Section 6-1

Introduction to Normal Distributions

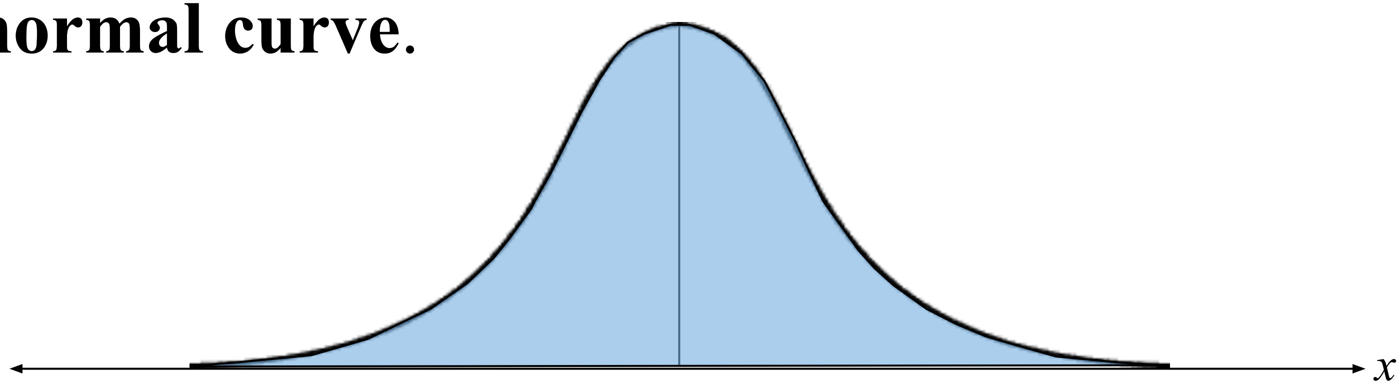
Section 6-1 Objectives

- Interpret graphs of normal probability distributions
- Find areas under the standard normal curve

Properties of Normal Distributions

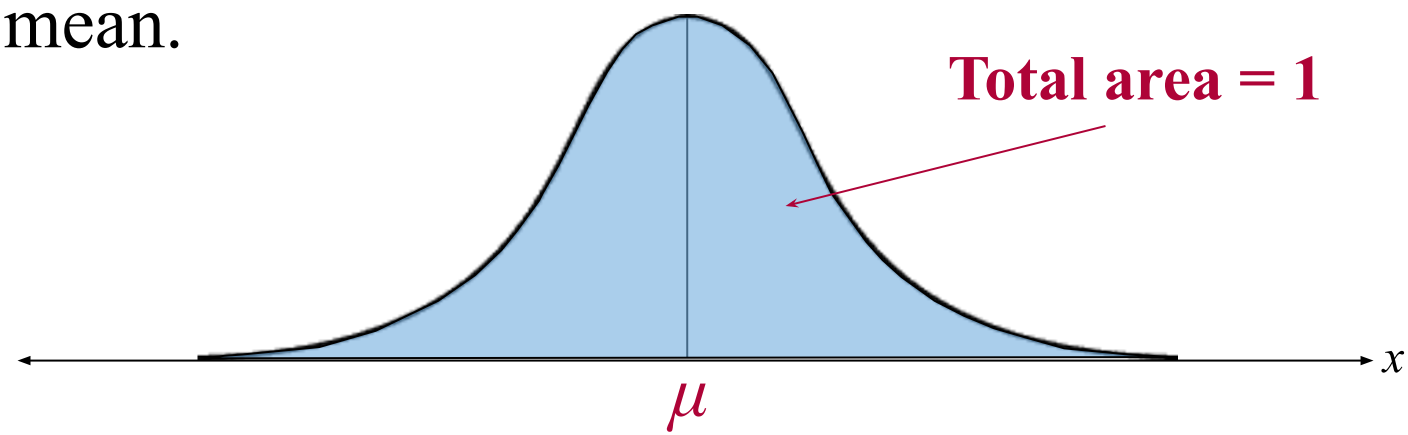
Normal distribution

- A continuous probability distribution for a random variable, x.
- The most important continuous probability distribution in statistics.
- The graph of a normal distribution is called the **normal curve**.



