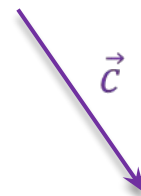
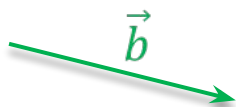
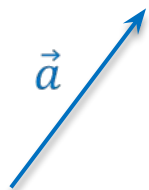
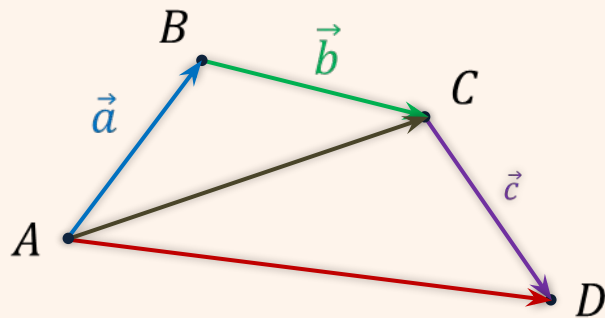


Сумма нескольких векторов

Сложение и вычитание векторов



Правило многоугольника



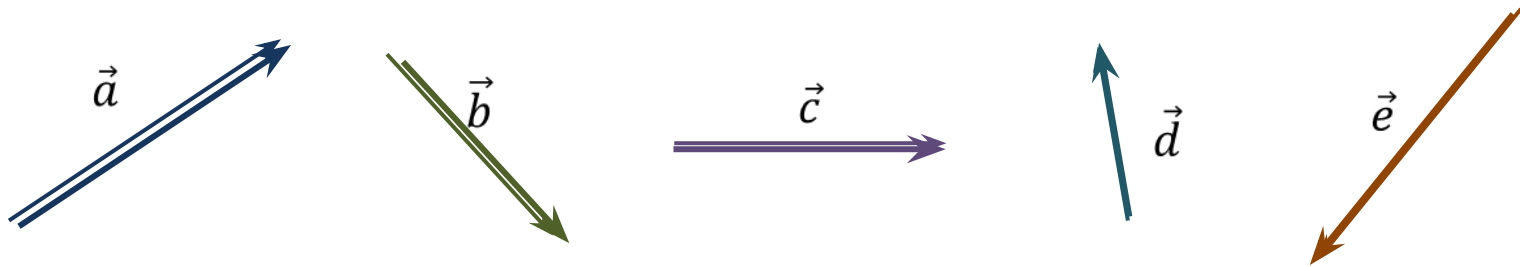
$$1. \overrightarrow{AB} = \vec{a}$$

$$2. \overrightarrow{BC} = \vec{b}$$

$$3. \overrightarrow{CD} = \vec{c}$$

$$4. \overrightarrow{AD} = \vec{a} + \vec{b} + \vec{c}$$

Задача. Построить вектор суммы попарно неколлинеарных векторов $\vec{a}$, $\vec{b}$, $\vec{c}$, $\vec{d}$ и $\vec{e}$.



Построение.

