IKKINCHI, UCHINCHI TARTIBLI DETERMINANTLAR VA ULARNING XOSSALARI. LAPLAS TEOREMASI. TESKARI MATRISA

Reja:

1. Ikkinchi, uchinchi tartibli determinantlar va ularning xossalari.
2. Laplas teoremasi.
3. Teskari matrisa.

Aytaylik, ikkinchi va uchinchi tartibli kvadrat matrisalar

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}, \quad B = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

berilgan bo‘lsin. Bu matrisalarga mos ravishda

$$a_{11}a_{22} - a_{21}a_{12} \text{ va}$$

$$a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} -$$
$$- a_{13}a_{22}a_{31} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33}$$

sonlarni mos qo‘yamiz. Odatda ular ikkinchi va uchinchi tartibli determinantlar deyiladi va quyidagicha belgilanadi.

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}, \quad \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

Bu ifodalar mos ravishda $\det A$, $\det B$ kabi ham belgilanadi.

Demak,

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}.$$

Masalan,

$$\begin{vmatrix} 2 & -3 \\ 5 & 6 \end{vmatrix} = 2\cdot 6 - 5\cdot(-3) = 12 + 15 = 27.$$

Demak,

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} =$$

$$= a_{11}a_{22}a_{13} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} -$$

$$- a_{13}a_{22}a_{31} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33}.$$

Uchinchi tartibli determinantning qiymati 6 ta had yig‘indisidan iborat bo‘lib, ulardan uchtasi musbat ishorali, qolgan uchtasi esa manfiy ishorali bo‘ladi.

Masalan,

$$\begin{vmatrix} 5 & -2 & 1 \\ 3 & 1 & -4 \\ 6 & 0 & -3 \end{vmatrix} =$$

$$= 5 \cdot 1 \cdot (-3) + (-2) \cdot (-4) \cdot 6 + 3 \cdot 0 \cdot 1 -$$

$$-6 \cdot 1 \cdot 1 - 3 \cdot (-2) \cdot (-3) - 0 \cdot (-4) \cdot 5 =$$

$$= -15 + 48 - 6 - 18 = 48 - 39 = 9.$$

Eslatma. Yuqoridagidek, $n-$tartibli $(n > 3)$ determinant

$$\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}$$

tushunchasi kiritiladi.

Aytaylik, uchinchi tartibli determinant

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \qquad (2)$$

berilgan bo‘lsin. Bu determinantning biror

$$a_{ij} \quad (i = 1, 2, 3; \quad j = 1, 2, 3)$$

elementini olib, shu element joylashgan yo‘lni hamda ustunni o‘chiramiz. Qolgan elementlari ikkinchi tartibli determinantni hosil qiladi. Uni a_{ij} element minori deyiladi va u M_{ij} kabi belgilanadi.

Masalan,

$$\begin{vmatrix} -2 & 5 & 3 \\ 1 & 2 & 0 \\ 3 & 0 & 1 \end{vmatrix}$$

determinantning $a_{12} = 5$ elementning minori

$$M_{12} = \begin{vmatrix} 1 & 0 \\ 3 & 1 \end{vmatrix}$$

bo‘ladi.

Uchinchi tartibli determinant 9 ta minorga ega bo‘ladi.

Ushbu

$$(-1)^{i+j} \cdot M_{ij}$$

miqdor (2) determinant a_{ij} elementining algebraik to‘ldiruvchisi deyiladi va A_{ij} orqali belgilanadi:

$$A_{ij} = (-1)^{i+j} \cdot M_{ij}.$$

Masalan,

$$\begin{vmatrix} 2 & 0 & 3 \\ 1 & 2 & 0 \\ 3 & 0 & 1 \end{vmatrix}$$

determinant $a_{13} = 3$ elementining algebraik to‘ldiruvchisi

$$A_{13} = (-1)^{1+3} \begin{vmatrix} 1 & 2 \\ 3 & 0 \end{vmatrix} = 1 \cdot (1 \cdot 0 - 2 \cdot 3) = -6$$

bo‘ladi.

