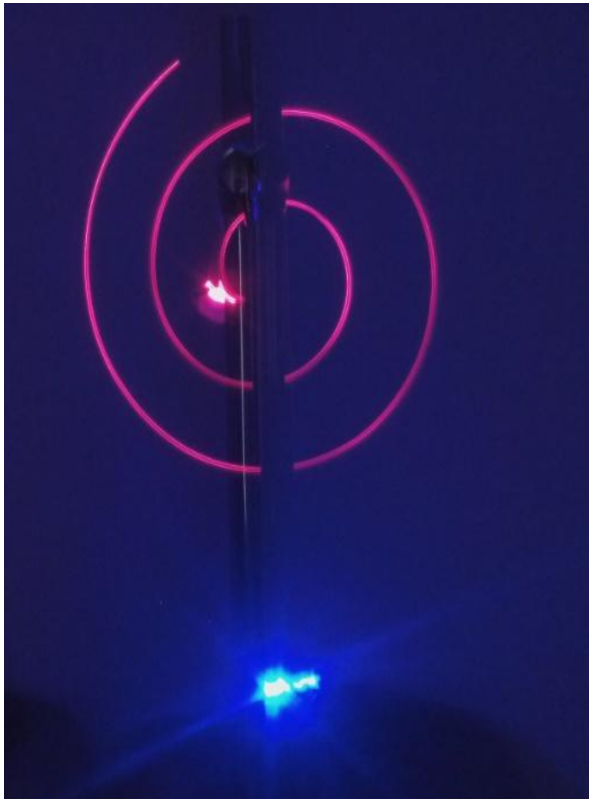
Problem №14

Looping pendulum

Connect two loads, one heavy and one light, with a string over a horizontal rod and lift up the heavy load by pulling down the light one. Release the light load and it will sweep around the rod, keeping the heavy load from falling to the ground. Investigate this phenomenon.



Team Russia

Reporter: Ivan Polonik

Plan

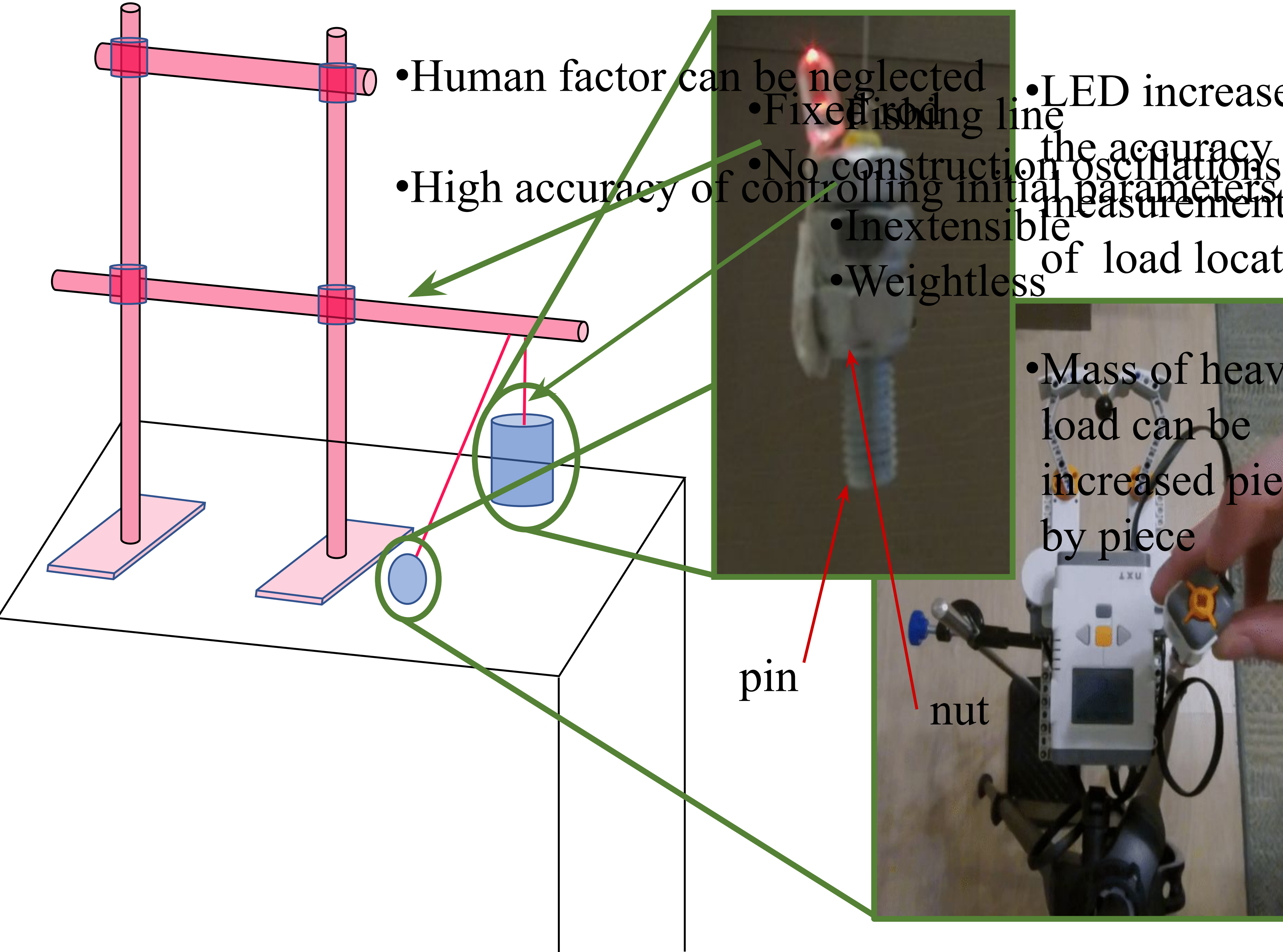
Qualitative explanation: Energy transfer

Parametric investigation

Boundary conditions: Mass, initial angle and length relationship

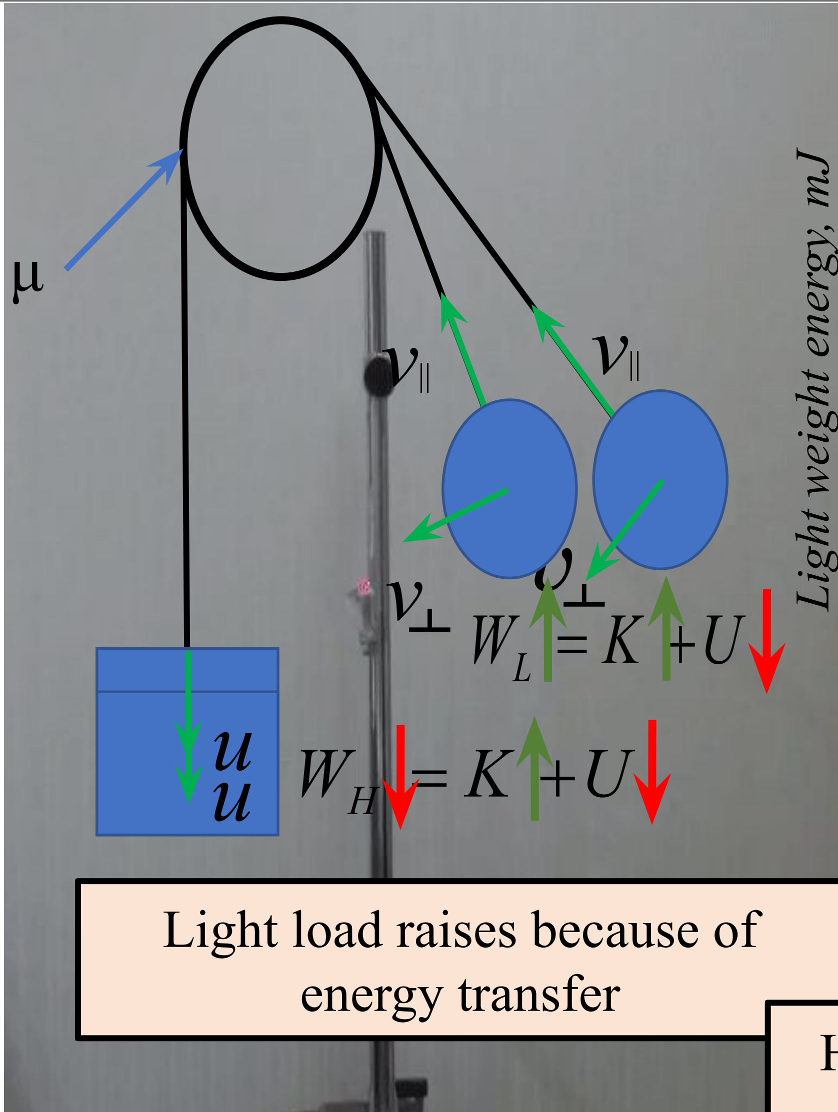
Different mode: step falling

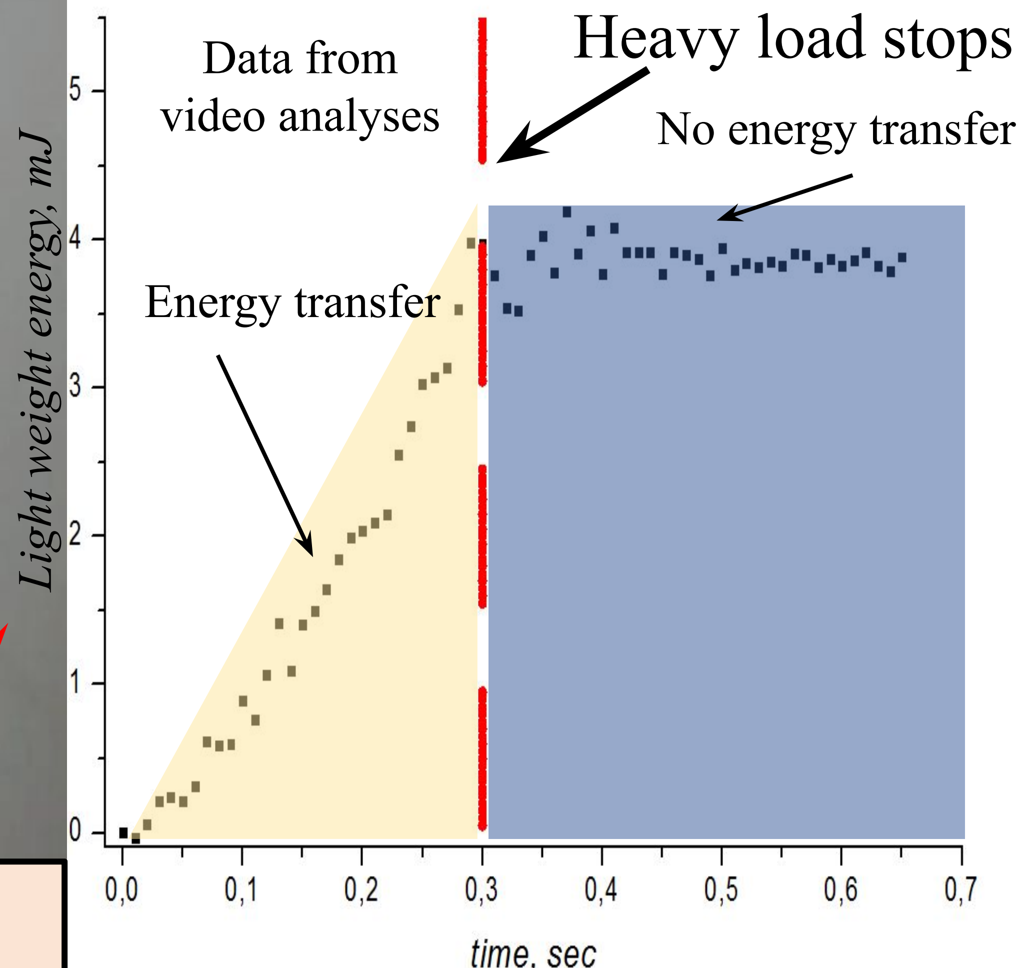
Setup scheme



- Human factor can be neglected
- High accuracy of controlling initial parameters
- Fixed rod
- No construction oscillations
- Fishing line
- Inextensible
- Weightless
- LED increases the accuracy of measurements of load location
- Mass of heavy load can be increased piece by piece