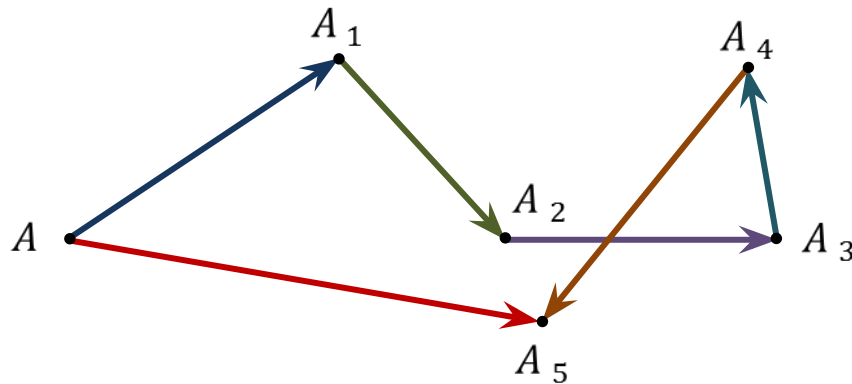
$$\overrightarrow{AA_1} = \vec{a}$$

$$\overrightarrow{A_1A_2} = \vec{b}$$

$$\overrightarrow{A_2A_3} = \vec{c}$$

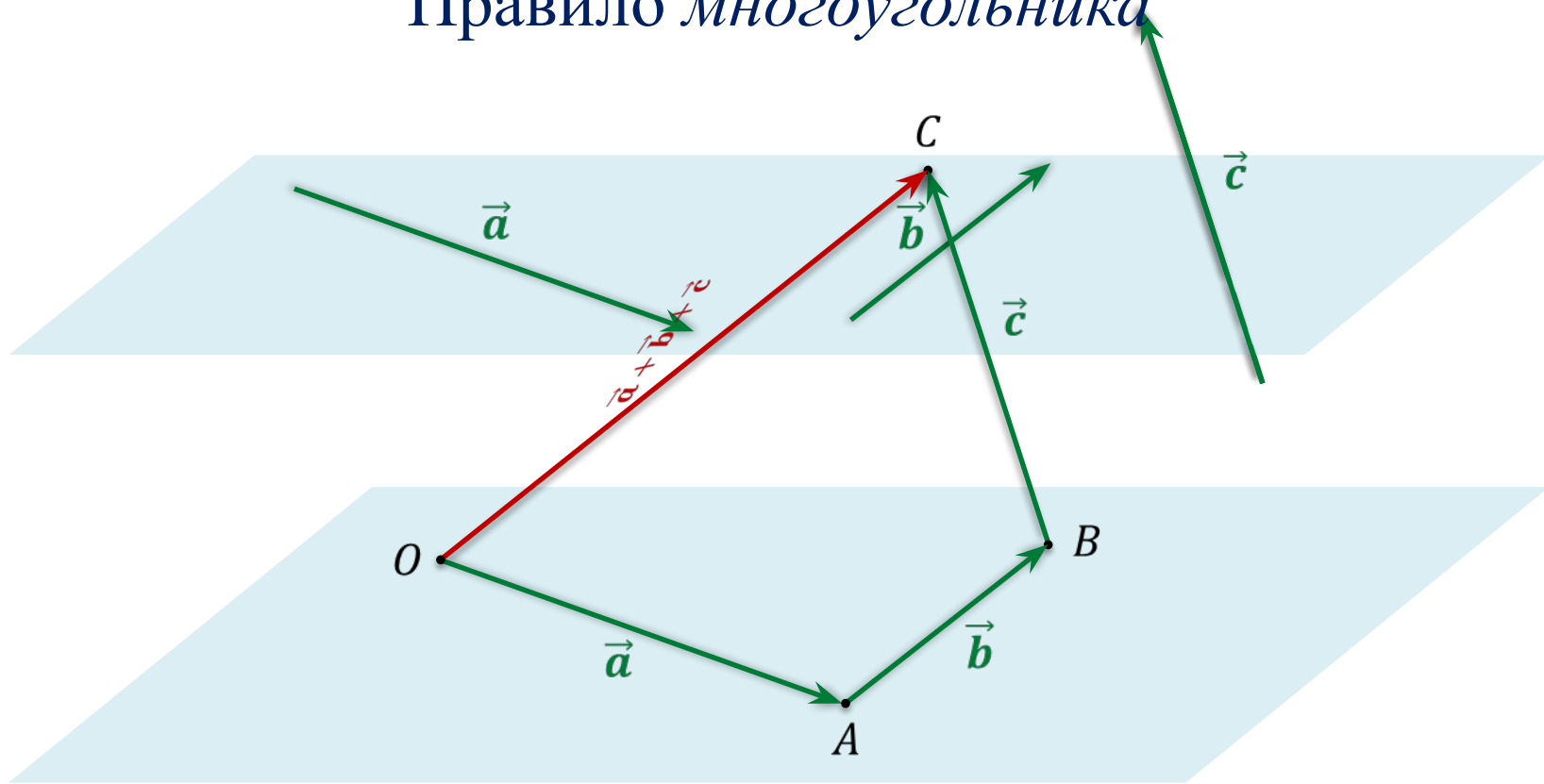
$$\overrightarrow{A_3A_4} = \vec{d}$$

$$\overrightarrow{A_4A_5} = \vec{e}$$

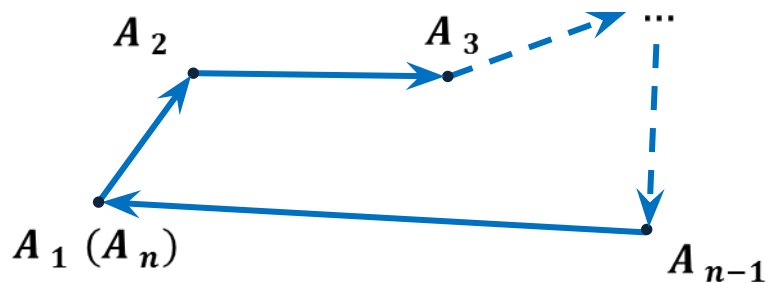
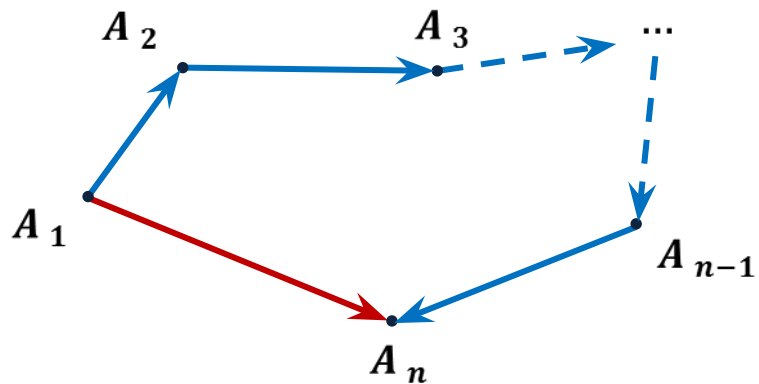


$$\overrightarrow{AA_1} + \overrightarrow{A_1A_2} + \overrightarrow{A_2A_3} + \overrightarrow{A_3A_4} + \overrightarrow{A_4A_5} = \overrightarrow{AA_5} \Rightarrow \vec{a} + \vec{b} + \vec{c} + \vec{d} + \vec{e} = \overrightarrow{AA_5}$$