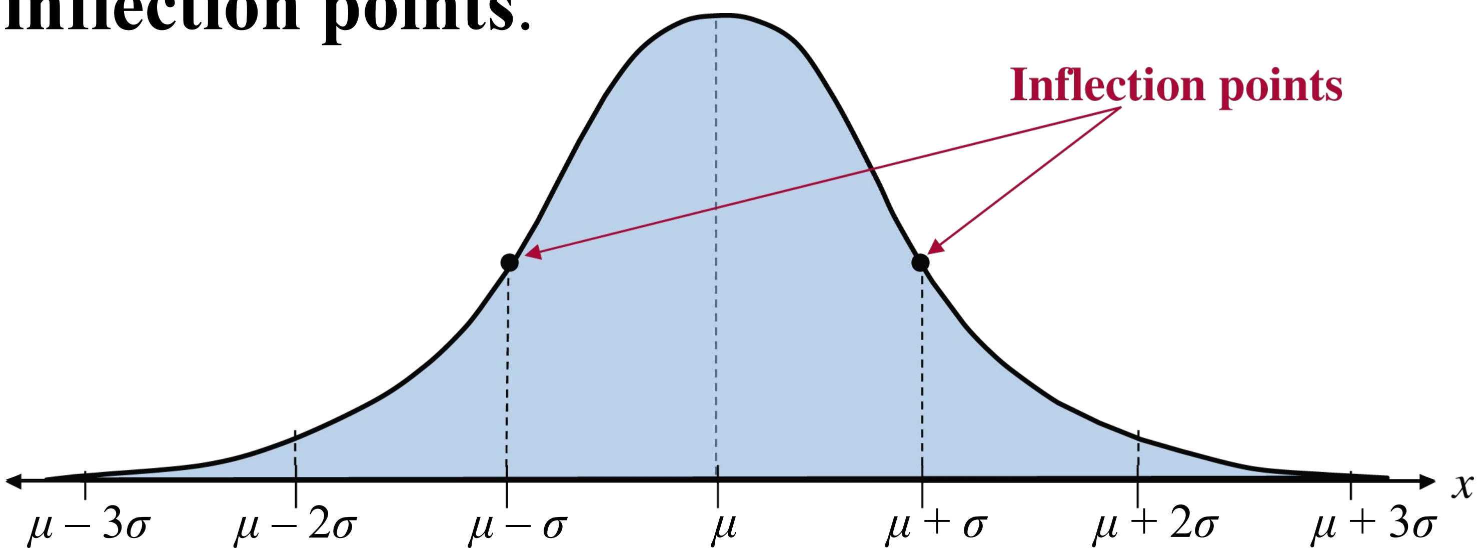
Properties of Normal Distributions

1. The mean, median, and mode are equal.
2. The normal curve is bell-shaped and is symmetric about the mean.
3. The total area under the normal curve is equal to 1.
4. The normal curve approaches, but never touches, the x-axis as it extends farther and farther away from the mean.



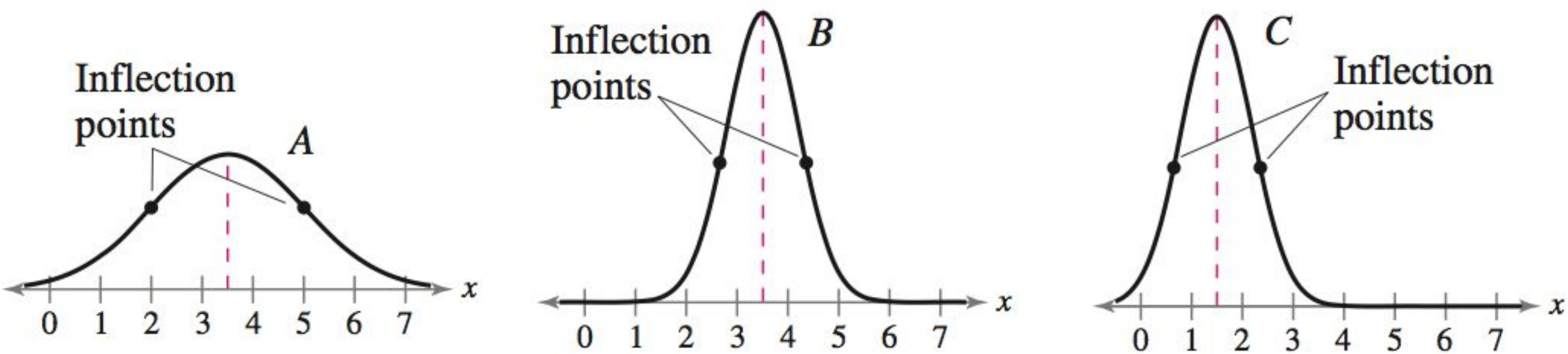
Properties of Normal Distributions

5. Between $\mu - \sigma$ and $\mu + \sigma$ (in the center of the curve), the graph curves downward. The graph curves upward to the left of $\mu - \sigma$ and to the right of $\mu + \sigma$. The points at which the curve changes from curving upward to curving downward are called the **inflection points**.



Means and Standard Deviations

- A normal distribution can have any mean and any positive standard deviation.
- The mean gives the location of the line of symmetry.
- The standard deviation describes the spread of the data.



