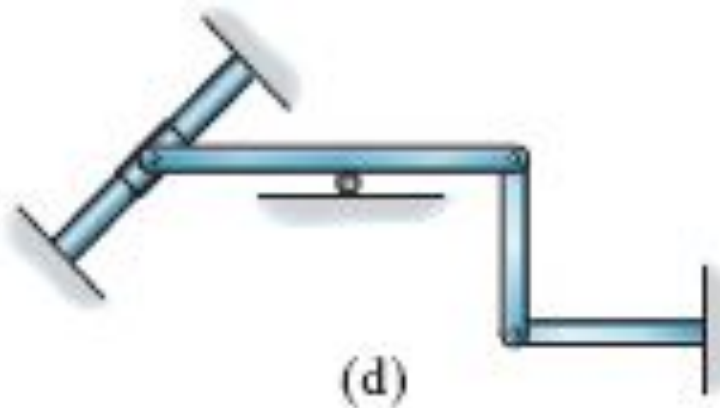
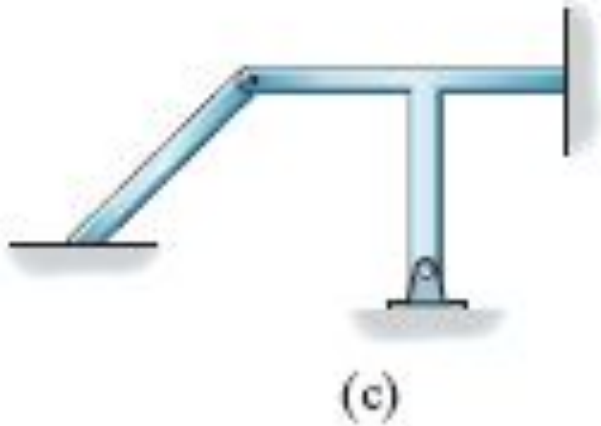


Statically indeterminate to the first degree

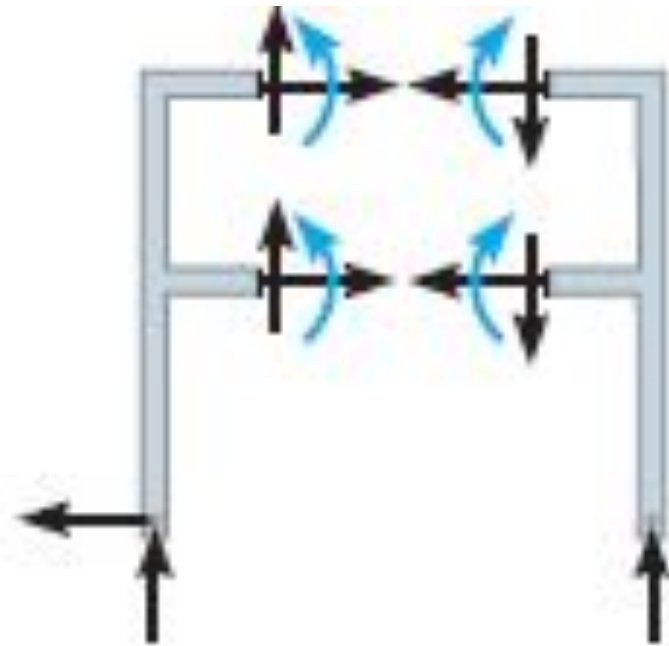
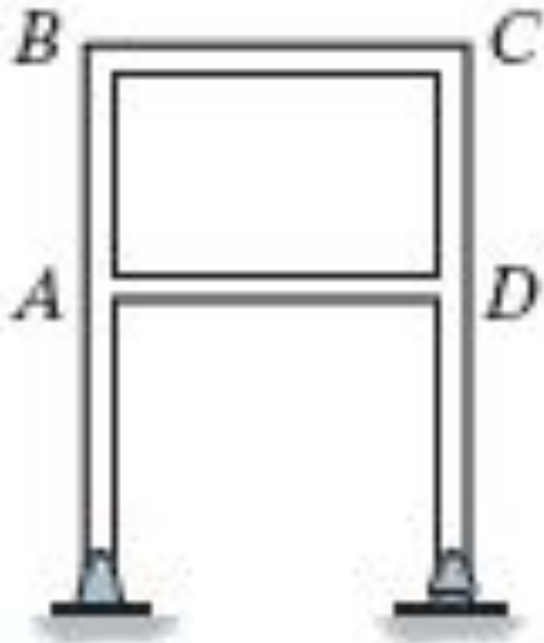
Determinate vs Indeterminate – Examples (Pin-connected structures)