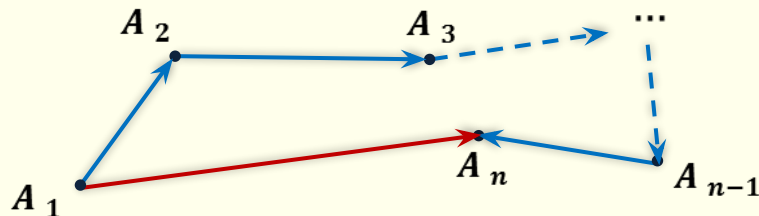
Правило многоугольника



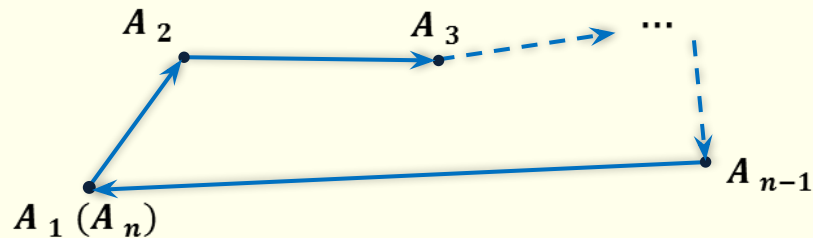
Правило многоугольника



$$\overrightarrow{A_1 A_2} + \overrightarrow{A_2 A_3} + \cdots + \overrightarrow{A_{n-1} A_n} = \vec{0}$$



$$\overrightarrow{A_1A_2} + \overrightarrow{A_2A_3} + \dots + \overrightarrow{A_{n-1}A_n} = \overrightarrow{A_1A_n}$$



$$\overrightarrow{A_1A_2} + \overrightarrow{A_2A_3} + \dots + \overrightarrow{A_{n-1}A_n} = \vec{0}$$

Задача. Упростить выражения.