Qualitative explanation





Light load raises because of energy transfer

Heavy load is accelerated by gravity force and is decelerated by friction force

Components of the system

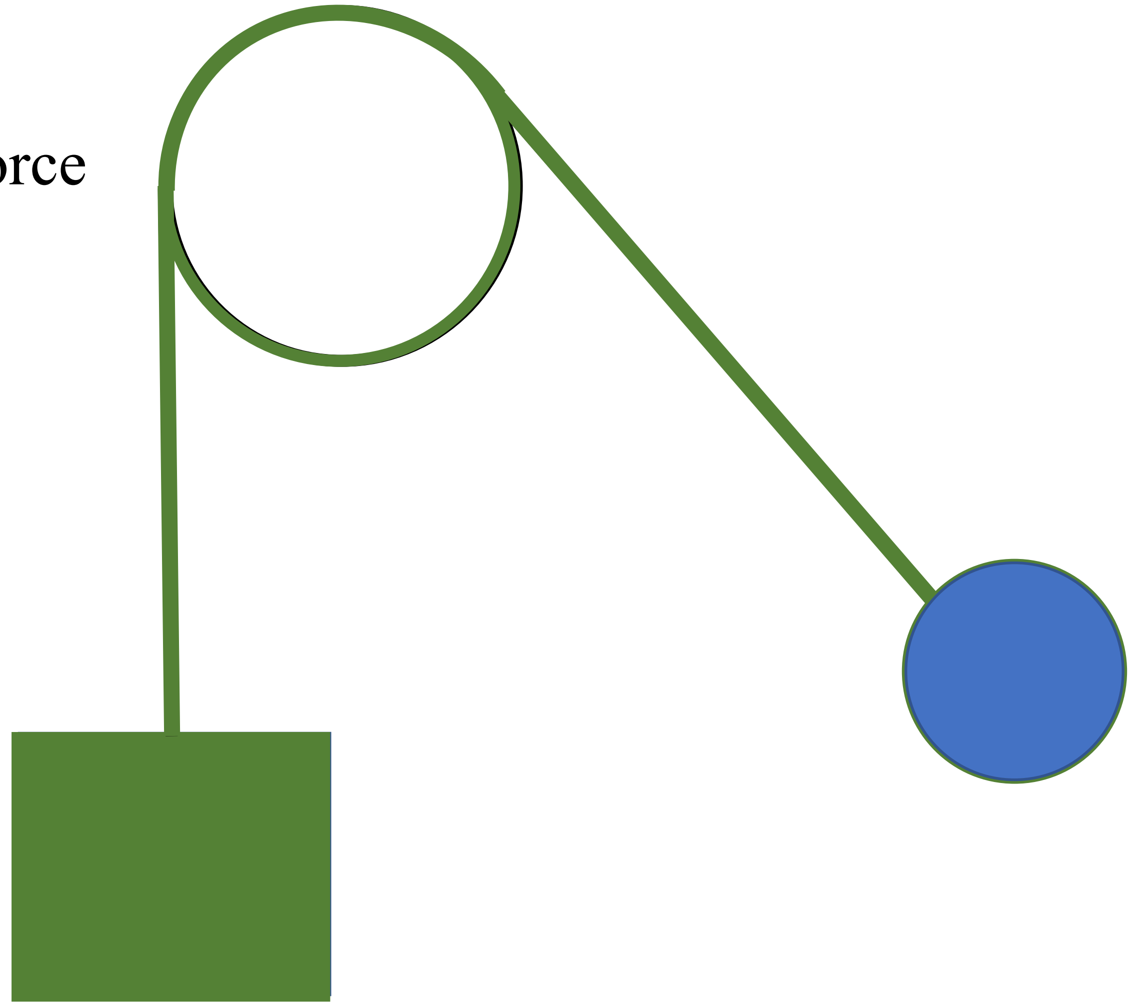
Things to describe

Rod + string – friction force

String – kinematic ratio

Light load – dynamics

Heavy load - dynamics



Qualitative explanation ✓ | Quantitative model • | Parametric investigation

Theory assumptions:

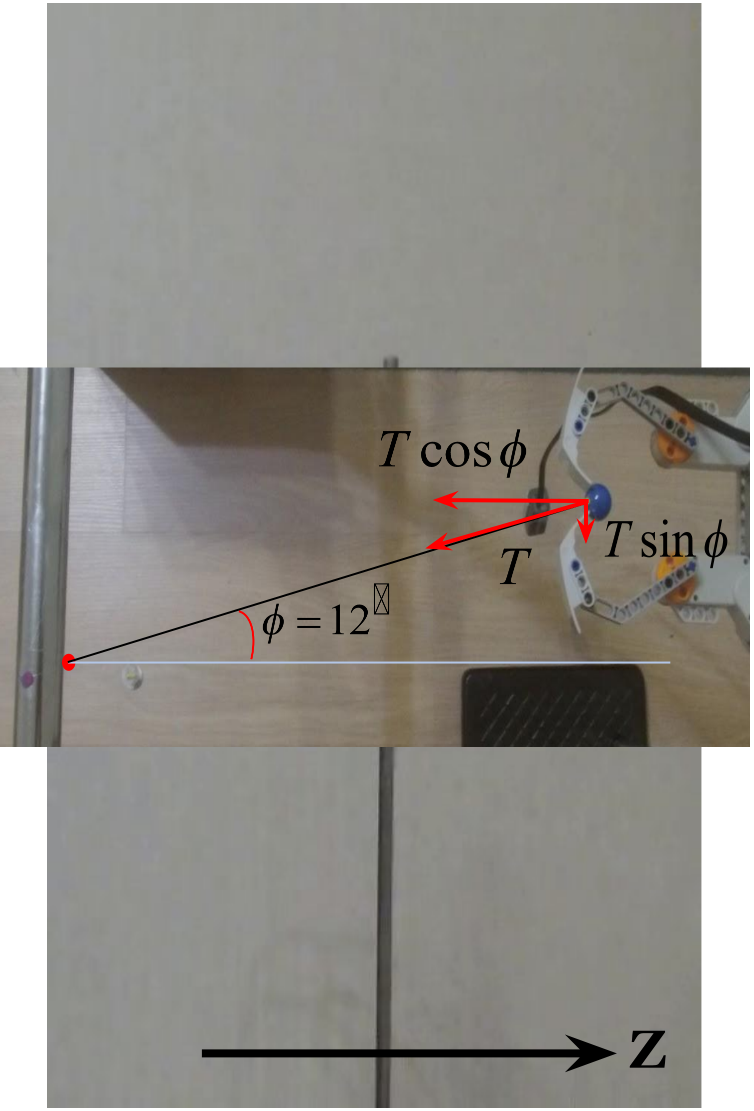
Drag force is neglectable

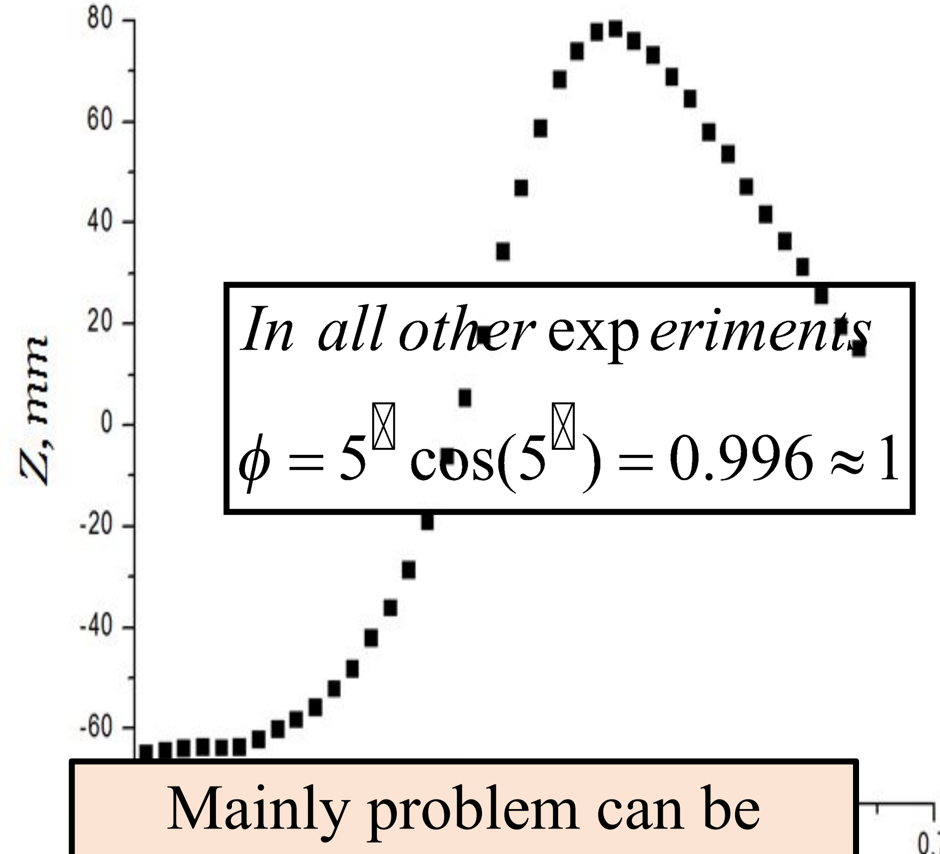
Heavy load falls vertically

String: weightless inextensible

String lays turn to turn

3 - dimensional movement





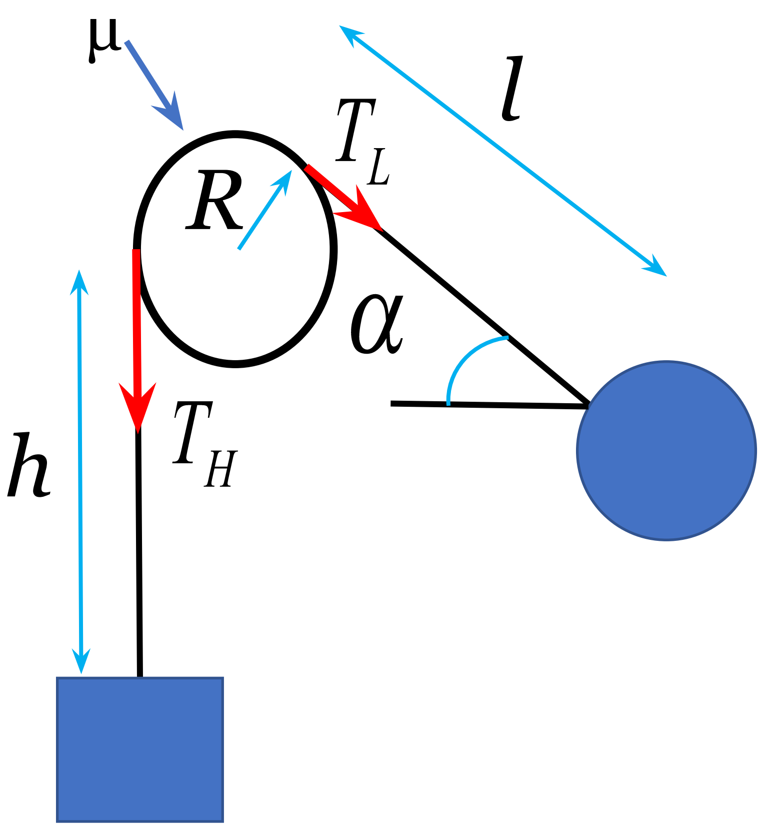
In all other experiments

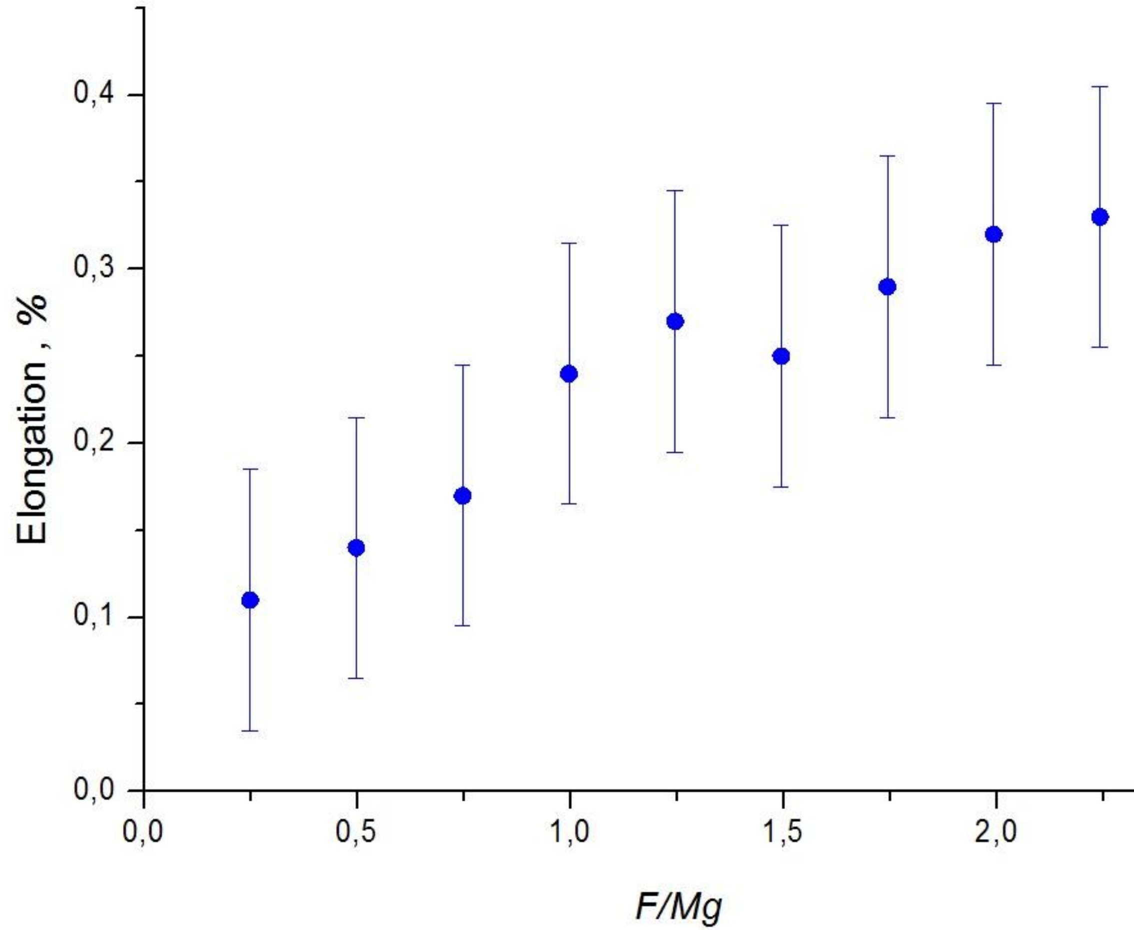
$\phi = 5^{\circ}$ $\cos(5^{\circ}) = 0.996 \approx 1$