Determinate vs Indeterminate – Examples (Pin-connected structures)



Determinate vs Indeterminate Structure – Examples (Frame)

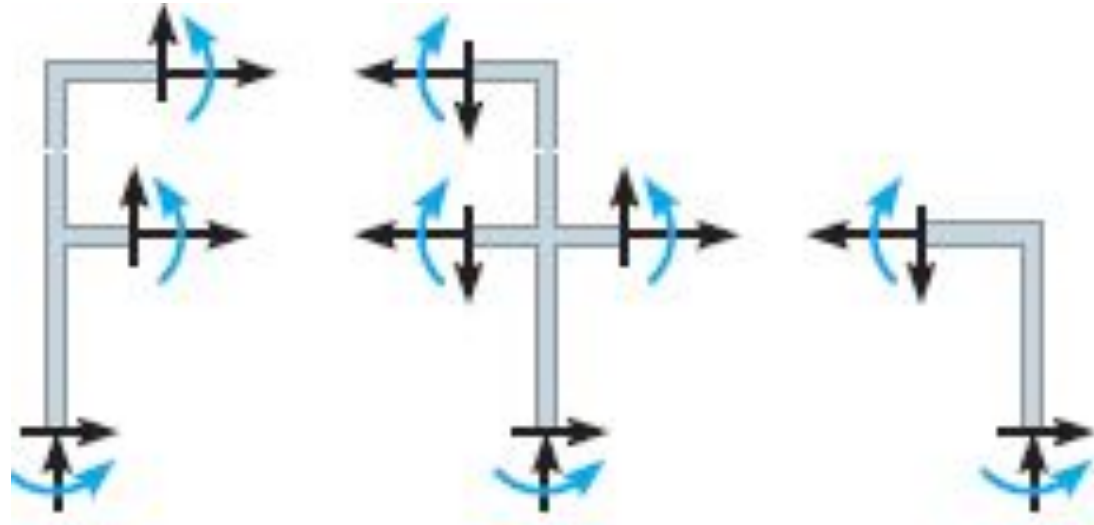
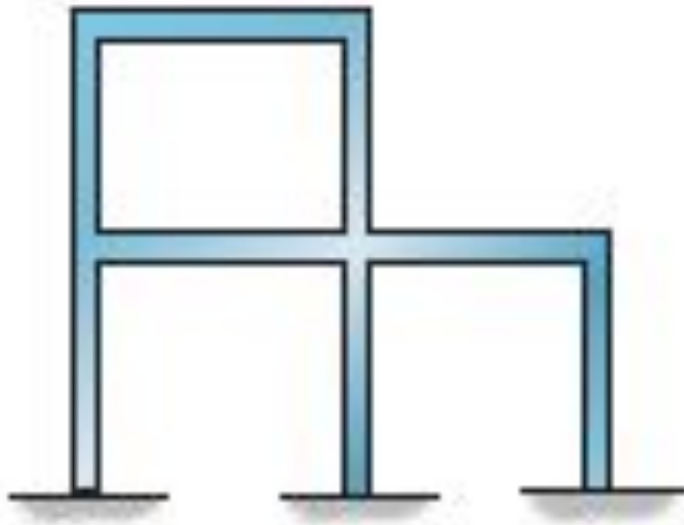


$$r = 9, n = 2, 9 > 6,$$

Statically indeterminate to the
third degree

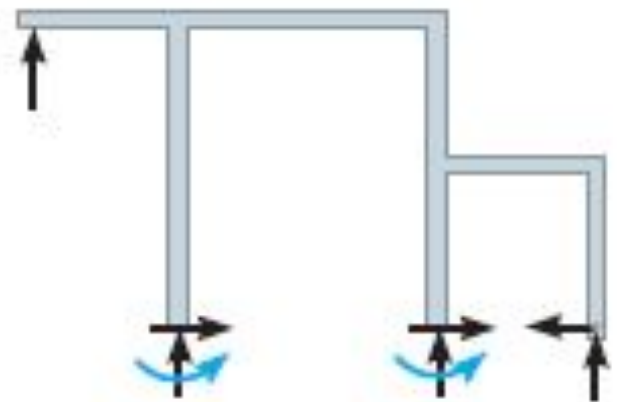
Ans.