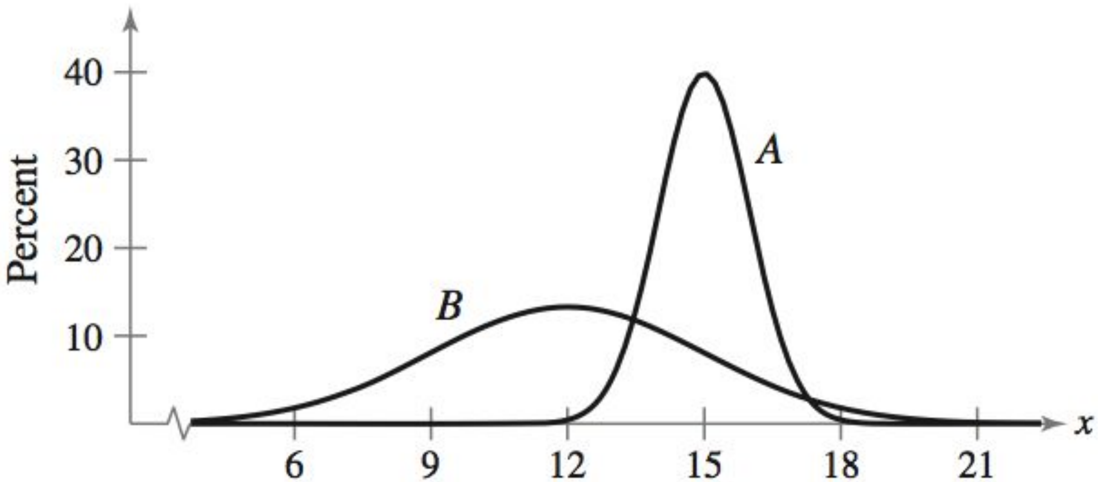
$\mu = 3.5$, $\sigma = 1.5$

$\mu = 3.5$, $\sigma = 0.7$

$\mu = 1.5$, $\sigma = 0.7$

Example: Understanding Mean and Standard Deviation

1. Which normal curve has the greater mean?

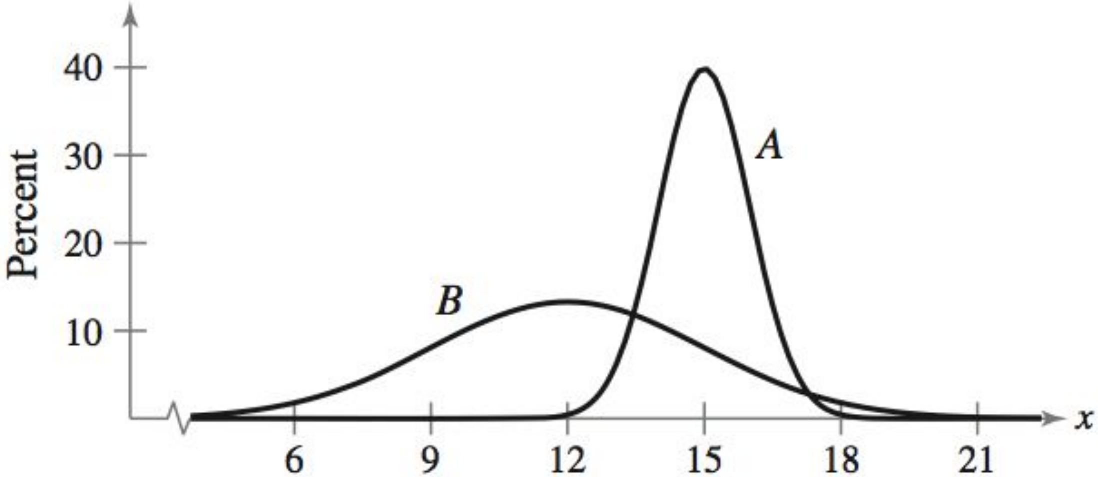


Solution:

Curve *A* has the greater mean (The line of symmetry of curve *A* occurs at $x = 15$. The line of symmetry of curve *B* occurs at $x = 12$.)

Example: Understanding Mean and Standard Deviation

2. Which curve has the greater standard deviation?



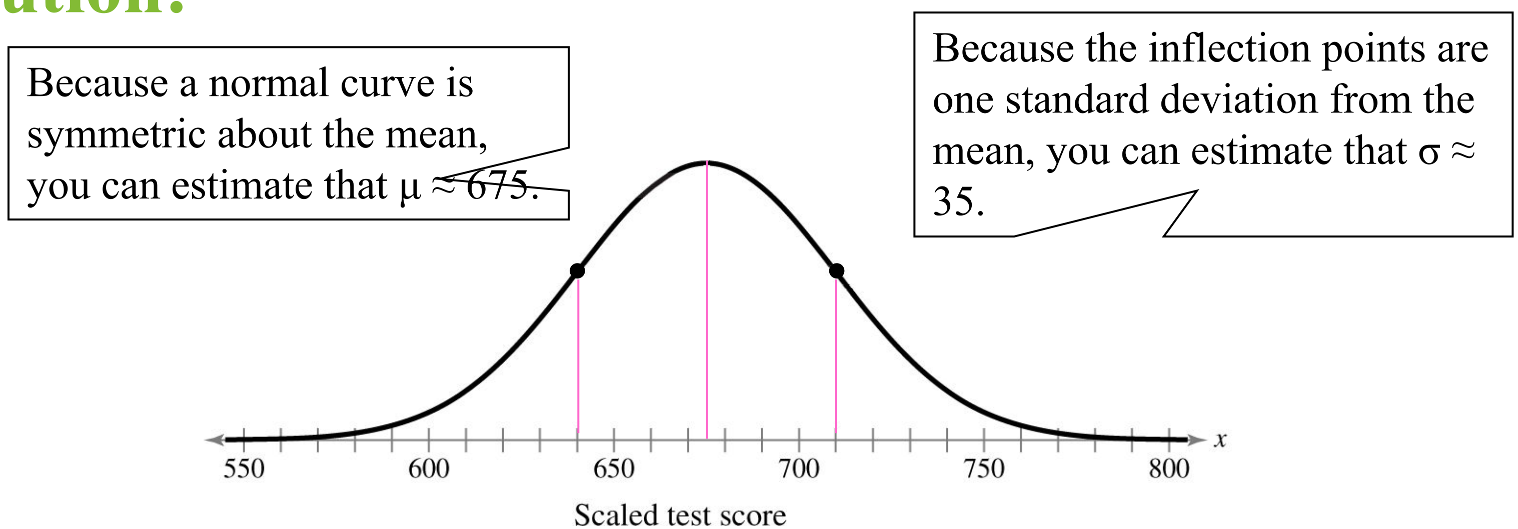
Solution:

Curve *B* has the greater standard deviation (Curve *B* is more spread out than curve *A*.)