а) $\overrightarrow{AB} + \overrightarrow{BA} + \overrightarrow{CD} + \overrightarrow{MN} + \overrightarrow{DC} + \overrightarrow{NM} = \overrightarrow{AB} + \overrightarrow{BA} + \overrightarrow{CD} + \overrightarrow{DC} + \overrightarrow{MN} + \overrightarrow{NM} = \vec{0} + \vec{0} + \vec{0} = \vec{0}$

б) $\overrightarrow{KM} + \overrightarrow{DF} + \overrightarrow{AC} + \overrightarrow{FK} + \overrightarrow{CD} + \overrightarrow{CA} + \overrightarrow{MP} = \overrightarrow{CD} + \overrightarrow{DF} + \overrightarrow{FK} + \overrightarrow{KM} + \overrightarrow{MP} = \overrightarrow{CP}$

в) $\overrightarrow{OP} - \overrightarrow{EP} + \overrightarrow{KD} - \overrightarrow{KA} = \overrightarrow{OP} + (-\overrightarrow{EP}) + \overrightarrow{KD} + (-\overrightarrow{KA}) = \overrightarrow{OP} + \overrightarrow{PE} + \overrightarrow{AK} + \overrightarrow{KD} = \overrightarrow{OE} + \overrightarrow{AD}$

г) $\overrightarrow{AC} - \overrightarrow{BC} - \overrightarrow{PM} - \overrightarrow{AP} + \overrightarrow{BM} = \overrightarrow{AC} + \overrightarrow{CB} + \overrightarrow{MP} + \overrightarrow{PA} + \overrightarrow{BM} = \overrightarrow{AC} + \overrightarrow{CB} + \overrightarrow{BM} + \overrightarrow{MP} + \overrightarrow{PA} = \vec{0}$

Задача. A, B, C, D — произвольные точки пространства.

Представить вектор $\overrightarrow{AB}$ в виде алгебраической суммы векторов:

а) $\overrightarrow{AC}, \overrightarrow{DC}, \overrightarrow{BD}$

б) $\overrightarrow{DA}, \overrightarrow{DC}, \overrightarrow{CB}$

в) $\overrightarrow{DA}, \overrightarrow{CD}, \overrightarrow{BC}$

Решение.

а) $\overrightarrow{AC}, \overrightarrow{DC}, \overrightarrow{BD}$

$$\overrightarrow{AB} = \overrightarrow{AC} + \overrightarrow{CD} + \overrightarrow{DB} = \overrightarrow{AC} + (-\overrightarrow{DC}) + (-\overrightarrow{BD}) = \overrightarrow{AC} - \overrightarrow{DC} - \overrightarrow{BD}$$

б) $\overrightarrow{DA}, \overrightarrow{DC}, \overrightarrow{CB}$

$$\overrightarrow{AB} = \overrightarrow{AD} + \overrightarrow{DC} + \overrightarrow{CB} = -\overrightarrow{DA} + \overrightarrow{DC} + \overrightarrow{CB}$$

в) $\overrightarrow{DA}, \overrightarrow{CD}, \overrightarrow{BC}$

$$\overrightarrow{AB} = \overrightarrow{AD} + \overrightarrow{DC} + \overrightarrow{CB} = -\overrightarrow{DA} + (-\overrightarrow{CD}) + (-\overrightarrow{BC}) = -\overrightarrow{DA} - \overrightarrow{CD} - \overrightarrow{BC}$$

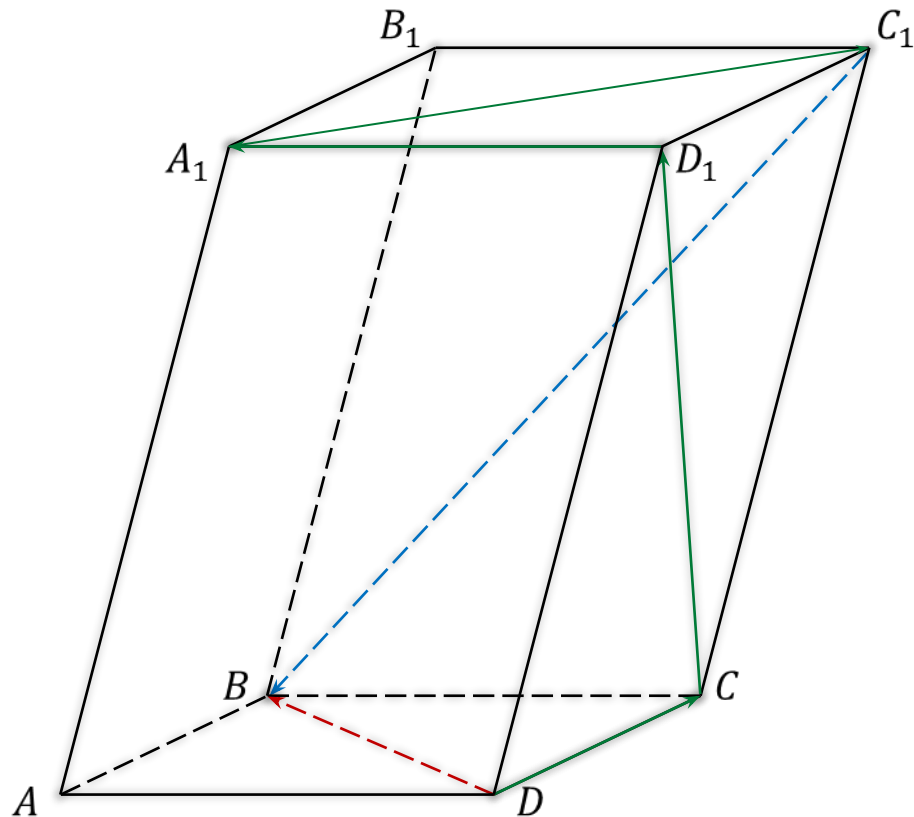
$ABCD A_1 B_1 C_1 D_1$ – параллелепипед.