Determinate vs Indeterminate Structure – Examples (Frame)

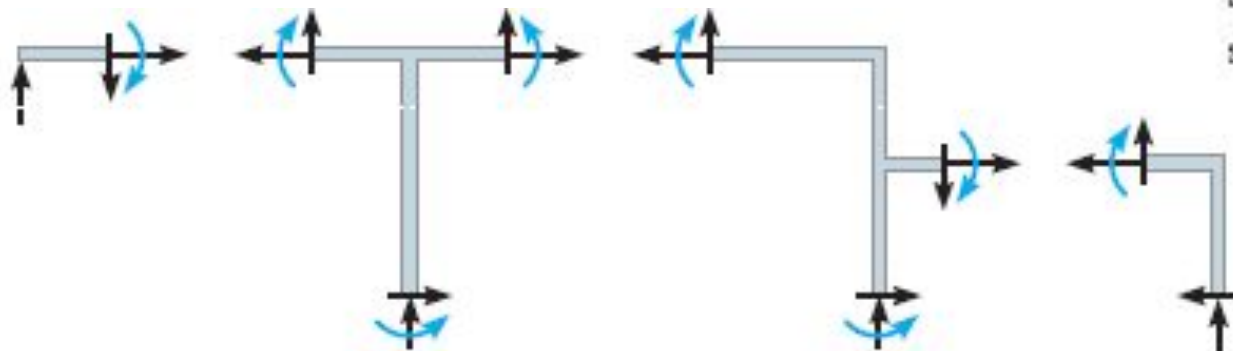


$r = 18, n = 3, 18 > 9,$
Statically indeterminate to the
ninth degree *Ans.*

Determinate vs Indeterminate Structure – Examples (Frame)



$r = 9, n = 1, 9 > 3,$
Statically indeterminate to the
sixth degree *Ans.*



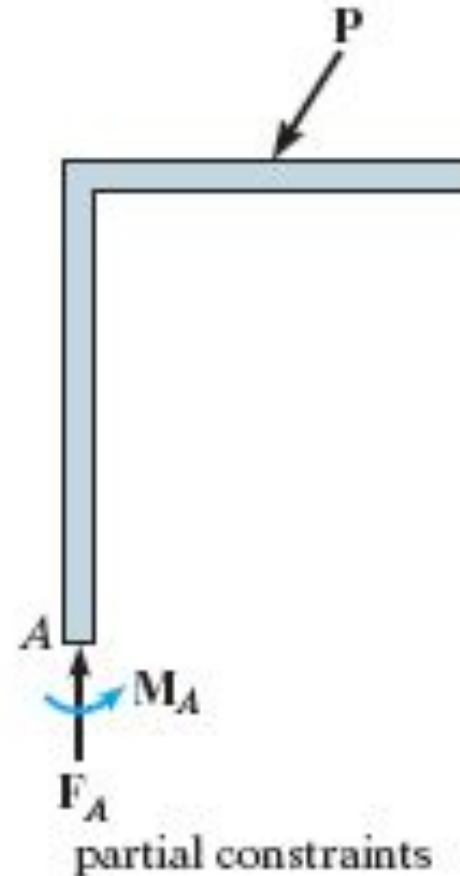
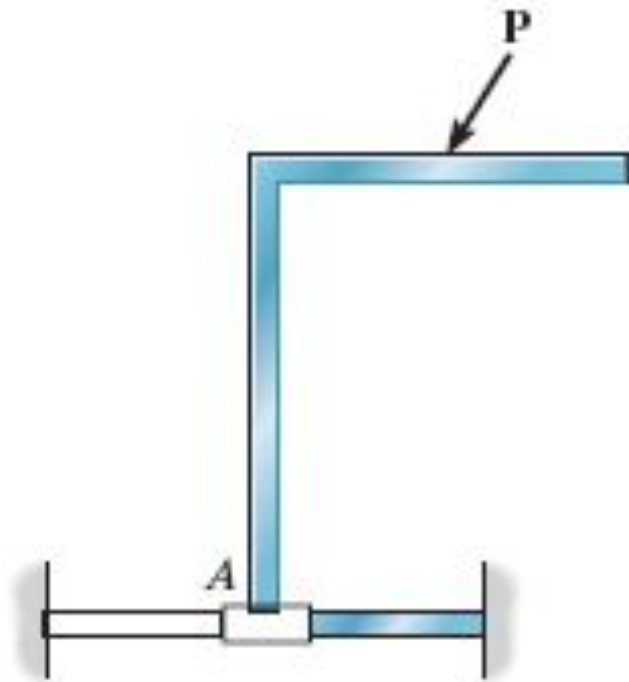
(c)