Example: Interpreting Graphs

The scaled test scores for the New York State Grade 8 Mathematics Test are normally distributed. The normal curve shown below represents this distribution. What is the mean test score? Estimate the standard deviation.

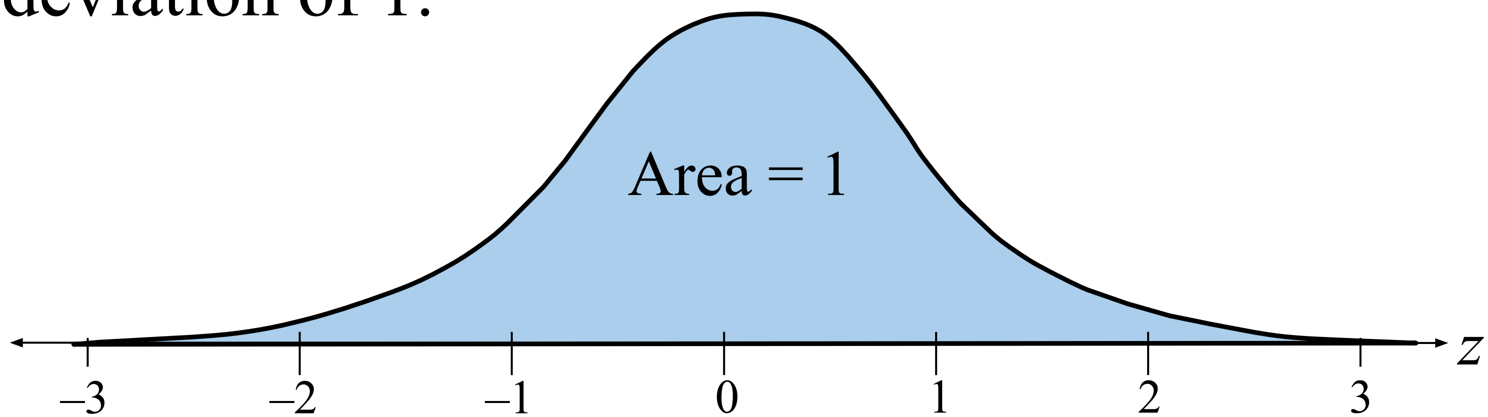
Solution:



The Standard Normal Distribution

Standard normal distribution

- A normal distribution with a mean of 0 and a standard deviation of 1.

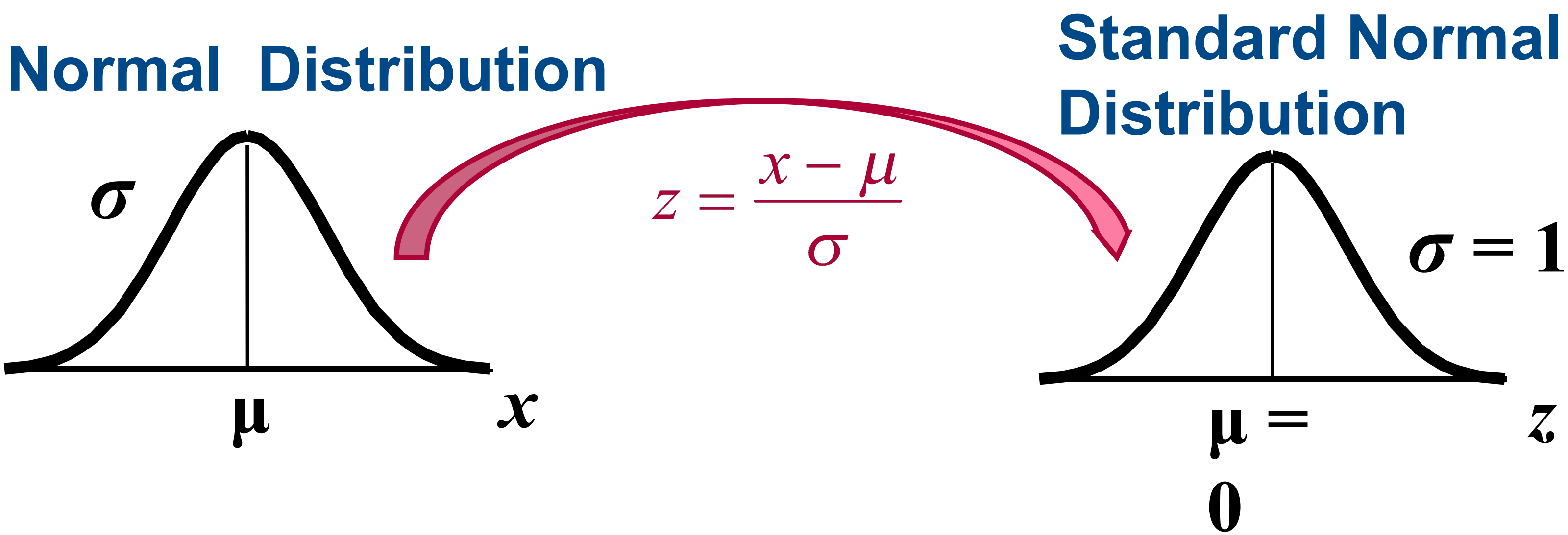


- Any x-value can be transformed into a z-score by using the formula

$$z = \frac{\text{Value} - \text{Mean}}{\text{Standard deviation}} = \frac{x - \mu}{\sigma}$$