Mainly problem can be solved as 2-dimensional

Qualitative explanation ✓	Quantitative model •	Parametric investigation

Rod and string description





Rod – string friction while heavy load falls:

Qualitative explanation ✓

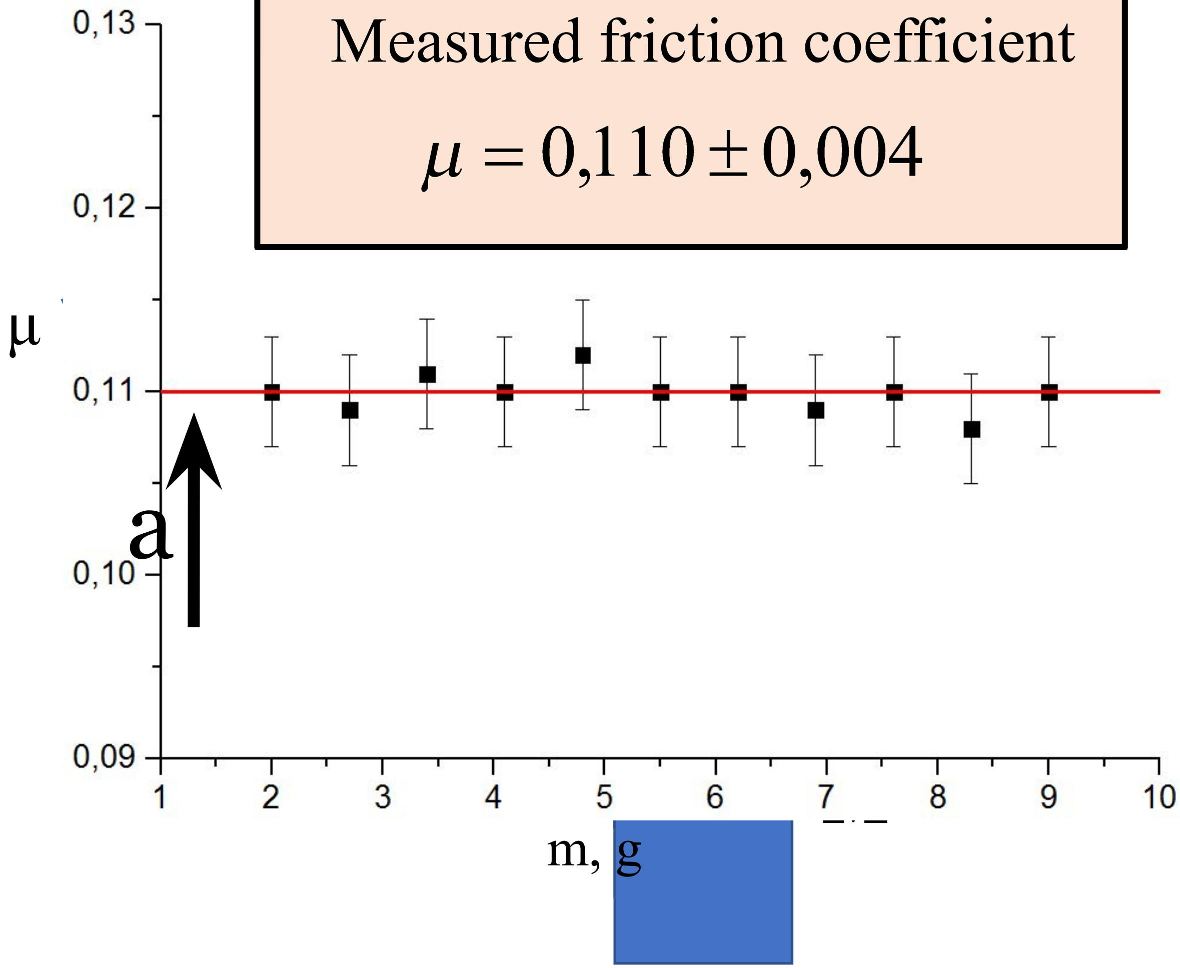
Quantitative model •

Parametric investigation

Friction coefficient measurements

Measured friction coefficient

$$\mu = 0{,}110 \pm 0{,}004$$



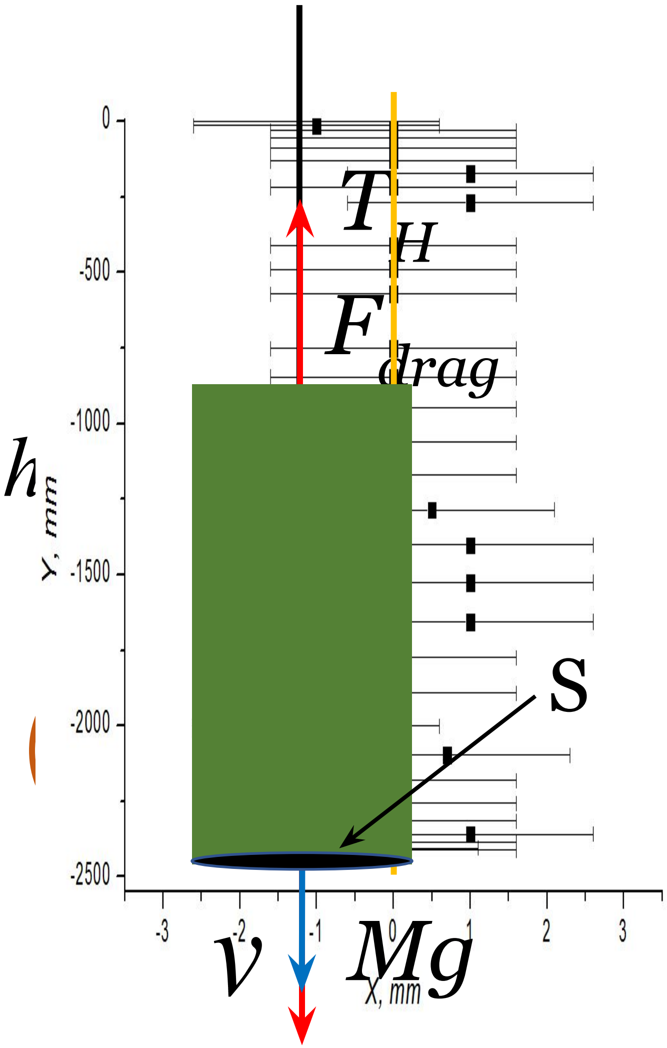
Acceleration was found from video analyses

Using Euler's formula

$$\boxed{\mu = \frac{1}{\pi} \ln \frac{M(g-a)}{m(g+a)}}$$

Qualitative explanation ✓ | Quantitative model • | Parametric investigation

Heavy load movement



$$F_{drag} = \frac{1}{2} C_x \rho v^2 S \qquad C_x = 0.85$$

$$F_{drag} \approx 7\times10^{-5}\ N \qquad Mg \approx 3\times10^{-1}\ N$$

Heavy load falls vertically

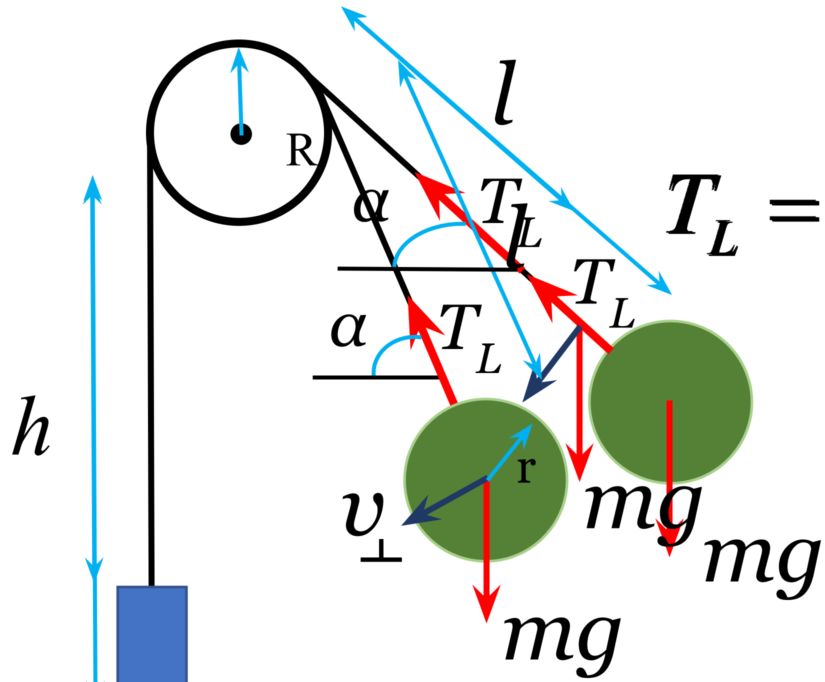
$$\frac{Mg}{F_{drag}^{\max}} \approx 4000$$

Drag force can be neglected

II Newton law on vertical axis : $M\ddot{h} = Mg - T_H$

Qualitative explanation ✓ | Quantitative model • | Parametric investigation

Tension force acting on the light load



$$T_L = m(g\sin\alpha + \ddot{h} + R\ddot{\alpha} + \dot{\alpha}^2 l)$$

$\upsilon_\perp = \dot{\alpha}\, l$

Gravity component

Inextensibility component

Centrifugal component

$$\frac{T_L^{\max}}{F_{drag}^{\max}} \approx 90$$

$$F_{drag} = \frac{1}{2} C_x \rho v^2 S \quad C_x = 0.47 \quad S = 4\pi r^2$$

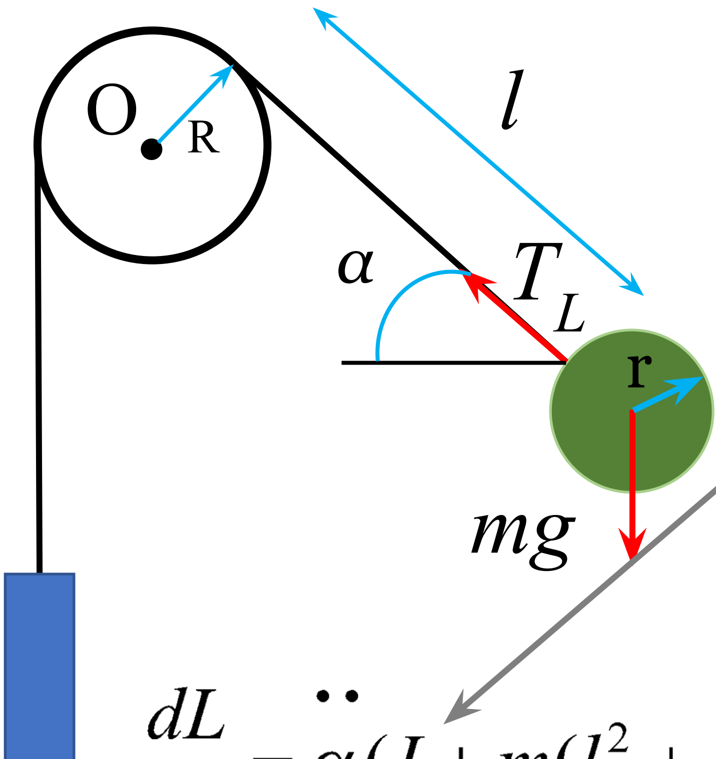
Drag force can be neglected

$\ddot{h}$

Found value of Tension force

Qualitative explanation ✓ | Quantitative model • | Parametric investigation

Rotation of light load



Torque equation
about point O :

$$I = \frac{2}{5}mr^2$$

Inconstant moment of inertia

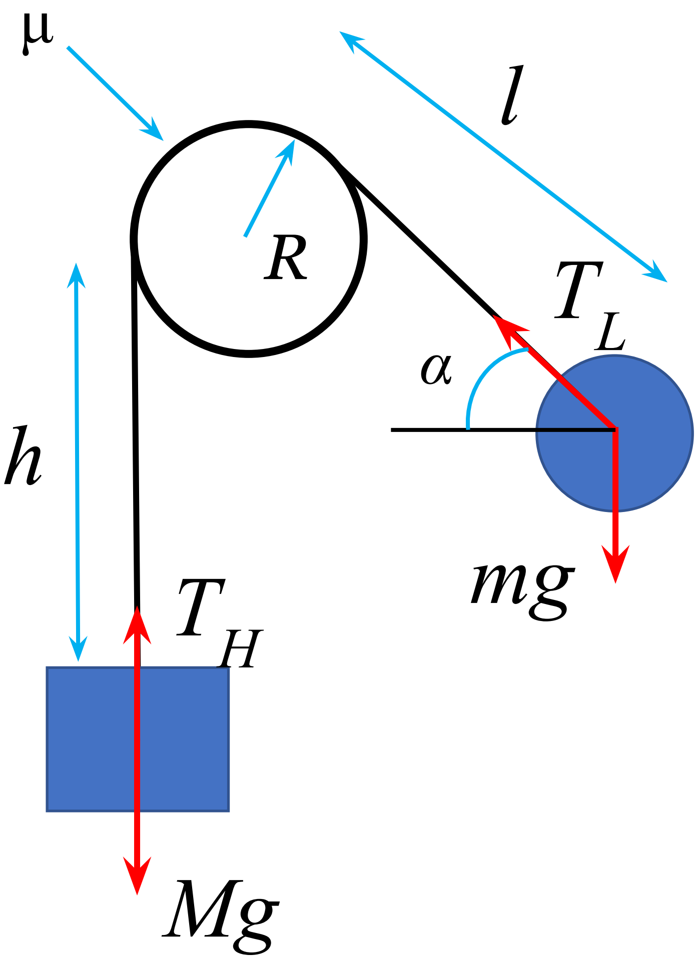
Tension force torque

$$\frac{dL}{dt} = \ddot{\alpha}\left(I + m(l^2 + R^2)\right) + \boxed{2ml\dot{l}\dot{\alpha}} = mg(l+R)\cos\alpha - \boxed{T_L R}$$