Determinantning xossalari

Determinant qator xossalarga ega. Quyida ularni keltiramiz.

1) Determinantning yo‘llarini mos ustunlari bilan almashtirilsa determinantning qiymati o‘zgarmaydi:

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{vmatrix},$$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{vmatrix}.$$

2) Determinantning ixtiyoriy ikki yo‘lini (ikki ustunini) o‘zaro almashtirilsa, determinantning qiymati o‘zgarmasdan, uni ishorasi esa qarama-qarshisiga o‘zgaradi.

3) Determinantning ikki yo‘li (ustuni) bir xil bo‘lsa, determinantning qiymati nolga teng bo‘ladi.

4) Determinantning ixtiyoriy yo‘lida (ustunida) turgan barcha elementlari o‘zgarmas songa ko‘paytirilsa, determinantning qiymati ham k soniga ko‘payadi.

5) Determinantning biror yo‘li (ustuni)da turgan barcha elementlarning ularga mos algebraik to‘ldiruvchilari bilan ko‘paytmasidan tashkil topgan yig‘indi shu determinantning qiymatiga teng.

Ikkinchi va uchinchi tartibli determinantlar bevosita ta’rifga ko‘ra hisoblanadi.

Masalan

$$\begin{vmatrix} 1 & 2 \\ -3 & 4 \end{vmatrix} = 1\cdot 4 - 2\cdot(-3) = 10,$$

$$\begin{vmatrix} 1 & 2 & 3 \\ 0 & -1 & 4 \\ 2 & 3 & 0 \end{vmatrix} =$$

$$= 1\cdot(-1)\cdot 0 + 2\cdot 4\cdot 2 + 3\cdot 0\cdot 3 - 3\cdot(-1)\cdot 2$$

$$- 2\cdot 0\cdot 0 - 1\cdot 4\cdot 3 = 10.$$

Uchinchi tartibli determinantlarni hisoblashda shuningdek ushbu

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} =$$

$$= a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} +$$

$$+ a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

munosabatdan foydalanish mumkin.

Yuqori tartibli determinantlarni hisoblash birmuncha murakkab bo'ladi. Ularni hisoblashda yuqorida keltirilgan xossalardan foydalaniladi.

Misol. Ushbu

$$\begin{vmatrix} 3 & 5 & 7 & 8 \\ 1 & 7 & 0 & 1 \\ 0 & 5 & 3 & 2 \\ 1 & -1 & 7 & 4 \end{vmatrix}$$

determinantni hisoblang.

Bu determinantni hisoblashda yuqorida keltirilgan 5) xossadan foydalanamiz.

$$\begin{vmatrix} 3 & 5 & 7 & 8 \\ 1 & 7 & 0 & 1 \\ 0 & 5 & 3 & 2 \\ 1 & -1 & 7 & 4 \end{vmatrix} =$$

$$= 3 \cdot \begin{vmatrix} 7 & 0 & 1 \\ 5 & 3 & 2 \\ -1 & 7 & 4 \end{vmatrix} - 1 \cdot \begin{vmatrix} 5 & 7 & 8 \\ 5 & 3 & 2 \\ -1 & 7 & 4 \end{vmatrix} +$$

$$+ 0 \cdot \begin{vmatrix} 5 & 7 & 8 \\ 7 & 0 & 1 \\ -1 & 7 & 4 \end{vmatrix} - 1 \cdot \begin{vmatrix} 5 & 7 & 8 \\ 7 & 0 & 1 \\ 5 & 3 & 2 \end{vmatrix} =$$