The Standard Normal Distribution

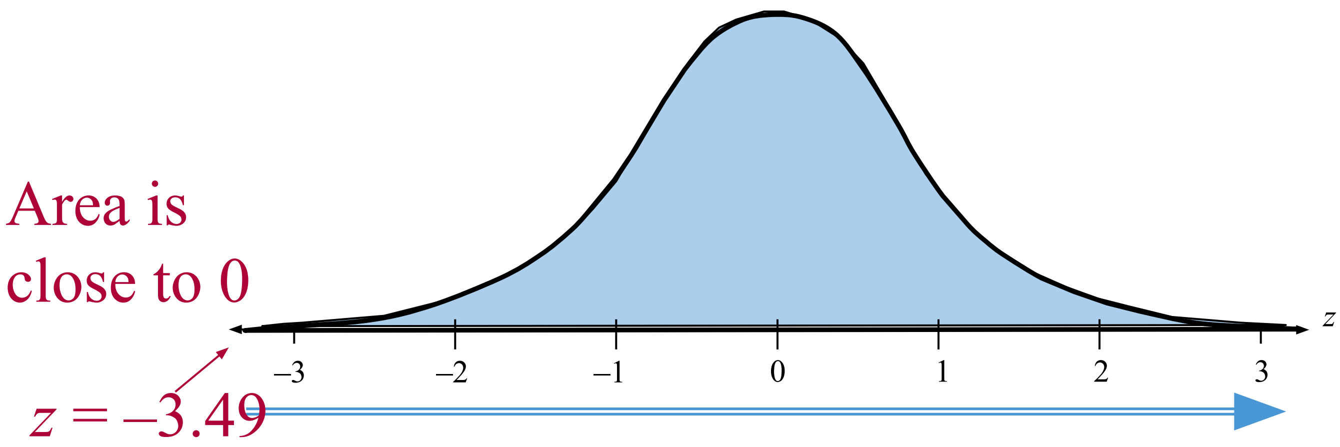
- If each data value of a normally distributed random variable x is transformed into a z-score, the result will be the standard normal distribution.



- Use the Standard Normal Table to find the cumulative area under the standard normal curve.

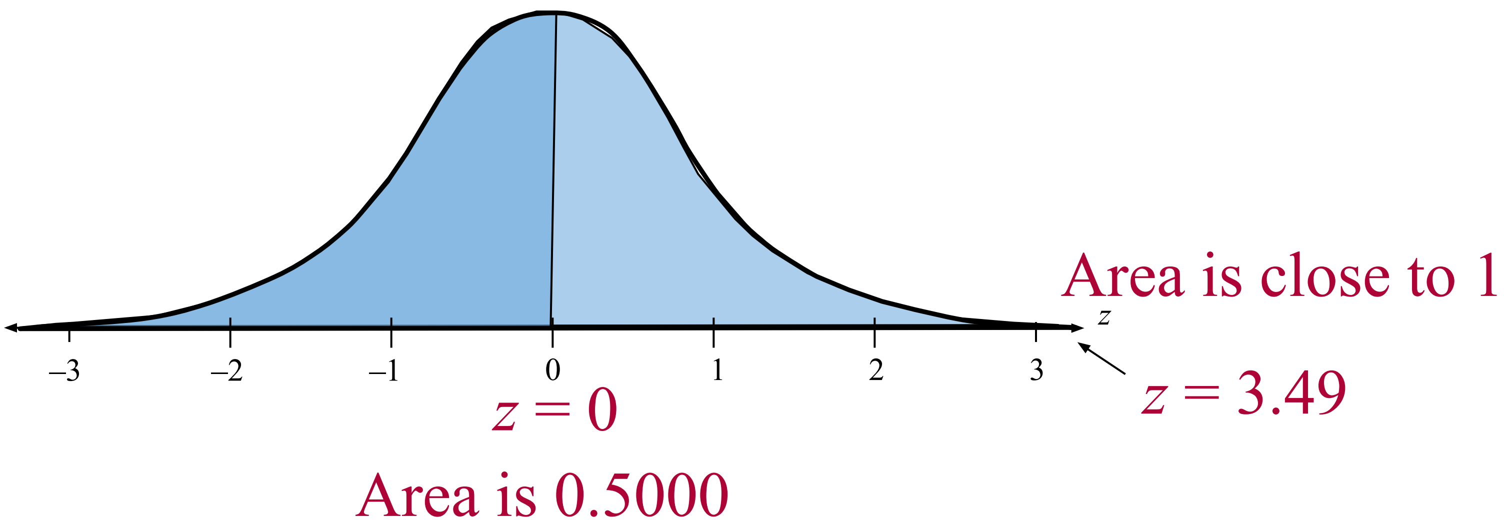
Properties of the Standard Normal Distribution

1. The cumulative area is close to 0 for z-scores close to $z = -3.49$.
2. The cumulative area increases as the z-scores increase.