Described light load movement

Qualitative explanation ✓	Quantitative model •	Parametric investigation

Numerical solution



$$T_L = m((\dot{\alpha})^2 l + \ddot{h} + R\ddot{\alpha} + g\sin\alpha)$$

$$\ddot{\alpha} = \frac{m((l+R)g\cos\alpha - 2l\dot{l}\dot{\alpha}) - T_L R}{m(l^2+R^2)+I}$$

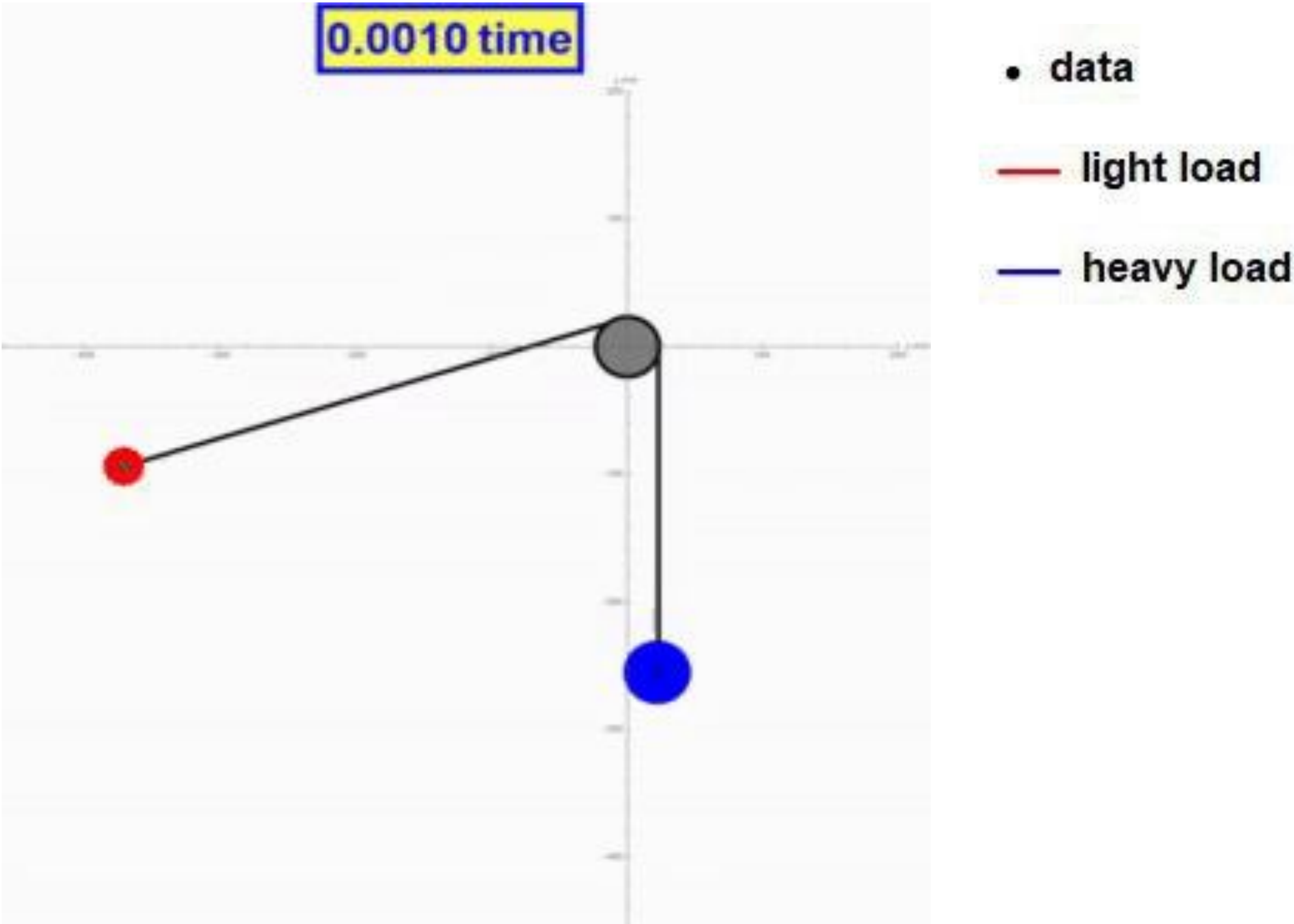
$$T_L(t=0) = 0$$

$$\ddot{h} = g - \frac{T_L e^{\mu(\alpha+\frac{\pi}{2})}}{M}$$

Set of equations was solved numerically (iteratively)

Qualitative explanation ✓ | Quantitative model • | Parametric investigation

Comparing the dynamics of the system

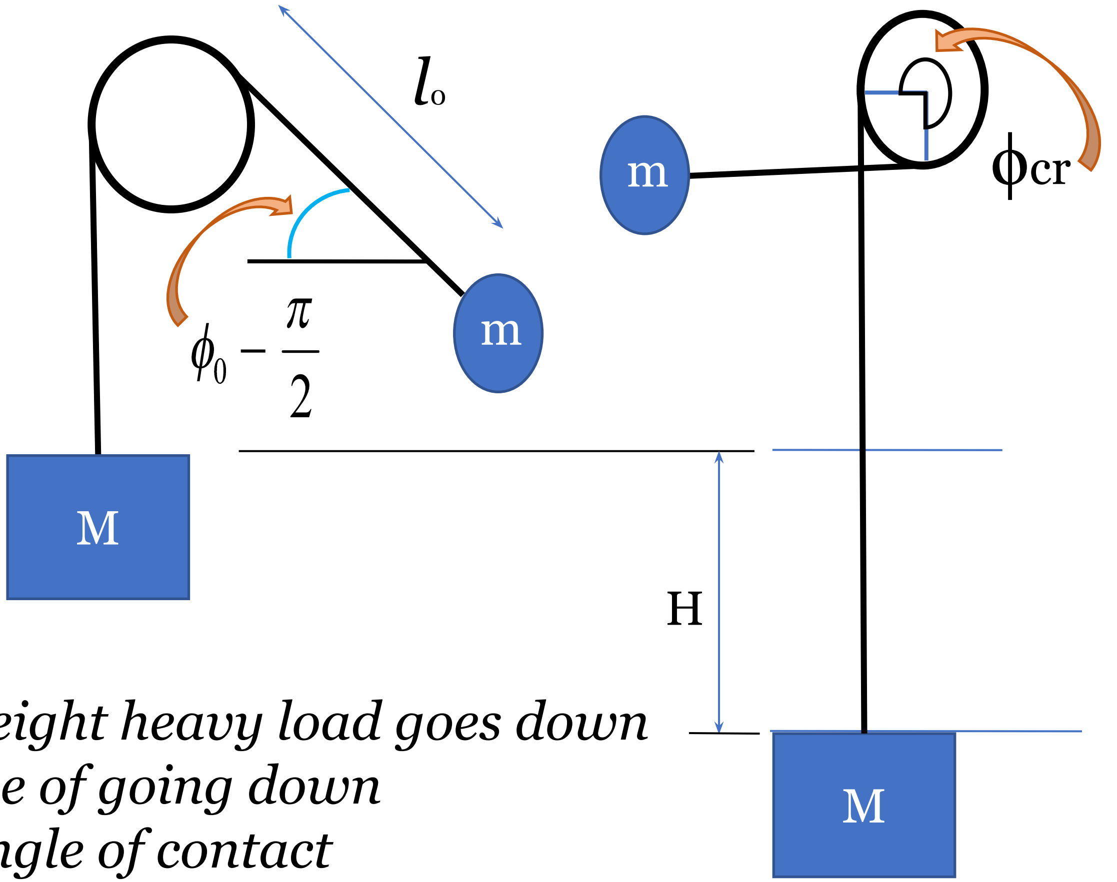


Qualitative explanation ✓

Quantitative model ✓

Parametric investigation •

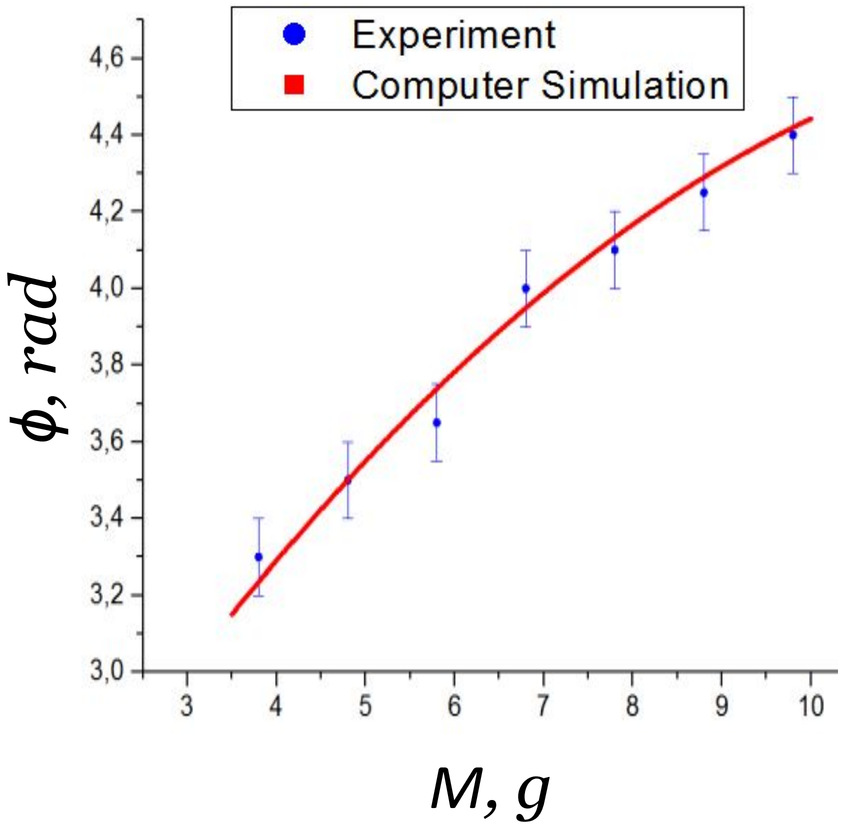
Legend

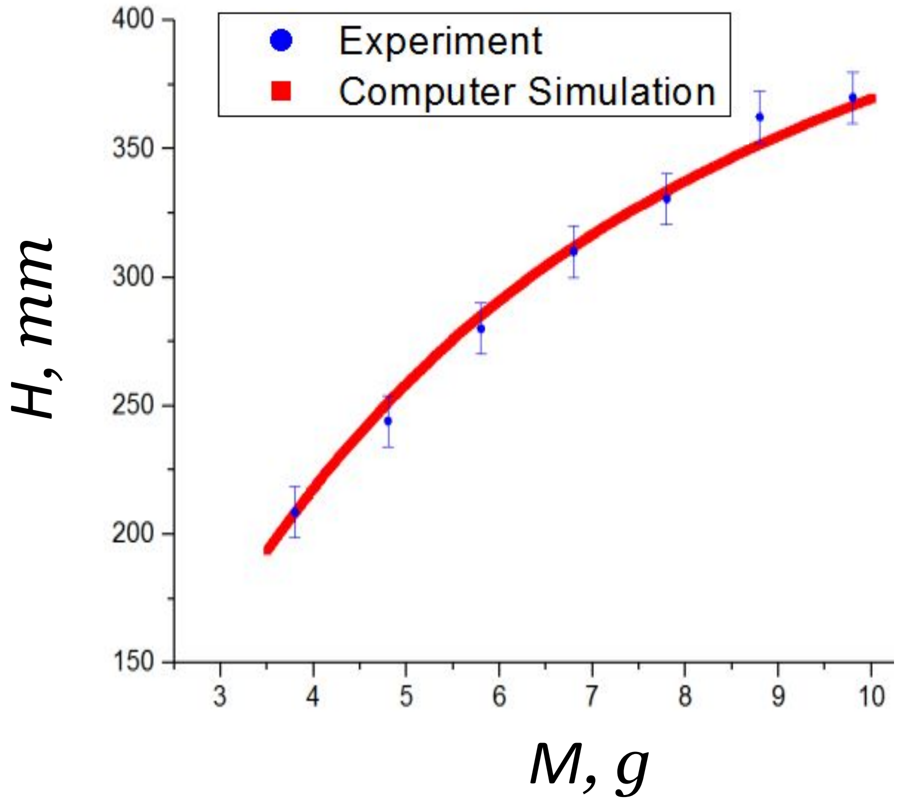


m – mass of light load
M – mass of heavy load
l – distance between light Load and the rod
μ – friction coefficient

H- height heavy load goes down
t- time of going down
φ- angle of contact between string and rod
φ_{crit}- φ at the moment of heavy load stopping

Heavy load mass influence





The higher heavy load mass the higher critical angle.