$$= 3(7 \cdot 3 \cdot 4 + (-1) \cdot 0 \cdot 2 + 5 \cdot 7 \cdot 1 -$$

$$- 1(5 \cdot 3 \cdot 4 - 7 \cdot 2 \cdot (-1) + 5 \cdot 7 \cdot 8) +$$

$$+ 0(5 \cdot 0 \cdot 4 + 7 \cdot 1 \cdot (-1) + 7 \cdot 7 \cdot 8) +$$

$$+ (5 \cdot 0 \cdot 2 + 7 \cdot 1 \cdot 5 + 7 \cdot 3 \cdot 8) = 3 \cdot 119 - 326 - 203 =$$

$$= 357 - 529 = 172.$$

A matrisa

$$a_{ij} \quad (i = 1,2,\ldots,n, \quad j = 1,2\ldots,n)$$

elementining algebraik to‘ldiruvchisini (u ham determinant elementining algebraik to‘ldiruvchisi kabi ta’riflanadi)

$$A_{ij} \quad (i = 1,2,\ldots,n, \quad j = 1,2,\ldots,n)$$

deb belgilaymiz. Bu A_{ij} lardan tuzilgan matrisani A^* orqali belgilaymiz:

$$A^* = \begin{pmatrix} A_{11} & A_{21} & \cdots & A_{n1} \\ A_{12} & A_{22} & \cdots & A_{n2} \\ \cdots & \cdots & \cdots & \cdots \\ A_{1n} & A_{2n} & \cdots & A_{nn} \end{pmatrix}.$$

Berilgan A matrisa bilan birga ushbu

$$E = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 1 \end{pmatrix}$$

birlik matrisani qaraymiz.

Agar

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix}, \quad B = \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ b_{n1} & b_{n2} & \cdots & b_{nn} \end{pmatrix},$$

matrisalar uchun

$$A \cdot B = B \cdot A = E$$

bo‘lsa, B matrisa A matrisaga teskari matrisa deyiladi va u A^{-1} kabi belgilanadi.

Masalan, ushbu

$$A = \begin{pmatrix} 1 & -2 & 1 \\ 2 & 0 & 1 \\ -2 & 1 & 1 \end{pmatrix}$$

matrisaga teskari matrisa

$$A^{-1} = \begin{pmatrix} \frac{1}{3} & 1 & \frac{2}{3} \\ 0 & 1 & 1 \\ \frac{2}{3} & 1 & \frac{4}{3} \end{pmatrix}$$

bo‘ladi, chunki

$$A \cdot A^{-1} = \begin{pmatrix} 1 & -2 & 1 \\ 2 & 0 & 1 \\ -2 & 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} \frac{1}{3} & 1 & \frac{2}{3} \\ 0 & 1 & 1 \\ \frac{2}{3} & 1 & \frac{4}{3} \end{pmatrix} =$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Teskari matrisa quyidagi xossalarga ega:

1) $\det(A^{-1}) = \frac{1}{\det A}$.

2) $(A \cdot B)^{-1} = B^{-1} \cdot A^{-1}$,

3) $(A^{-1})^{T} = (A^{T})^{-1}$

Misol. Agar

$$A = \begin{pmatrix} 2 & 3 \\ -1 & 1 \end{pmatrix}$$

bo‘lsa, uning teskarisi A^{-1} topilsin.

Bu misolni quyidagicha yechamiz:

1) A matrisaning determinantini topamiz:

$$\det A = \begin{vmatrix} 2 & 3 \\ -1 & 1 \end{vmatrix} = 2 + 3 = 5 \neq 0.$$

2) A^* matrisani topamiz:

$$A_{11} = 1, \quad A_{21} = -3, \quad A_{12} = -(-1) = 1, \quad A_{22} = 2.$$

Demak,

$$A^* = \begin{pmatrix} 1 & -3 \\ 1 & 2 \end{pmatrix}.$$

3) A matrisaning teskari matrisasi A^{-1} ni topamiz:

$$A^{-1} = \frac{1}{5}\begin{pmatrix} 1 & -3 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} \frac{1}{5} & -\frac{3}{5} \\ \frac{1}{5} & \frac{2}{5} \end{pmatrix}.$$

E’TIBORLARINGIZ UCHUN RAHMAT!!!