Properties of the Standard Normal Distribution

3. The cumulative area for $z = 0$ is 0.5000.
4. The cumulative area is close to 1 for z-scores close to $z = 3.49$.



Example: Using The Standard Normal Table

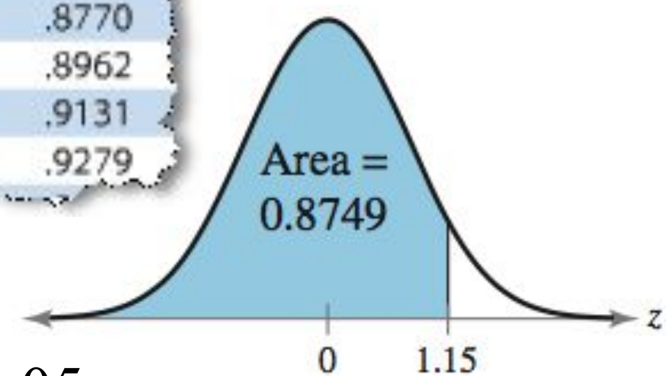
Find the cumulative area that corresponds to a *z*-score of 1.15.

z	.00	.01	.02	.03	.04	.05	.06
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279

Solution:

Find 1.1 in the left hand column.

Move across the row to the column under 0.05

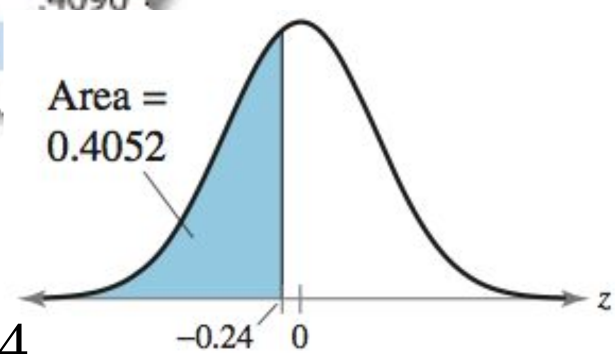


The area to the left of *z* = 1.15 is 0.8749.

Example: Using The Standard Normal Table

Find the cumulative area that corresponds to a *z*-score of –0.24.

z	.09	.08	.07	.06	.05	.04	.03
−3.4	.0002	.0003	.0003	.0003	.0003	.0003	.0003
−3.3	.0003	.0004	.0004	.0004	.0004	.0004	.0004
−3.2	.0005	.0005	.0005	.0006	.0006	.0006	.0006
−0.5	.2776	.2810	.2843	.2877	.2912	.2946	.2981
−0.4	.3121	.3156	.3192	.3228	.3264	.3300	.3336
−0.3	.3483	.3520	.3557	.3594	.3632	.3669	.3707
−0.2	.3859	.3897	.3936	.3974	.4013	.4052	.4090
−0.1	.4247	.4286	.4325	.4364	.4404	.4443	
−0.0	.4641	.4681	.4721	.4761	.4801	.4840	



Solution:

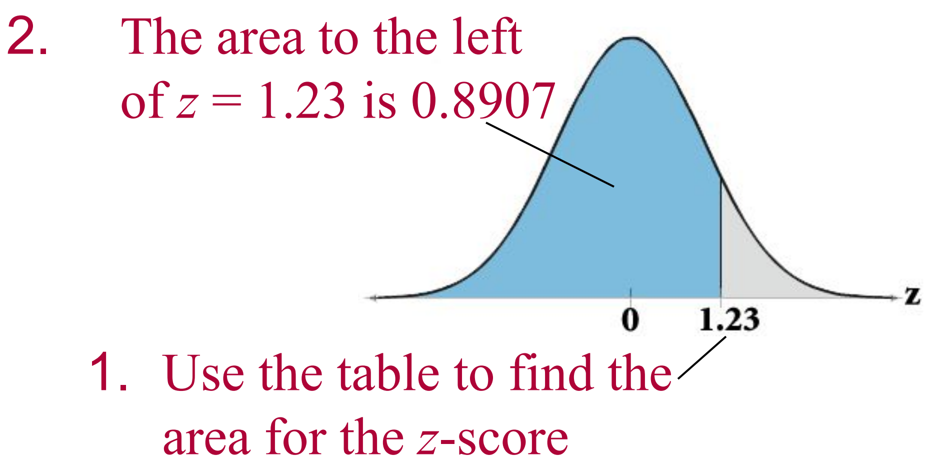
Find –0.2 in the left hand column.

Move across the row to the column under 0.04

The area to the left of $z = –0.24$ is 0.4052.