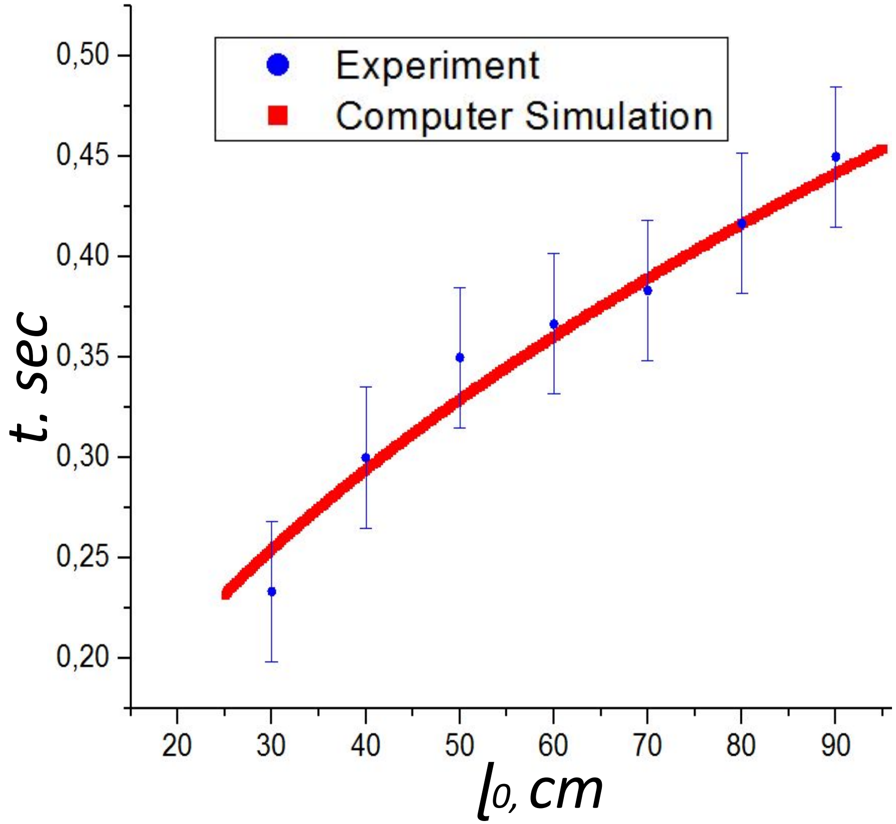
The higher heavy load mass the higher H.

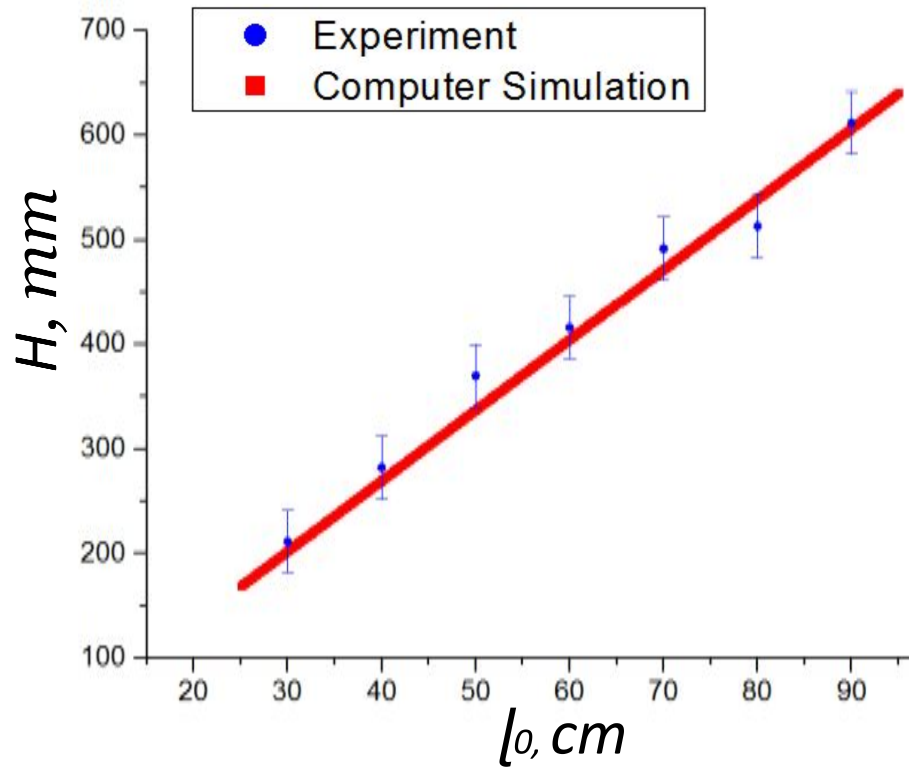
Qualitative explanation ✓

Quantitative model ✓

Parametric investigation •

Initial length of the string influence





The higher initial distance between light load and rod the higher time of falling

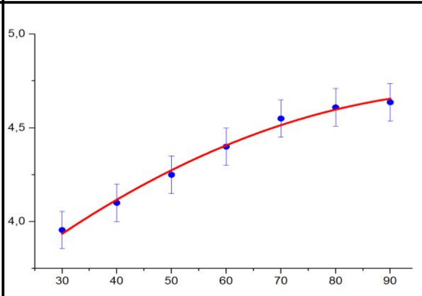
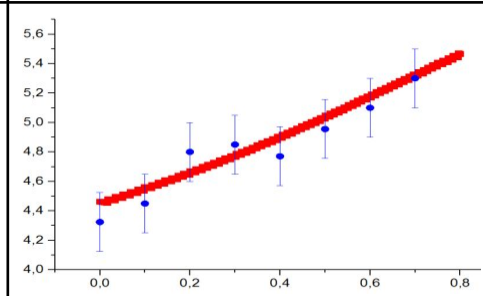
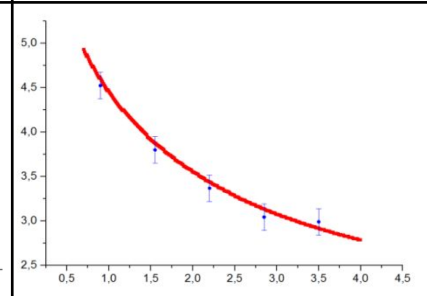
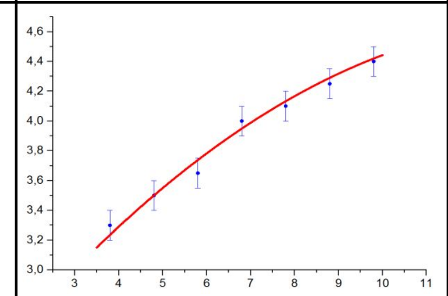
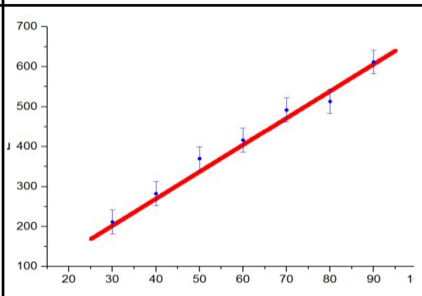
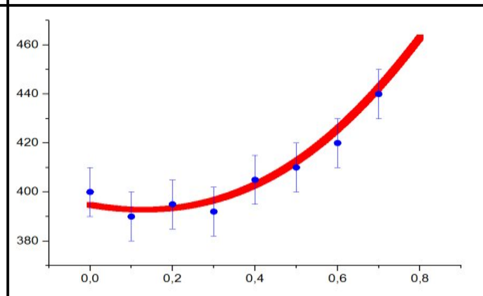
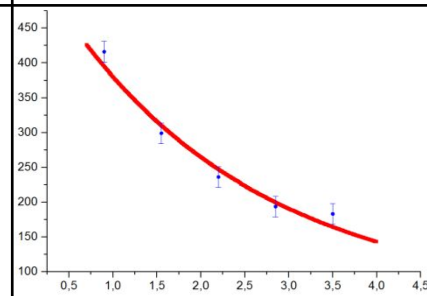
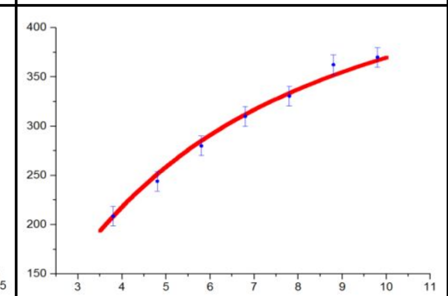
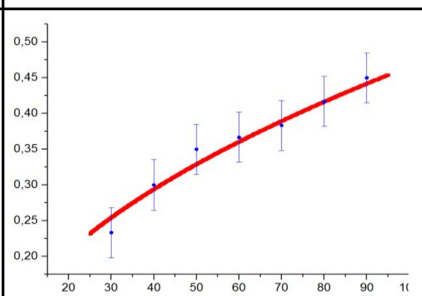
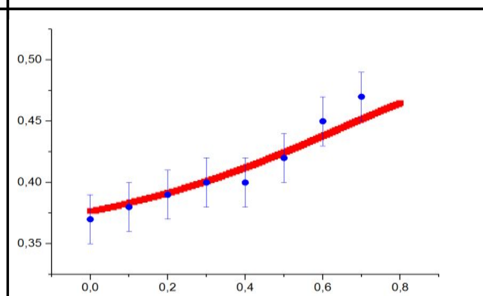
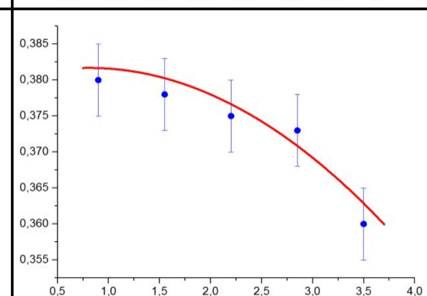
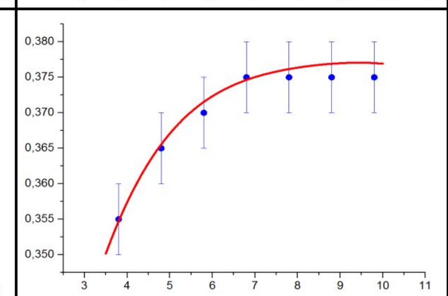
The higher initial distance between light load and rod the higher H

Qualitative explanation ✓

Quantitative model ✓

Parametric investigation •

Whole parametric investigation

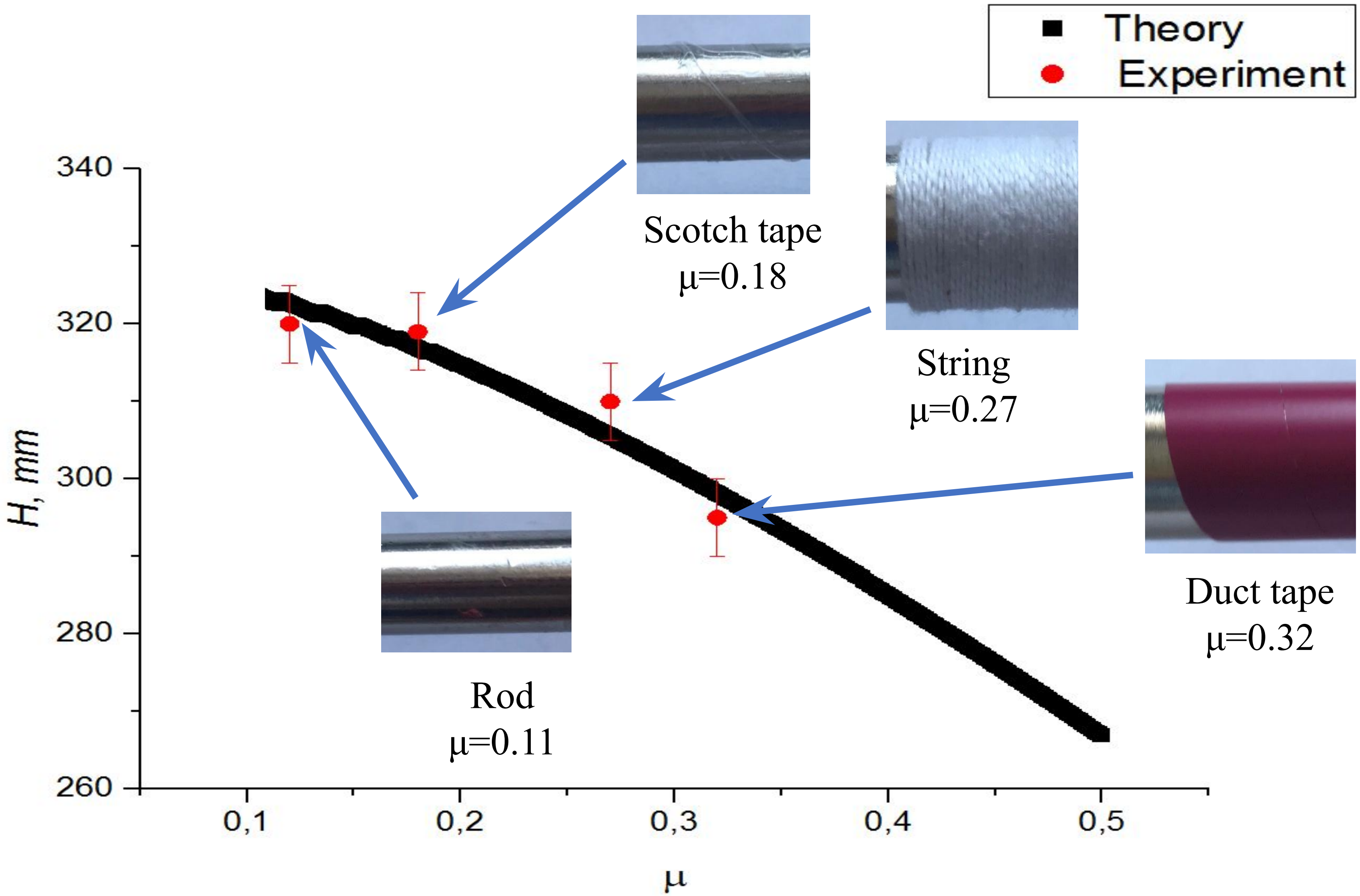
	$l_{0,\ cm}$	$\alpha_{0,\ rad}$	$m_{,\ g}$	$M_{,\ g}$
ϕ , *rad*				
H , *mm*				
t , *sec*				

