# Laplace Transform

In each of Problems 15 through 20, use integration by parts to find the Laplace transform of the given function; $n$ is a positive integer and $a$ is a real constant.

15. $f(t) = te^{at}$

16. $f(t) = t \sin at$

17. $f(t) = t \cosh at$

18. $f(t) = t^n e^{at}$

19. $f(t) = t^2 \sin at$

20. $f(t) = t^2 \sinh at$

# Laplace Transform

In each of Problems 21 through 24, find the Laplace transform of the given function.

21. $f(t) = \begin{cases} 1, & 0 \le t < \pi \\ 0, & \pi \le t < \infty \end{cases}$

22. $f(t) = \begin{cases} t, & 0 \le t < 1 \\ 0, & 1 \le t < \infty \end{cases}$

23. $f(t) = \begin{cases} t, & 0 \le t < 1 \\ 1, & 1 \le t < \infty \end{cases}$

24. $f(t) = \begin{cases} t, & 0 \le t < 1 \\ 2 - t, & 1 \le t < 2 \\ 0, & 2 \le t < \infty \end{cases}$

# Laplace Transform

In each of Problems 25 through 28, determine whether the given integral converges or diverges.

25. $\int_0^\infty (t^2+1)^{-1}\,dt$

26. $\int_0^\infty te^{-t}\,dt$

27. $\int_1^\infty t^{-2}e^{t}\,dt$

28. $\int_0^\infty e^{-t}\cos t\,dt$

# Laplace

In each of Problems 1 through 10, find the inverse Laplace transform of the given function.

1. $F(s) = \dfrac{3}{s^2 + 4}$

2. $F(s) = \dfrac{4}{(s-1)^3}$

3. $F(s) = \dfrac{2}{s^2 + 3s - 4}$

4. $F(s) = \dfrac{3s}{s^2 - s - 6}$

5. $F(s) = \dfrac{2s + 2}{s^2 + 2s + 5}$

6. $F(s) = \dfrac{2s - 3}{s^2 - 4}$

7. $F(s) = \dfrac{2s + 1}{s^2 - 2s + 2}$

8. $F(s) = \dfrac{8s^2 - 4s + 12}{s(s^2 + 4)}$

9. $F(s) = \dfrac{1 - 2s}{s^2 + 4s + 5}$

10. $F(s) = \dfrac{2s - 3}{s^2 + 2s + 10}$

# Laplace

In each of Problems 11 through 23, use the Laplace transform to solve the given initial value problem.

11. $y'' - y' - 6y = 0; \quad y(0) = 1, \quad y'(0) = -1$
12. $y'' + 3y' + 2y = 0; \quad y(0) = 1, \quad y'(0) = 0$
13. $y'' - 2y' + 2y = 0; \quad y(0) = 0, \quad y'(0) = 1$
14. $y'' - 4y' + 4y = 0; \quad y(0) = 1, \quad y'(0) = 1$
15. $y'' - 2y' + 4y = 0; \quad y(0) = 2, \quad y'(0) = 0$
16. $y'' + 2y' + 5y = 0; \quad y(0) = 2, \quad y'(0) = -1$
17. $y^{(4)} - 4y''' + 6y'' - 4y' + y = 0; \quad y(0) = 0, \quad y'(0) = 1, \quad y''(0) = 0, \quad y'''(0) = 1$
18. $y^{(4)} - y = 0; \quad y(0) = 1, \quad y'(0) = 0, \quad y''(0) = 1, \quad y'''(0) = 0$
19. $y^{(4)} - 4y = 0; \quad y(0) = 1, \quad y'(0) = 0, \quad y''(0) = -2, \quad y'''(0) = 0$
20. $y'' + \omega^2 y = \cos 2t, \quad \omega^2 \neq 4; \quad y(0) = 1, \quad y'(0) = 0$
21. $y'' - 2y' + 2y = \cos t; \quad y(0) = 1, \quad y'(0) = 0$
22. $y'' - 2y' + 2y = e^{-t}; \quad y(0) = 0, \quad y'(0) = 1$
23. $y'' + 2y' + y = 4e^{-t}; \quad y(0) = 2, \quad y'(0) = -1$

# Laplace Transform

In each of Problems 24 through 27, find the Laplace transform $Y(s) = \mathcal{L}\{y\}$ of the solution of the given initial value problem.

24. $y'' + 4y = \begin{cases} 1, & 0 \le t < \pi, \\ 0, & \pi \le t < \infty; \end{cases} \quad y(0) = 1, \quad y'(0) = 0$

25. $y'' + y = \begin{cases} t, & 0 \le t < 1, \\ 0, & 1 \le t < \infty; \end{cases} \quad y(0) = 0, \quad y'(0) = 0$

26. $y'' + 4y = \begin{cases} t, & 0 \le t < 1, \\ 1, & 1 \le t < \infty; \end{cases} \quad y(0) = 0, \quad y'(0) = 0$

27. $y'' + y = \begin{cases} t, & 0 \le t < 1, \\ 2 - t, & 1 \le t < 2, \\ 0, & 2 \le t < \infty; \end{cases} \quad y(0) = 0, \quad y'(0) = 0$