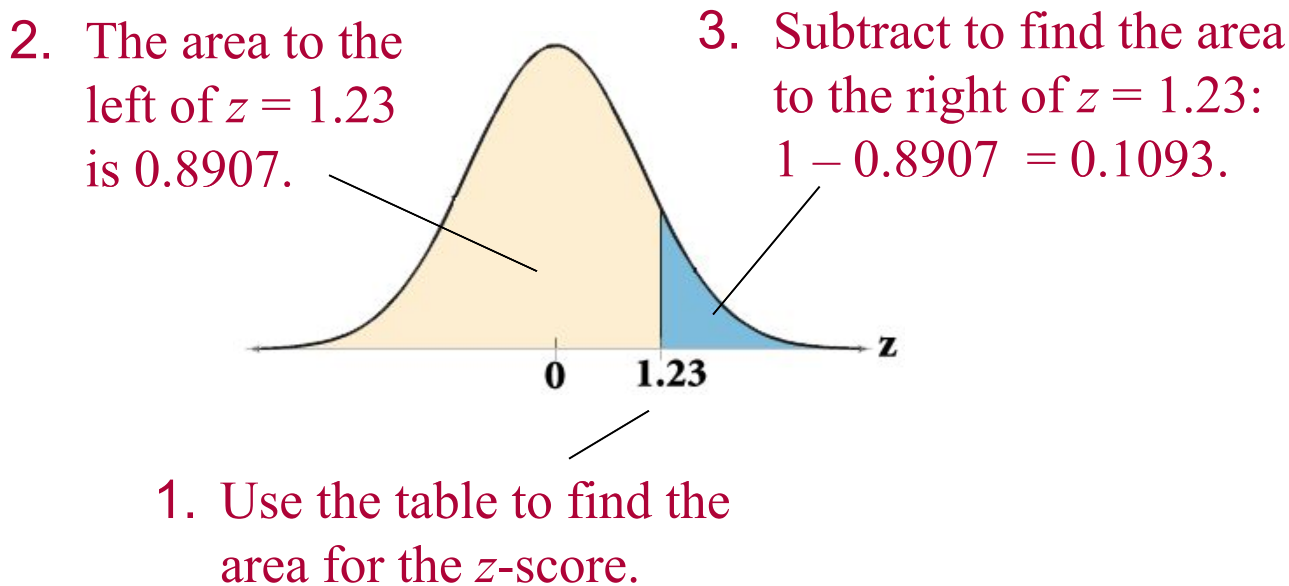
Finding Areas Under the Standard Normal Curve

1. Sketch the standard normal curve and shade the appropriate area under the curve.
2. Find the area by following the directions for each case shown.
   a. To find the area to the *left* of z, find the area that corresponds to z in the Standard Normal Table.



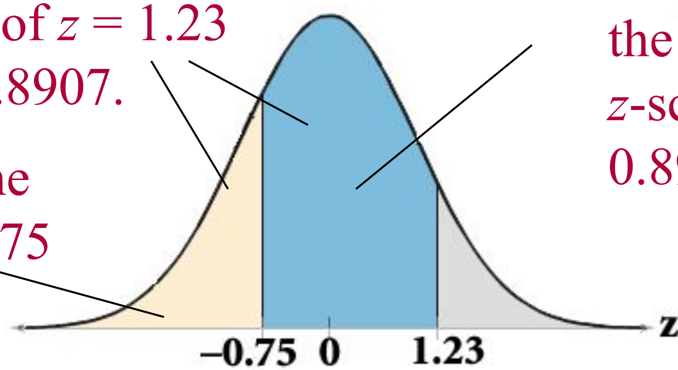
Finding Areas Under the Standard Normal Curve

b. To find the area to the *right* of z, use the Standard Normal Table to find the area that corresponds to z. Then subtract the area from 1.



Finding Areas Under the Standard Normal Curve

c. To find the area *between* two *z*-scores, find the area corresponding to each *z*-score in the Standard Normal Table. Then subtract the smaller area from the larger area.

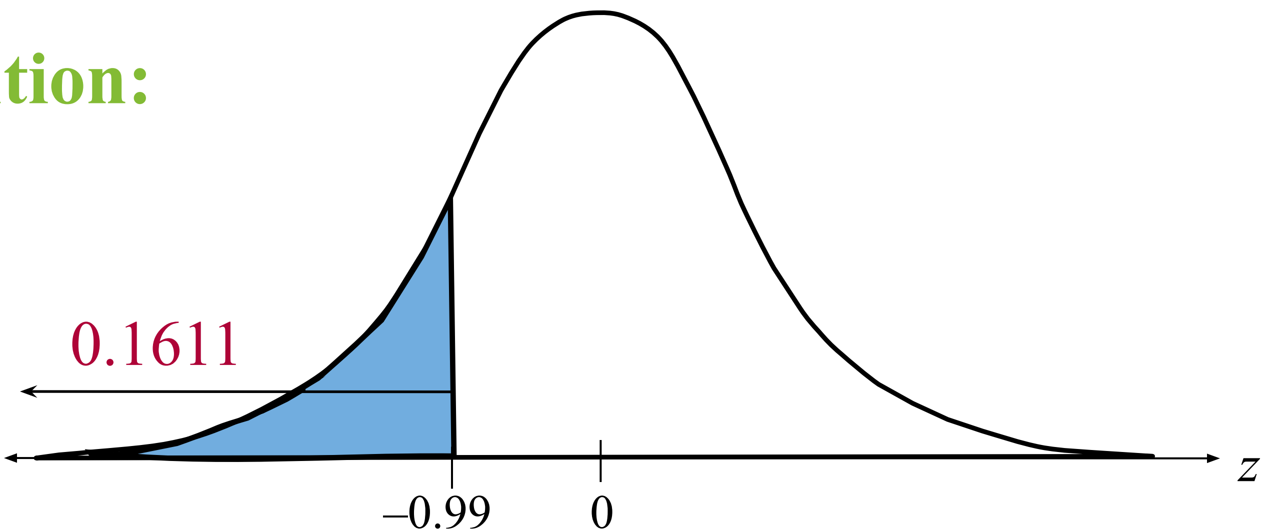


1. Use the table to find the area for the *z*-scores.
2. The area to the left of $z = 1.23$ is 0.8907.
3. The area to the left of $z = –0.75$ is 0.2266.
4. Subtract to find the area of the region between the two *z*-scores: $0.8907 – 0.2266 = 0.6641$.