Qualitative explanation ✓

Quantitative model ✓

Parametric investigation •

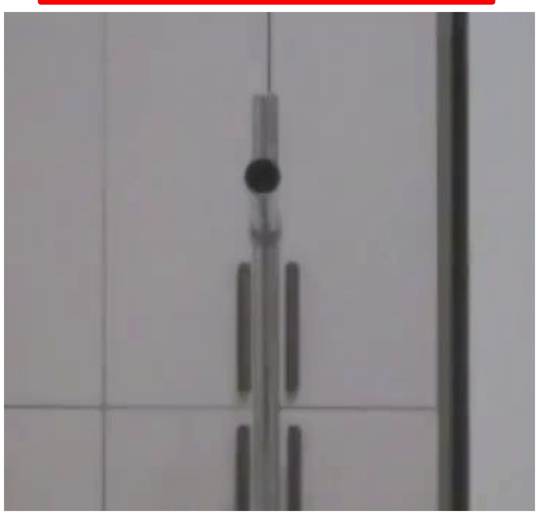
Influence of the friction coefficient



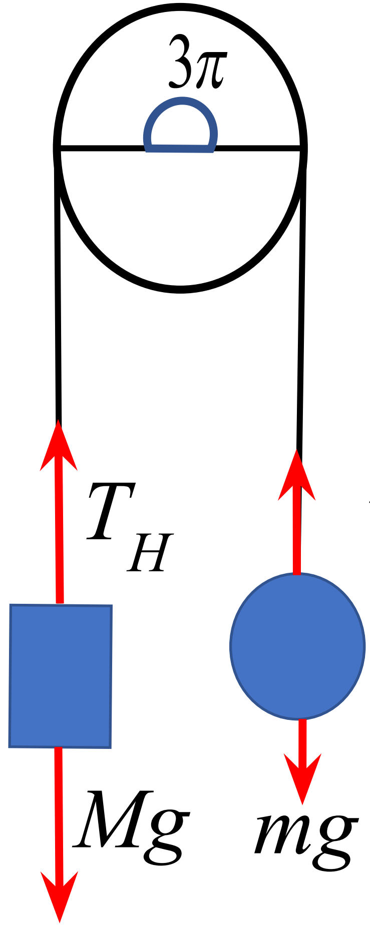
Qualitative explanation ✓ | Quantitative model ✓ | Parametric investigation •

Boundary conditions

M/m=2.75



M/m=2.95



$$T_H = Mg$$

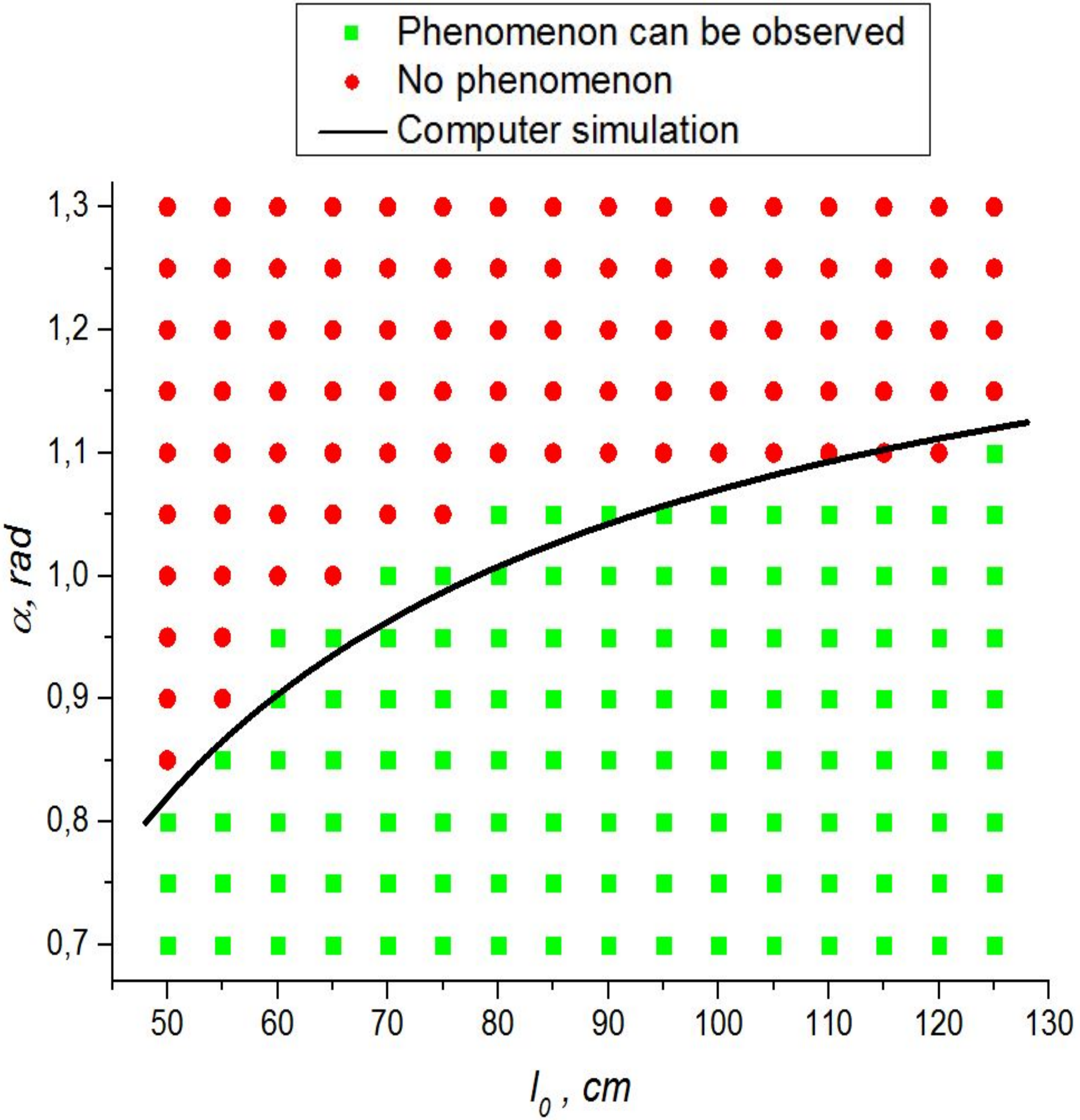
$$T_L = mg$$

$$\frac{M}{m} \geq e^{3\pi\mu}$$

$$\boxed{Theory\ prediction - \frac{M}{m} \geq 2.82}$$

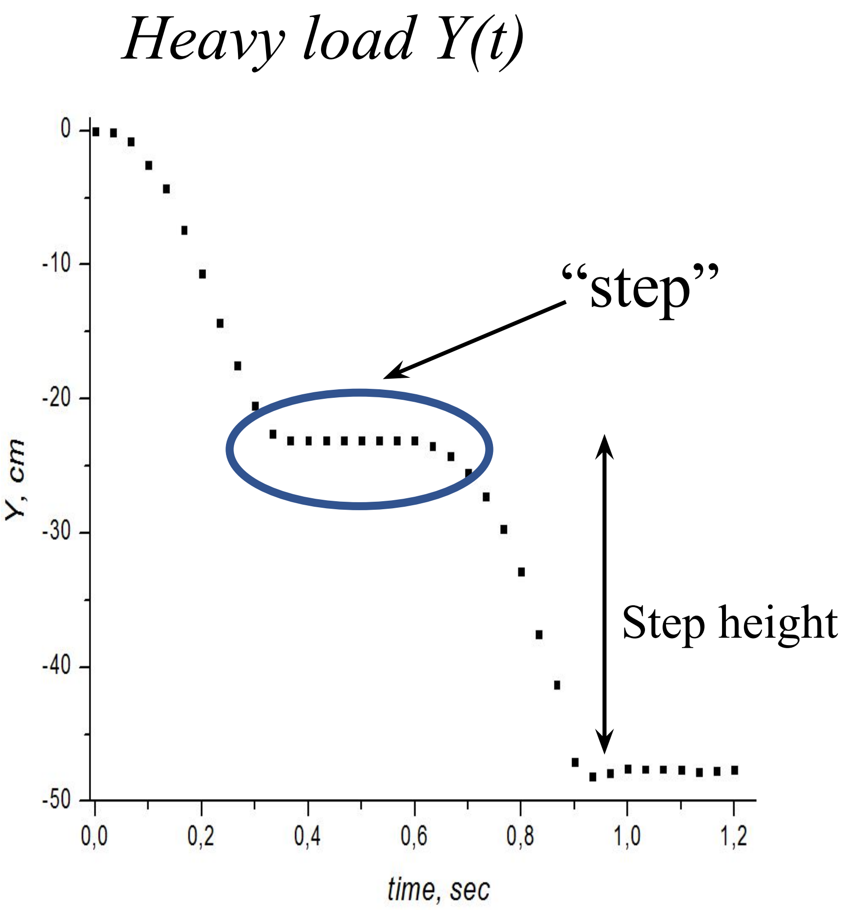
$$\boxed{\frac{M}{m} \leq 2.82 - no\ phenomenon}$$