$$\begin{aligned} \text{a) } \overrightarrow{DC} + \overrightarrow{D_1A_1} + \overrightarrow{CD_1} + \vec{x} + \overrightarrow{A_1C_1} &= \overrightarrow{DB} \\ \vec{x} &= \overrightarrow{C_1B} \end{aligned}$$

$$6) \overrightarrow{DA} + \vec{x} + \overrightarrow{D_1B} + \overrightarrow{AD_1} + \overrightarrow{BA} = \overrightarrow{DC}$$

$$\text{B) } \overrightarrow{AA_1} + \overrightarrow{B_1C} - \vec{x} = \overrightarrow{BA}$$

$$\Gamma) \overrightarrow{AC_1} - \overrightarrow{BB_1} + \vec{x} = \overrightarrow{AB}$$

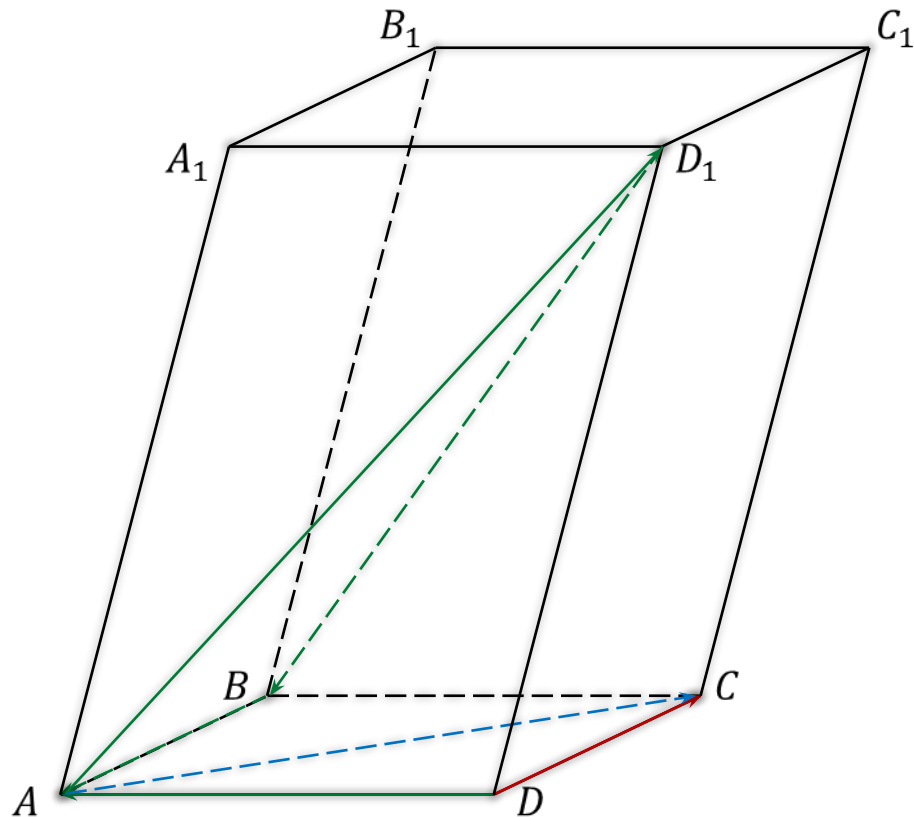


$ABCD A_1 B_1 C_1 D_1$ – параллелепипед.

