

Понятие бесконечно малой и бесконечно большой величины

$$\lim_{0} \frac{1}{0} = \infty$$

$$\lim_{\infty} \frac{1}{\infty} = 0$$

Примеры:

$$1. \lim_{x \rightarrow \sqrt{3}} \frac{x^2 - 3}{x^4 + x^2 + 1} = \frac{3 - 3}{9 + 3 + 1} = \frac{0}{13} = 0$$

$$2. \lim_{x \rightarrow -1} (x^3 - x^2 + 1) = -1 - 1 + 1 = -1$$

$$3. \lim_{x \rightarrow 0} (3x^3 + x^2 - 8x + 10) = 0 + 0 - 0 + 10 = 10$$

Раскрытие неопределенности

вида $\frac{0}{0}$

Например:

1.

$$\lim_{x \rightarrow -\frac{3}{2}} \frac{4x^2 - 9}{2x + 3} = \lim_{x \rightarrow -\frac{3}{2}} \frac{(2x - 3)(2x + 3)}{(2x + 3)} =$$
$$\lim_{x \rightarrow -\frac{3}{2}} 3(2x - 3) = -3 - 3 = -6$$

2.

$$\lim_{x \rightarrow 5} \frac{x^2 - 7x + 10}{x^2 - 9x + 20} = \lim_{x \rightarrow 5} \frac{(x - 5)(x - 2)}{(x - 5)(x - 4)} = \frac{3}{1} = 3$$

$$\begin{aligned}
3. \quad \lim_{x \rightarrow \frac{\pi}{4}} \frac{\sin x - \cos x}{\cos 2x} &= \lim_{x \rightarrow \frac{\pi}{4}} \frac{\sin x - \cos x}{\cos^2 x - \sin^2 x} = \\
&= -\lim_{x \rightarrow \frac{\pi}{4}} \frac{(\cos x - \sin x)}{(\cos x - \sin x)(\cos x + \sin x)} = -\frac{1}{\cos \frac{\pi}{4} + \sin \frac{\pi}{4}} = -\frac{1}{\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}} = -\frac{1}{\sqrt{2}} = \\
&= -\frac{\sqrt{2}}{2}
\end{aligned}$$

4.

$$\begin{aligned}
\lim_{x \rightarrow 0} \frac{x}{\sqrt{1+3x}-1} &= \lim_{x \rightarrow 0} \frac{x(\sqrt{1+3x}+1)}{(\sqrt{1+3x}-1)(\sqrt{1+3x}+1)} = \\
&= \lim_{x \rightarrow 0} \frac{x(\sqrt{1+3x}+1)}{1+3x-1} = \lim_{x \rightarrow 0} \frac{x(\sqrt{1+3x}+1)}{3x} = \frac{2}{3}
\end{aligned}$$

Раскрытие неопределенности

вида $\frac{\infty}{\infty}$

$$1. \lim_{x \rightarrow \infty} \frac{x^4 - x^2 + 2}{x^3 - x + 1} = \lim_{x \rightarrow \infty} \frac{1 - \frac{1}{x^2} + \frac{2}{x^4}}{\frac{1}{x} - \frac{1}{x^3} + \frac{1}{x^4}} = \frac{1}{0} = \infty$$

$$2. \lim_{x \rightarrow \infty} \frac{\frac{x^3}{x^6} + \frac{x^4}{x^6}}{\frac{x^5}{x^6} + \frac{x^6}{x^6}} = \lim_{x \rightarrow \infty} \frac{\frac{x^3}{x^6} + \frac{x^4}{x^6}}{\frac{x^5}{x^6} + \frac{x^6}{x^6}} = \lim_{x \rightarrow \infty} \frac{\frac{1}{x^3} + \frac{1}{x^2}}{\frac{1}{x} + 1} = \frac{0}{1} = 0$$

$$3. \lim_{x \rightarrow \infty} \frac{10x^2 - x - 6}{3x - x^2} = \lim_{x \rightarrow \infty} \frac{\frac{10-1}{x} - \frac{6}{x^2}}{\frac{3}{x} - 1} = -10$$

Первый замечательный предел

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

Например:

$$1. \quad \lim_{x \rightarrow 0} \frac{\sin x}{3x} = \frac{1}{3} \lim_{x \rightarrow 0} \frac{\sin x}{x} = \frac{1}{3} \cdot 1 = \frac{1}{3}$$

$$2. \quad \lim_{x \rightarrow 0} \frac{\sin 5x}{x} = 5 \cdot \lim_{x \rightarrow 0} \frac{\sin 5x}{5x} = 5$$

$$\begin{aligned}
 3. \quad \lim_{x \rightarrow 0} \frac{\operatorname{tg} 3x}{x} &= \lim_{x \rightarrow 0} \left(\frac{\sin 3x}{x} \cdot \frac{1}{\cos 3x} \right) \\
 &= \lim_{x \rightarrow 0} \frac{3 \sin 3x}{3x} \cdot \lim_{x \rightarrow 0} \frac{1}{\cos 3x} \\
 &= 3 \cdot 1 \cdot 1 = 3
 \end{aligned}$$

$$4. \quad \lim_{x \rightarrow 0} \frac{\sin 3x}{\sin 5x} = \lim_{x \rightarrow 0} \frac{\frac{\sin 3x}{x}}{\frac{\sin 5x}{x}} = \frac{\lim_{x \rightarrow 0} \frac{\sin 3x}{x}}{\lim_{x \rightarrow 0} \frac{\sin 5x}{x}} = \frac{3}{5}$$

Второй замечательный предел

$$\lim_{x \rightarrow 0} (1 + x)^{\frac{1}{x}} = e$$

$$\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = e$$

Например:

$$1. \quad \lim_{x \rightarrow \infty} \left(1 + \frac{3}{x}\right)^x = \left(\lim_{x \rightarrow \infty} \left(1 + \frac{3}{x}\right)^{\frac{x}{3}} \right)^3 = e^3$$

$$2. \quad \lim_{x \rightarrow \infty} \left(1 + \frac{1}{5x}\right)^x = \left(\lim_{x \rightarrow \infty} \left(1 + \frac{1}{5x}\right)^{5x} \right)^{\frac{1}{5}} = e^{\frac{1}{5}}$$

$$3. \lim_{x \rightarrow 0} (1 + 4x)^{\frac{3}{5x}} = \left(\lim_{x \rightarrow 0} (1 + 4x)^{\frac{1}{4x}} \right)^{\frac{3 \cdot 4}{5}} = e^{\frac{12}{5}}$$

$$4. \lim_{x \rightarrow \infty} \left(\frac{1+x}{x} \right)^{\frac{x}{2}} = \left(\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x} \right)^x \right)^{\frac{1}{2}} = e^{\frac{1}{2}}$$