Boundary conditions



$$\frac{M}{m} = 3.57$$

«Step» falling of heavy load

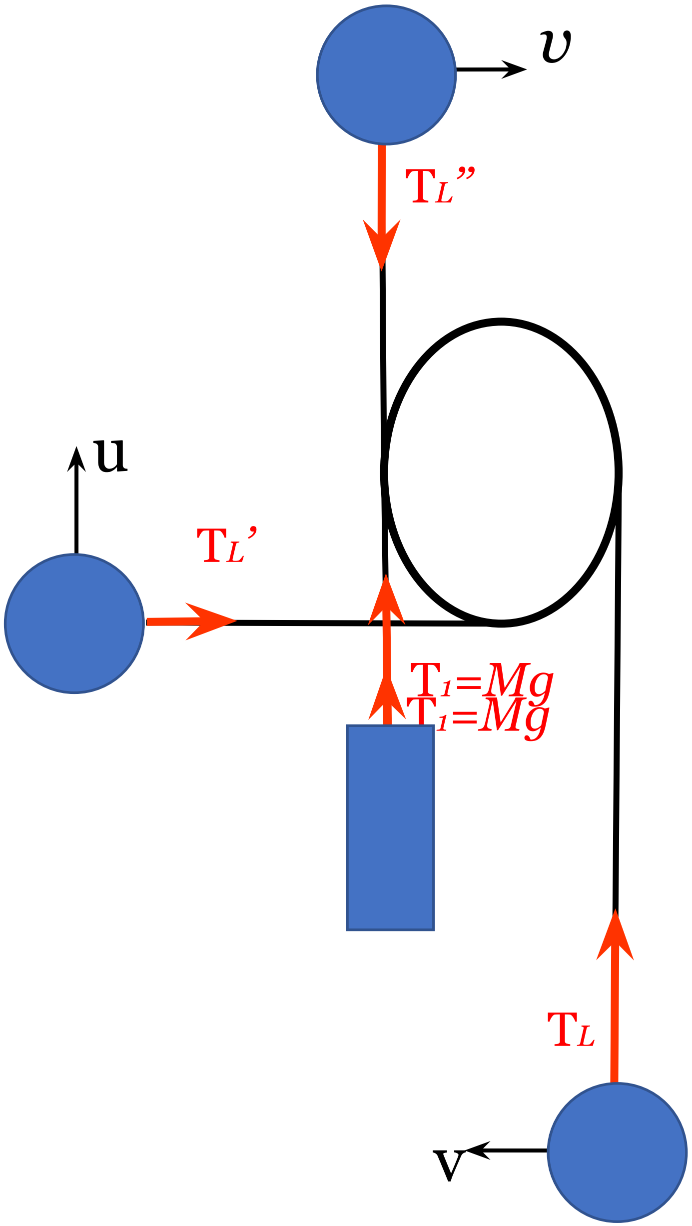


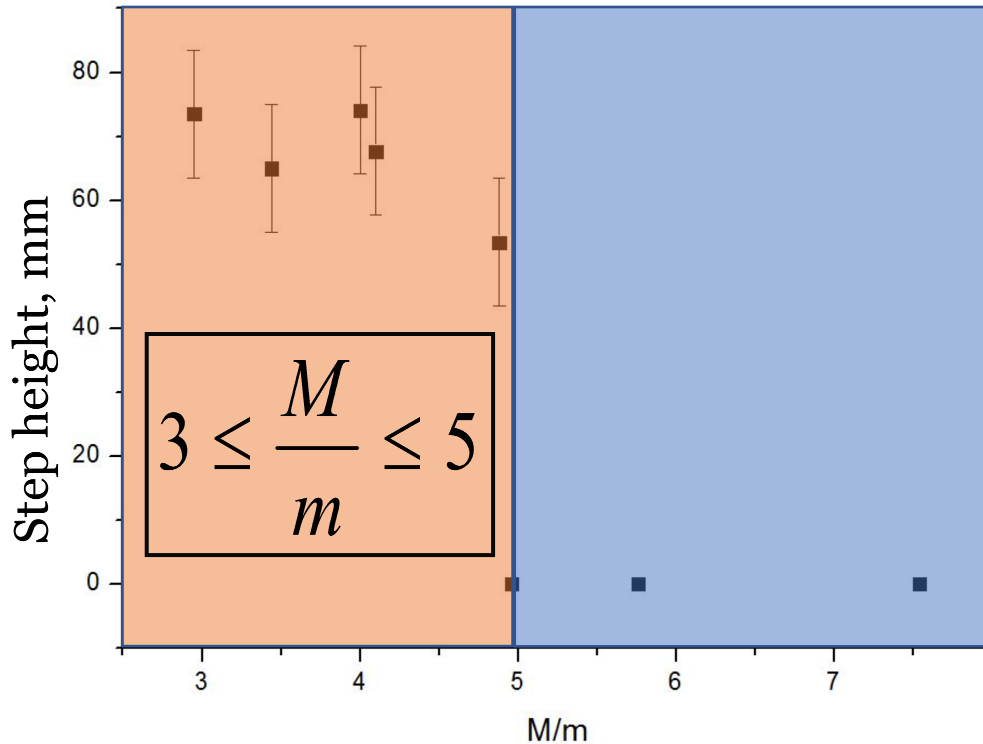
Qualitative explanation ✓

Quantitative model ✓

Parametric investigation •

«Step» falling

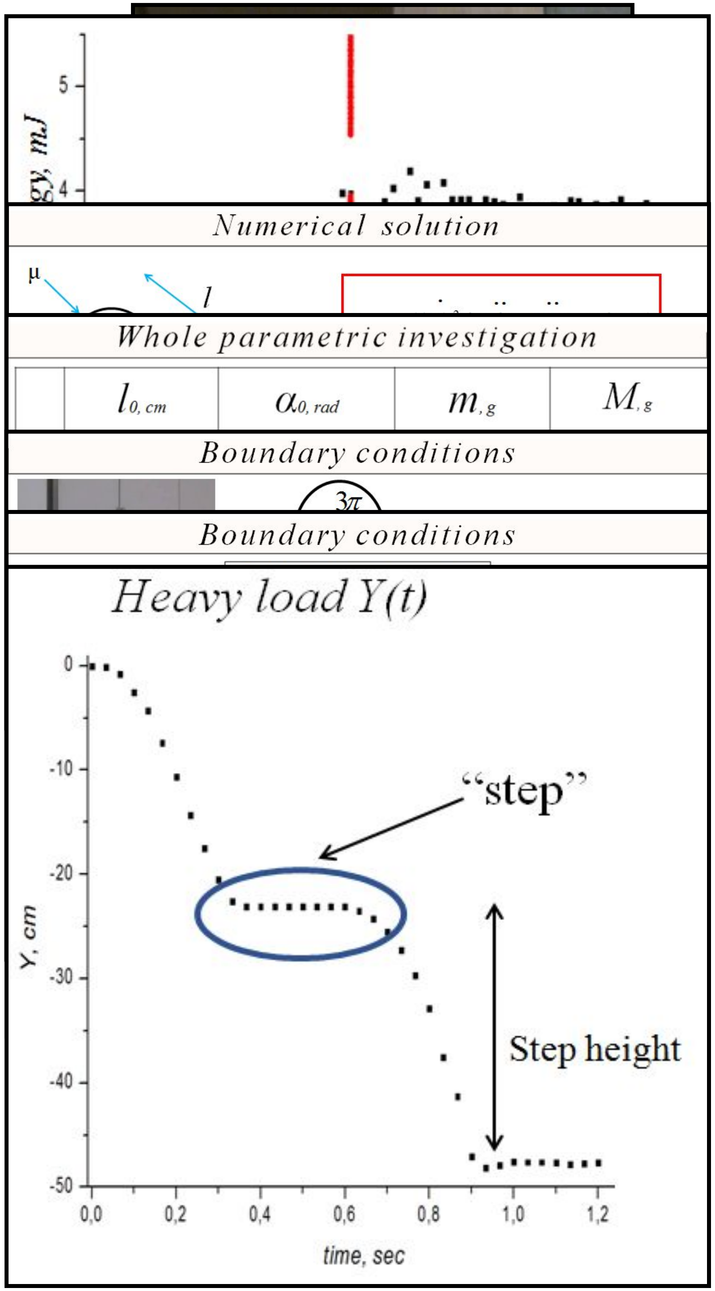




Step falling can be observed with small mass relationship

Conclusion

- Was built experimental setup excluding human factor and control of 3-dimensional effect
- Light load sweeps around because of the energy transfer
- Heavy load stops by friction force
- Built mathematical model based on inextensibility of the string, friction between string and cylindrical rod, 2-nd Newton’s laws and torque equation.
- Theory has a good agreement with experiment
- Found out minimal relationship between masses needed for phenomenon observation and relationship between
- Such mode as «step falling» was explained



Thank you for your attention!

Connect two loads, one heavy and one light, with a string over a horizontal rod and lift up the heavy load by pulling down the light one. Release the light load and it will sweep around the rod, keeping the heavy load from falling to the ground. Investigate this phenomenon.

Also was investigated:

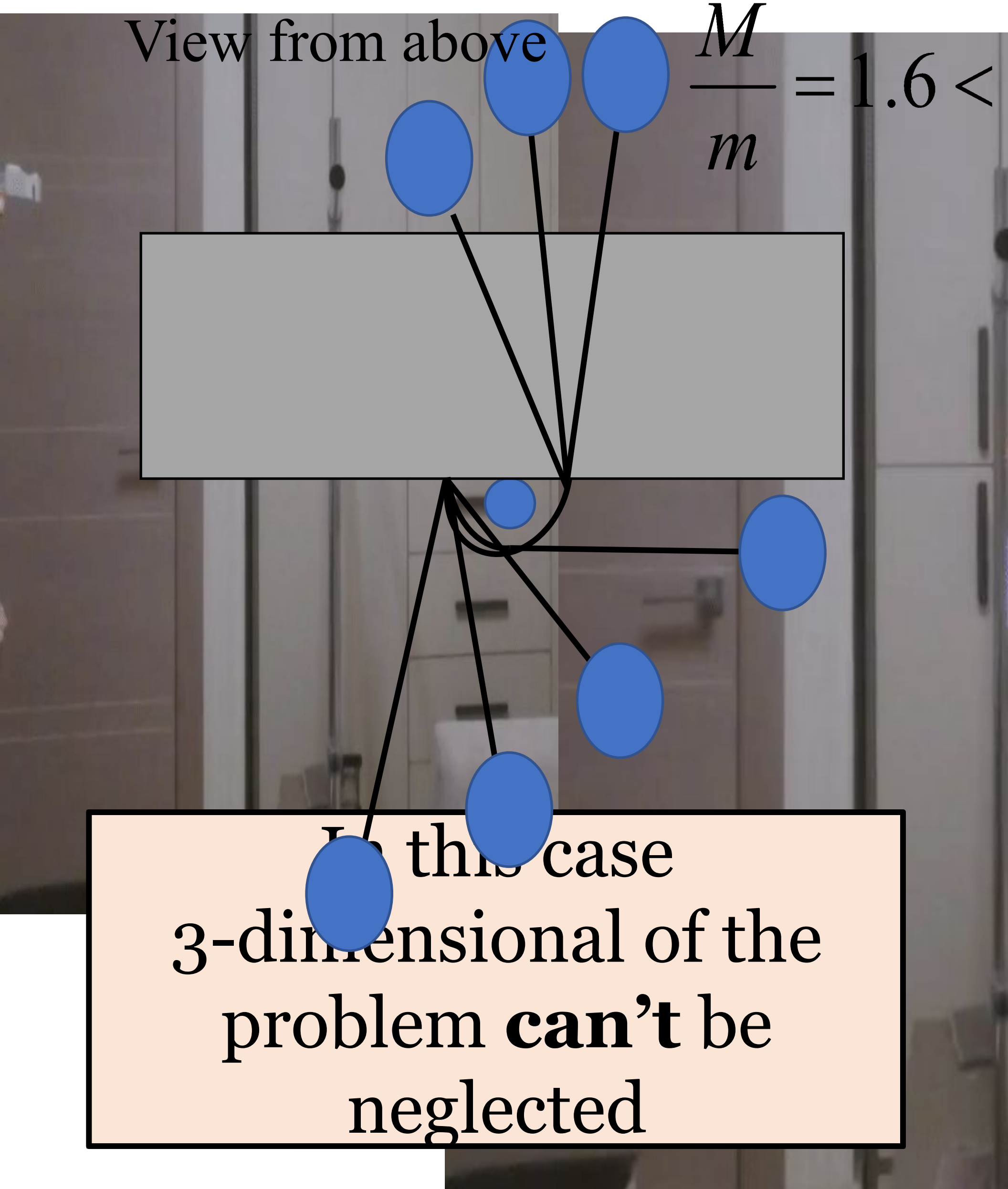
- Massive string
- Back sweeping
- Rod strike of light load
- Swinging of heavy load

Additional slides

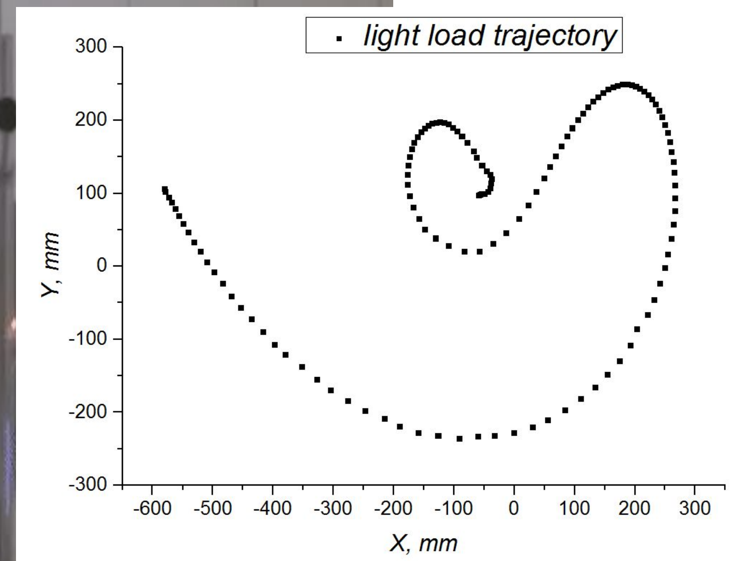
Back sweeping

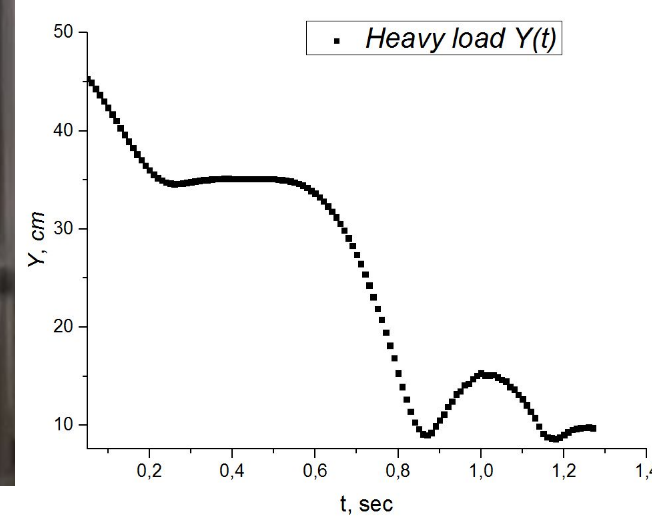
View from above

$$\frac{M}{m} = 1.6 < 2.82$$



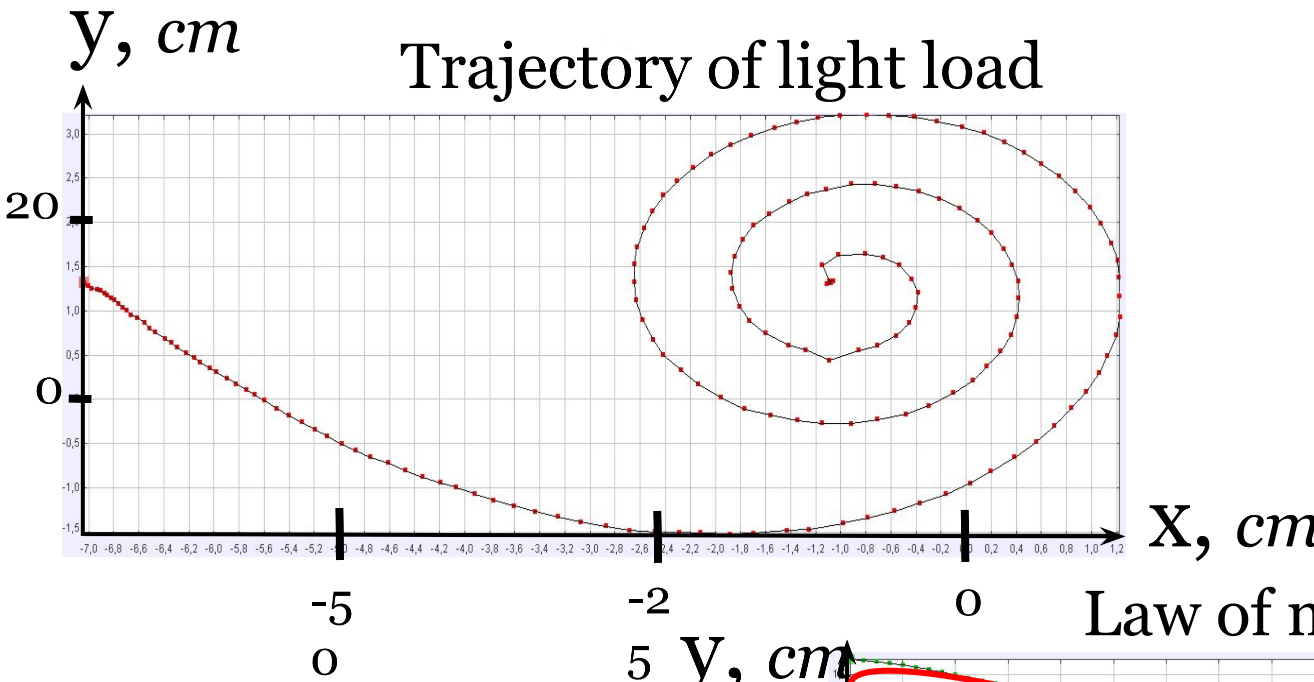
In this case 3-dimensional of the problem **can't** be neglected