$$\text{a) } \overrightarrow{DC} + \overrightarrow{D_1A_1} + \overrightarrow{CD_1} + \vec{x} + \overrightarrow{A_1C_1} = \overrightarrow{DB}$$

$$\vec{x} = \overrightarrow{C_1B}$$

$$\begin{aligned} 6) \quad & \overrightarrow{DA} + \vec{x} + \overrightarrow{D_1B} + \overrightarrow{AD_1} + \overrightarrow{BA} = \overrightarrow{DC} \\ & \vec{x} = \overrightarrow{AC} \end{aligned}$$

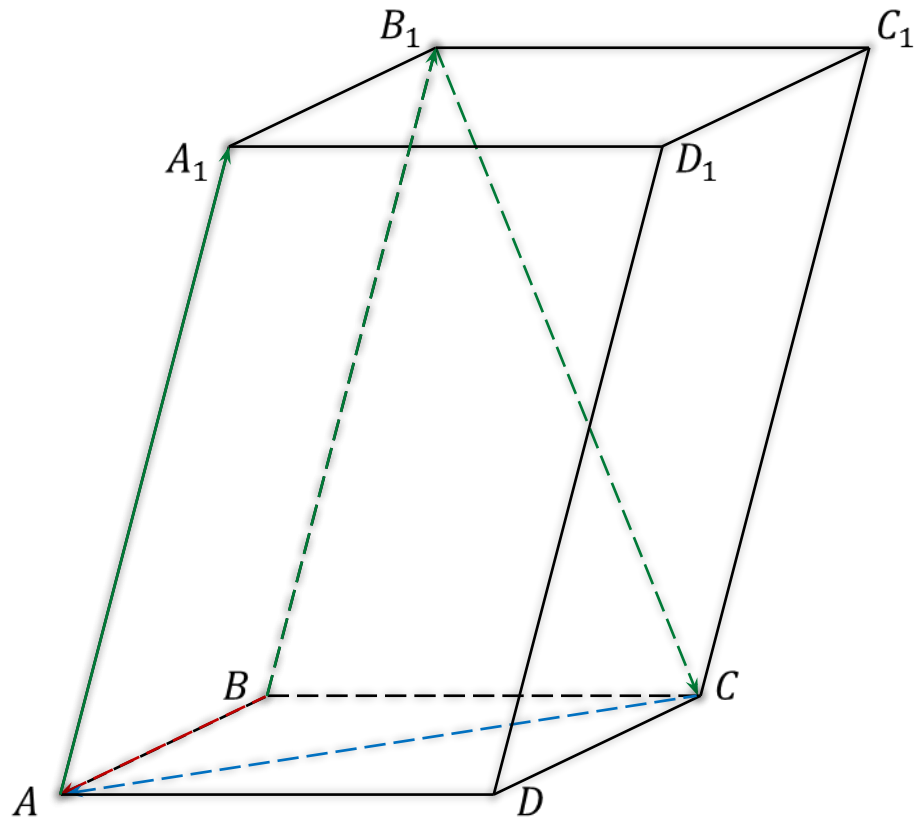


$ABCD A_1 B_1 C_1 D_1$ – параллелепипед.

$$\begin{aligned} \text{а) } \overrightarrow{DC} + \overrightarrow{D_1 A_1} + \overrightarrow{CD_1} + \vec{x} + \overrightarrow{A_1 C_1} &= \overrightarrow{DB} \\ \vec{x} &= \overrightarrow{C_1 B} \end{aligned}$$

$$\begin{aligned} \text{б) } \overrightarrow{DA} + \vec{x} + \overrightarrow{D_1 B} + \overrightarrow{AD_1} + \overrightarrow{BA} &= \overrightarrow{DC} \\ \vec{x} &= \overrightarrow{AC} \end{aligned}$$

$$\begin{aligned} \text{в) } \overrightarrow{AA_1} + \overrightarrow{B_1 C} - \vec{x} &= \overrightarrow{BA} \\ \overrightarrow{BB_1} + \overrightarrow{B_1 C} - \vec{x} &= \overrightarrow{BA} \\ -\vec{x} &= \overrightarrow{CA} \Rightarrow \vec{x} = \overrightarrow{AC} \end{aligned}$$