Example: Finding Area Under the Standard Normal Curve

Find the area under the standard normal curve to the left of $z = -0.99$.

Solution:

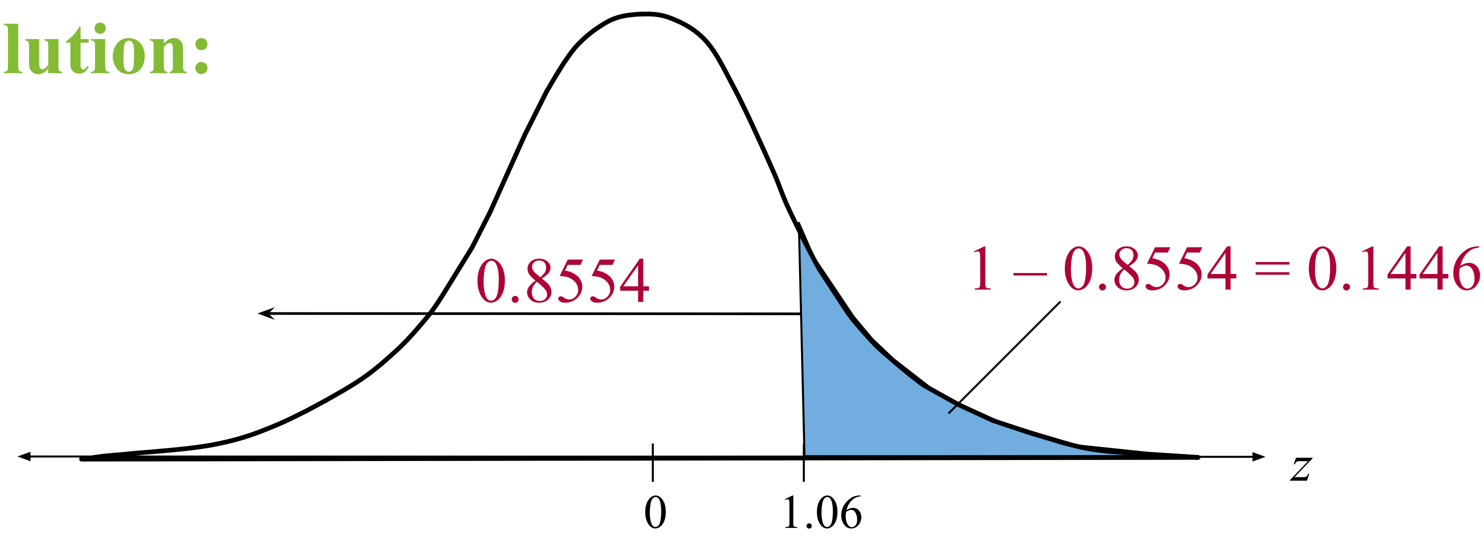


From the Standard Normal Table, the area is equal to 0.1611.

Example: Finding Area Under the Standard Normal Curve

Find the area under the standard normal curve to the right of $z = 1.06$.

Solution:

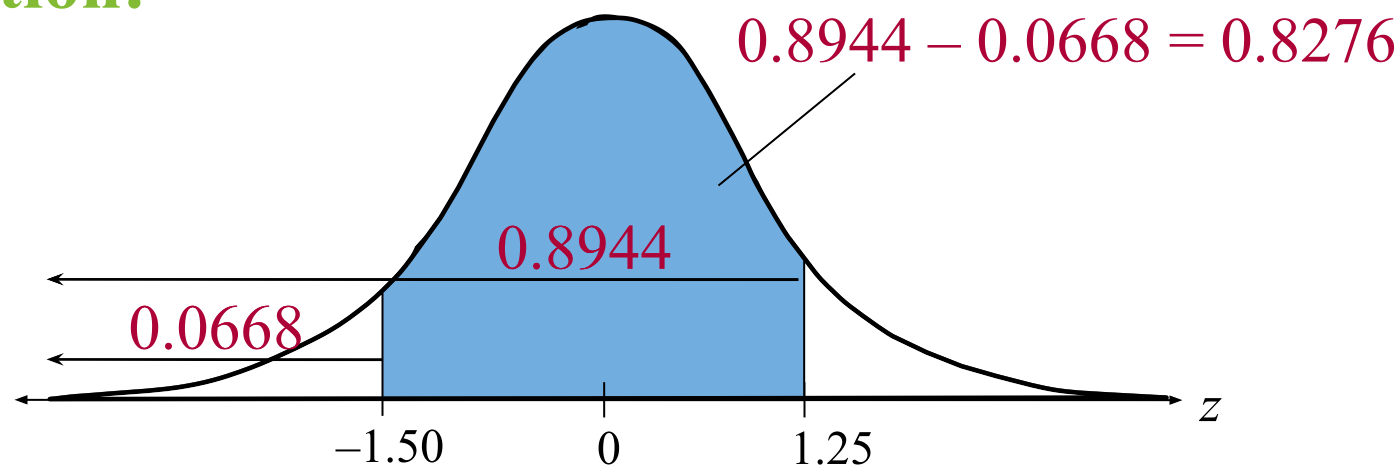


From the Standard Normal Table, the area is equal to 0.1446.

Example: Finding Area Under the Standard Normal Curve

Find the area under the standard normal curve between $z = -1.5$ and $z = 1.25$.

Solution:



From the Standard Normal Table, the area is equal to 0.8276.