$r = 18, n = 4, 18 > 12,$
Statically indeterminate to the
sixth degree *Ans.*

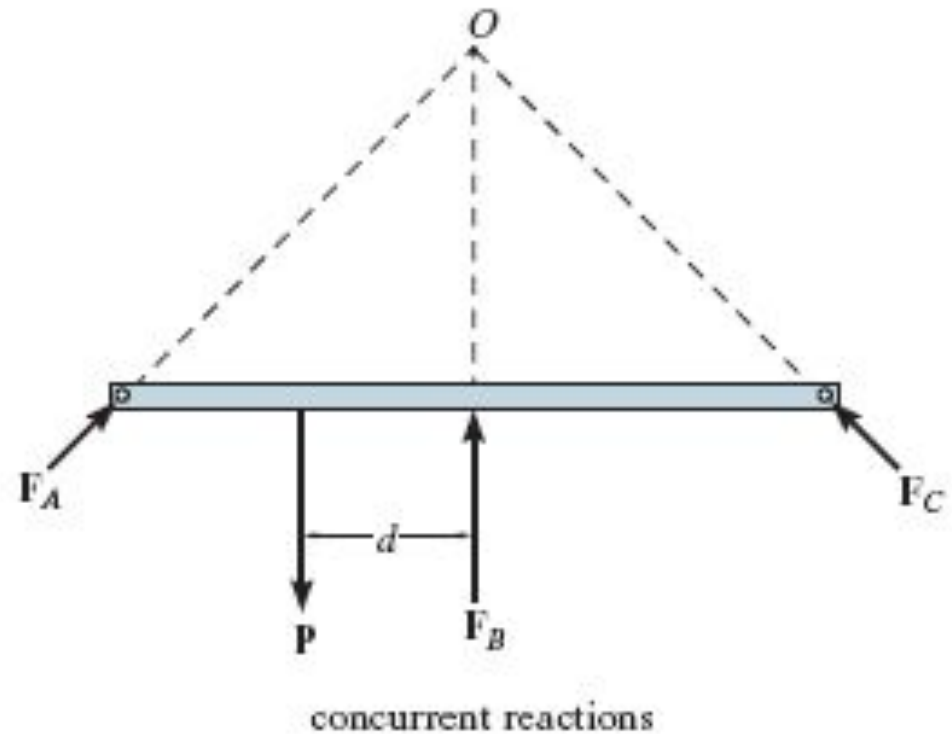
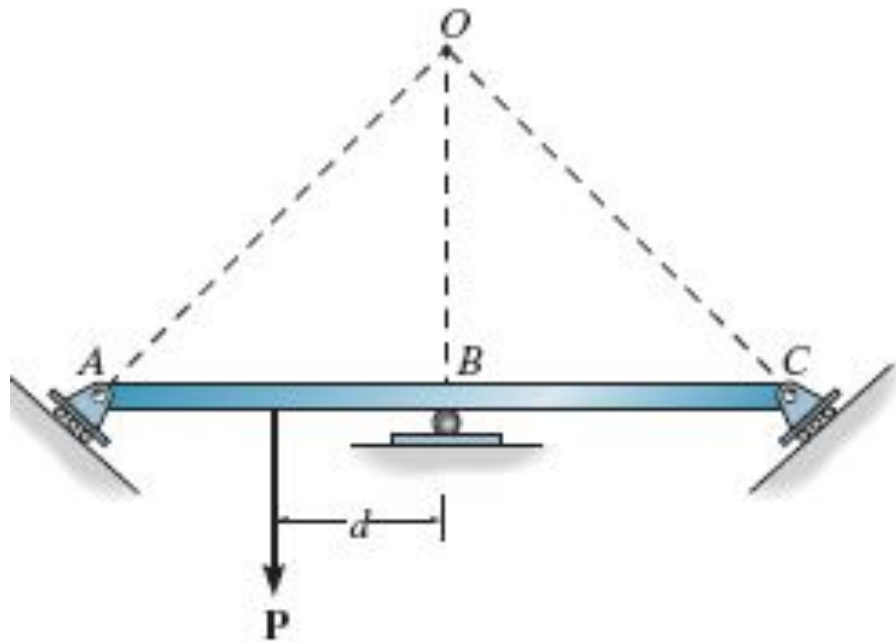
Stability