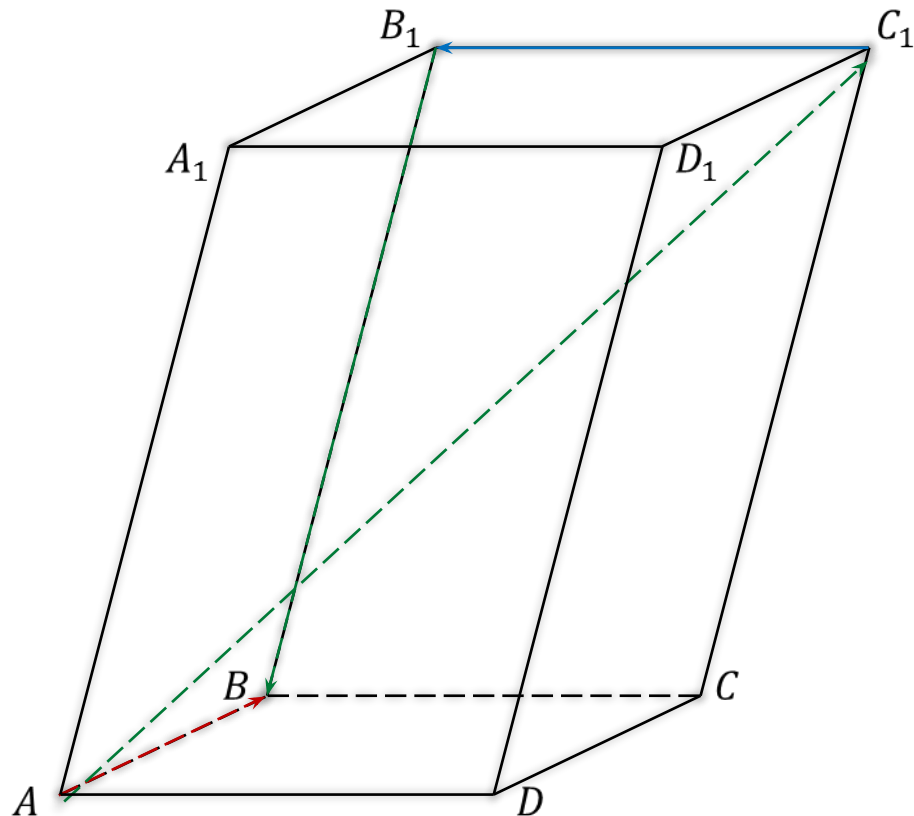
$ABCD A_1 B_1 C_1 D_1$ – параллелепипед.

$$\begin{aligned} \text{а) } \overrightarrow{DC} + \overrightarrow{D_1 A_1} + \overrightarrow{CD_1} + \vec{x} + \overrightarrow{A_1 C_1} &= \overrightarrow{DB} \\ \vec{x} &= \overrightarrow{C_1 B} \end{aligned}$$

$$\begin{aligned} \text{б) } \overrightarrow{DA} + \vec{x} + \overrightarrow{D_1 B} + \overrightarrow{AD_1} + \overrightarrow{BA} &= \overrightarrow{DC} \\ \vec{x} &= \overrightarrow{DC} \end{aligned}$$

$$\begin{aligned} \text{в) } \overrightarrow{AA_1} + \overrightarrow{B_1 C} - \vec{x} &= \overrightarrow{BA} \\ \overrightarrow{BB_1} + \overrightarrow{B_1 C} - \vec{x} &= \overrightarrow{BA} \\ -\vec{x} = \overrightarrow{CA} &\Rightarrow \vec{x} = \overrightarrow{AC} \end{aligned}$$

$$\begin{aligned} \text{г) } \overrightarrow{AC_1} - \overrightarrow{BB_1} + \vec{x} &= \overrightarrow{AB} \\ \overrightarrow{AC_1} + (-\overrightarrow{BB_1}) + \vec{x} &= \overrightarrow{AB} \\ \overrightarrow{AC_1} + \overrightarrow{B_1 B} + \vec{x} &= \overrightarrow{AB} \\ \vec{x} &= \overrightarrow{C_1 B_1} \end{aligned}$$



Сумма нескольких векторов