- What conditions are necessary To ensure equilibrium of a structure?
- A structure will be unstable if
- there are fewer reactive forces than equations of equilibrium
(Partial Constraints)
or
- there are enough reactions and instability will occur if the lines of action of reactive forces intersect at a common point or are parallel to one another **(Improper Constraints)**

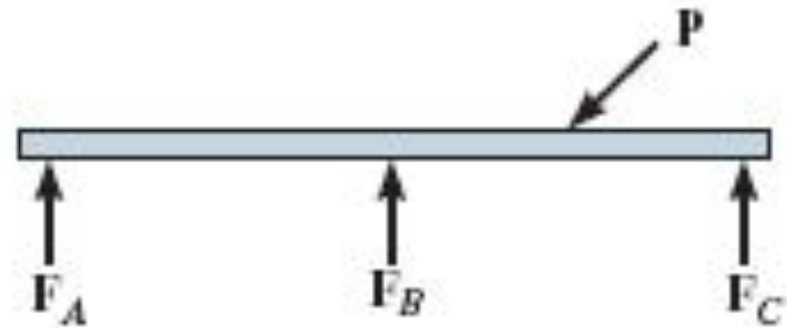
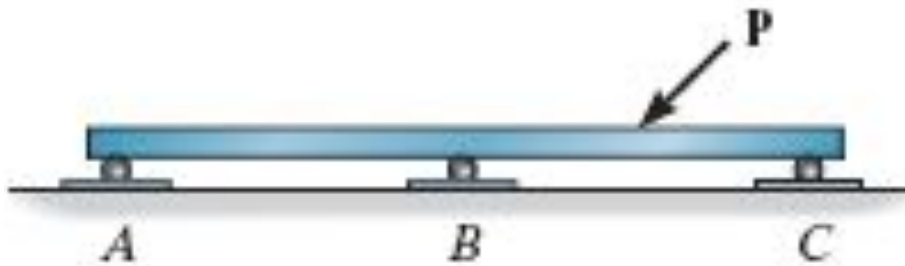
Stability – Example – Partial Constraints



Stability – Example – Improper Constraints



Stability – Example – Improper Constraints



parallel reactions

Stability

$r < 3n$ *unstable*

$r \geq 3n$ *unstable if member reactions
are concurrent or parallel or
some of the components form a
collapsible mechanism*

r --- *Unknown reactions*

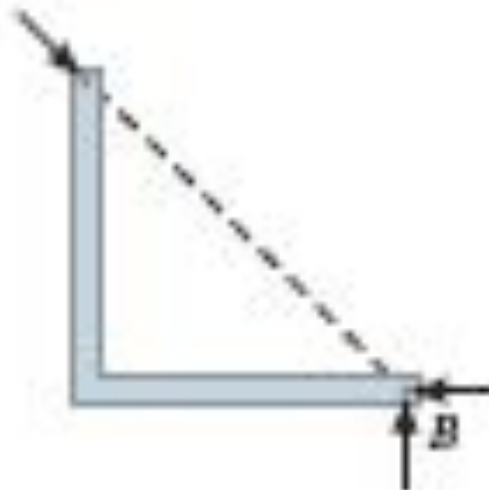
n --- *Members*

Unstable structures □ Must be avoided in practice

Stability – Examples



Stable



Unstable

Stability

$r < 3n$ *unstable*

$r \geq 3n$ *unstable if member reactions
are concurrent or parallel or
some of the components form
a collapsible mechanism*

r --- *Unknown reactions*

n --- *Members*

Summary

Now You should be able to:

- Idealize a structure
- Determine Determinacy and Stability of structure

Assignment 1

Issue Date 16-1-2017

Submission Date 23-1-2017

- Classify each of the structures as statically determinate, statically indeterminate, or unstable. If indeterminate, specify the degree of indeterminacy



Assignment 1

Issue Date 23-1-2017

Submission Date 30-1-2017

- Classify each of the structures as statically determinate, statically indeterminate, or unstable. If indeterminate, specify the degree of indeterminacy