Quality explanation | Quantitative model | Parametric investigation

Key observation



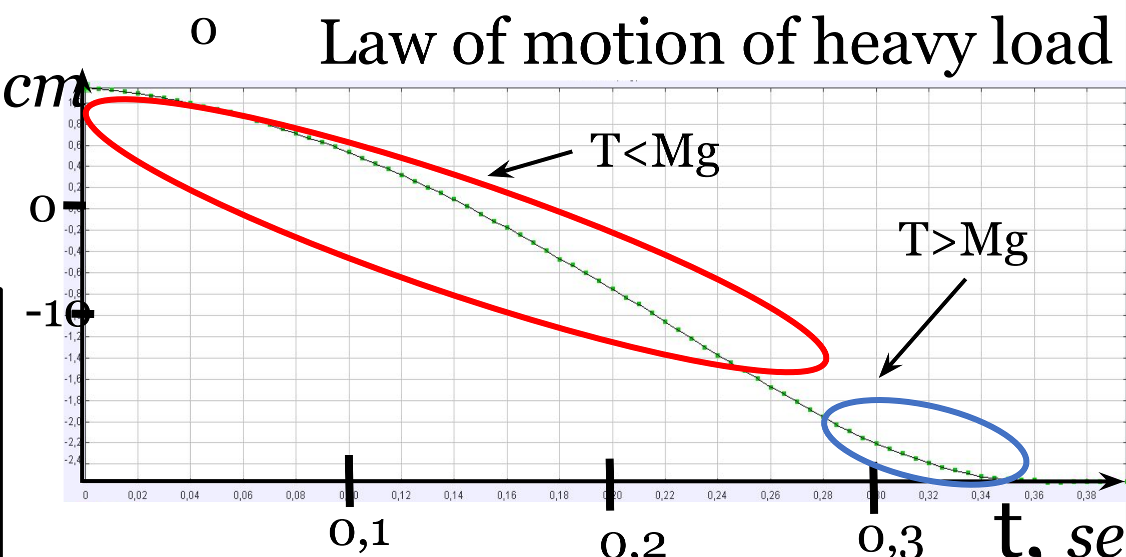
Parameters

$m = 1$ g

$M = 10$ g

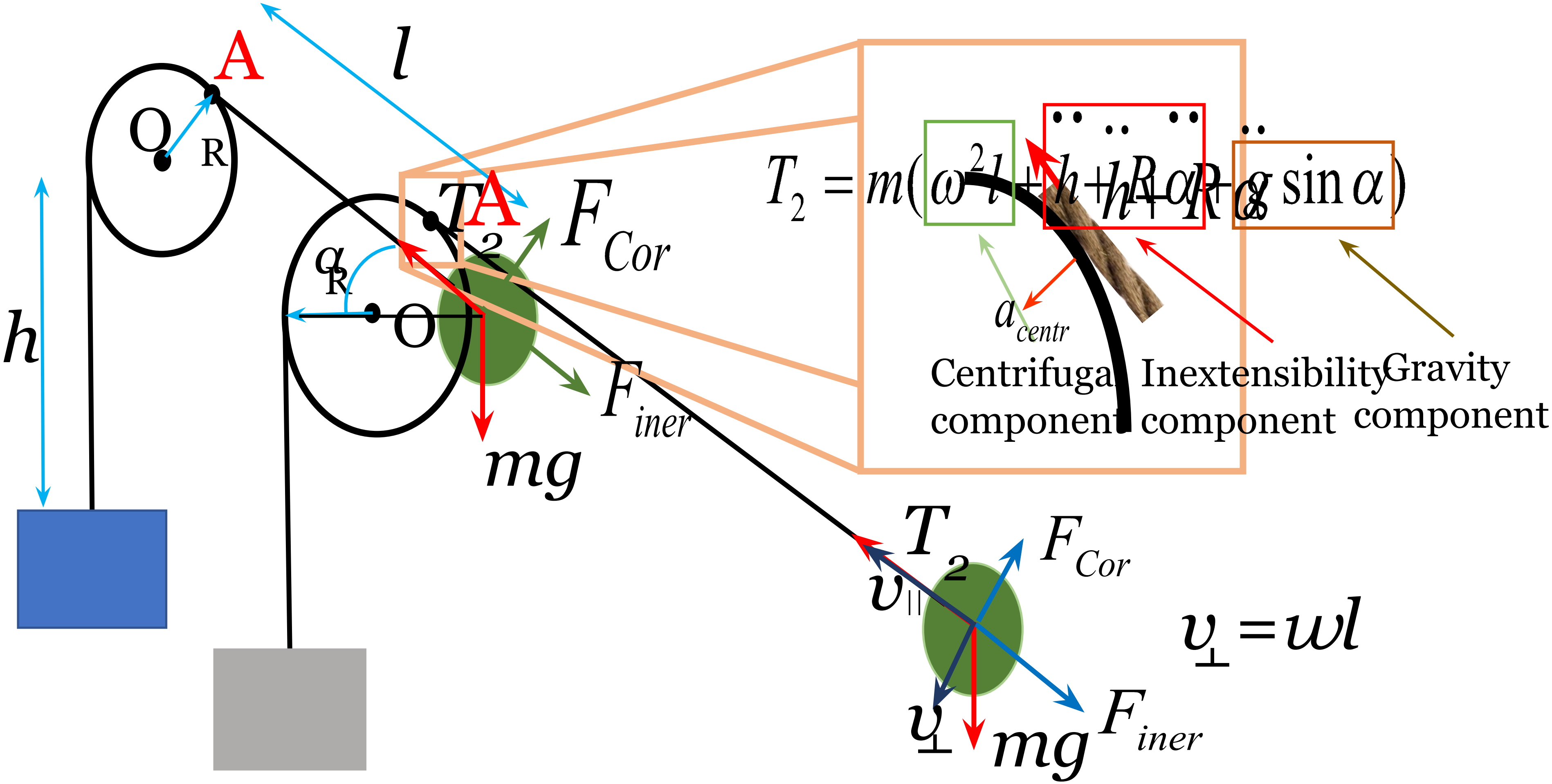
$l = 65$ cm

$\phi_0 = 90°$

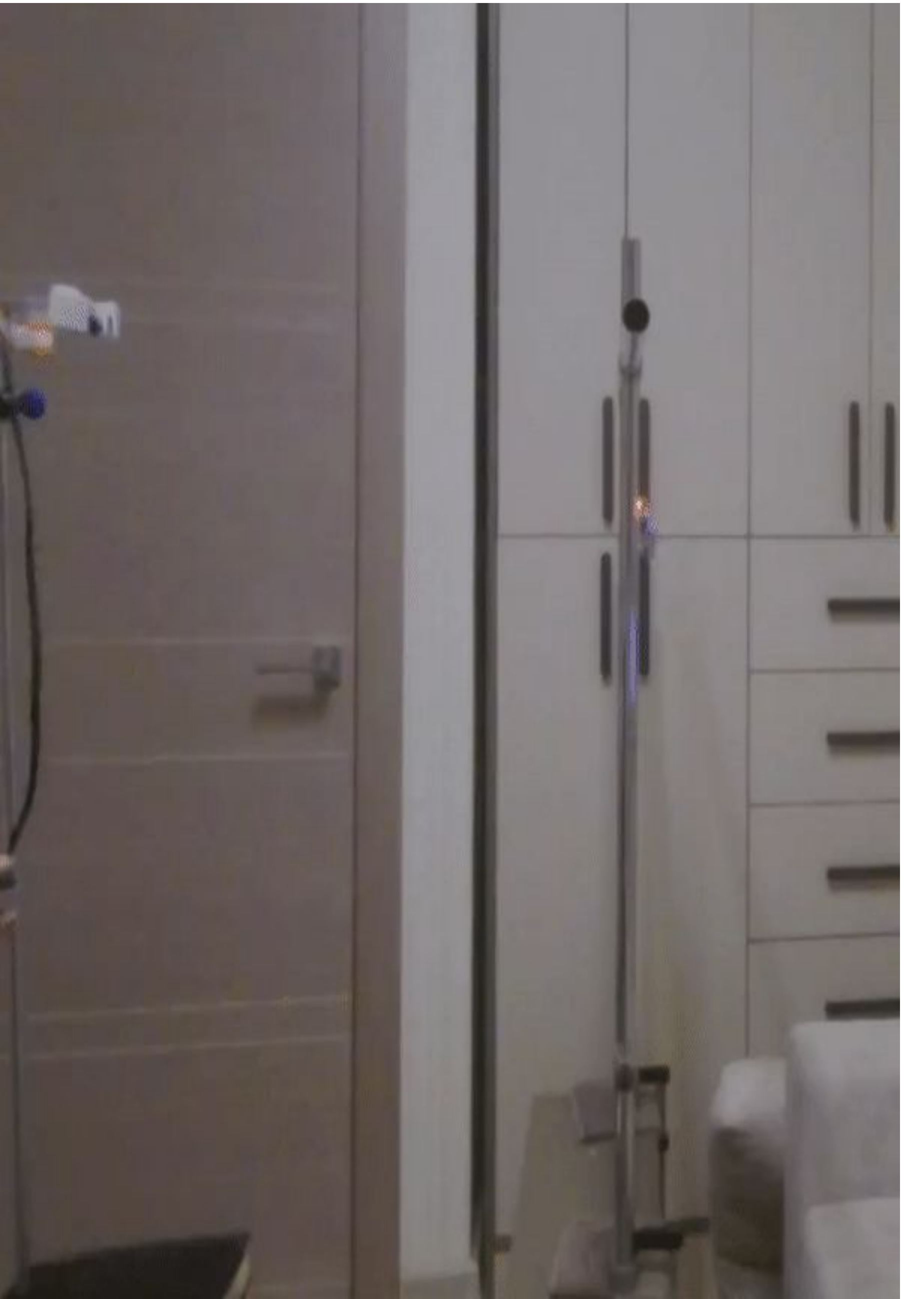


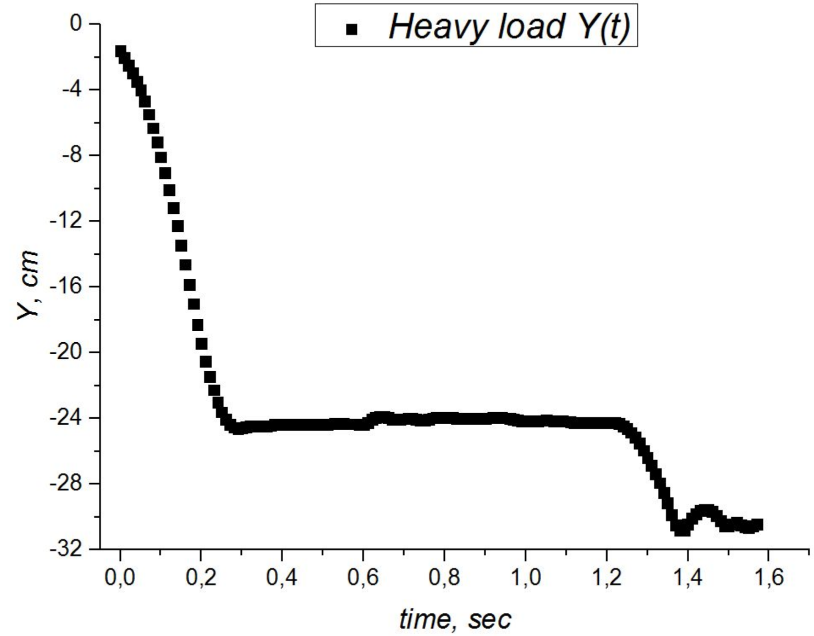
Trajectory of light load is a spiral. After heavy load stop spiral pitch becomes constant

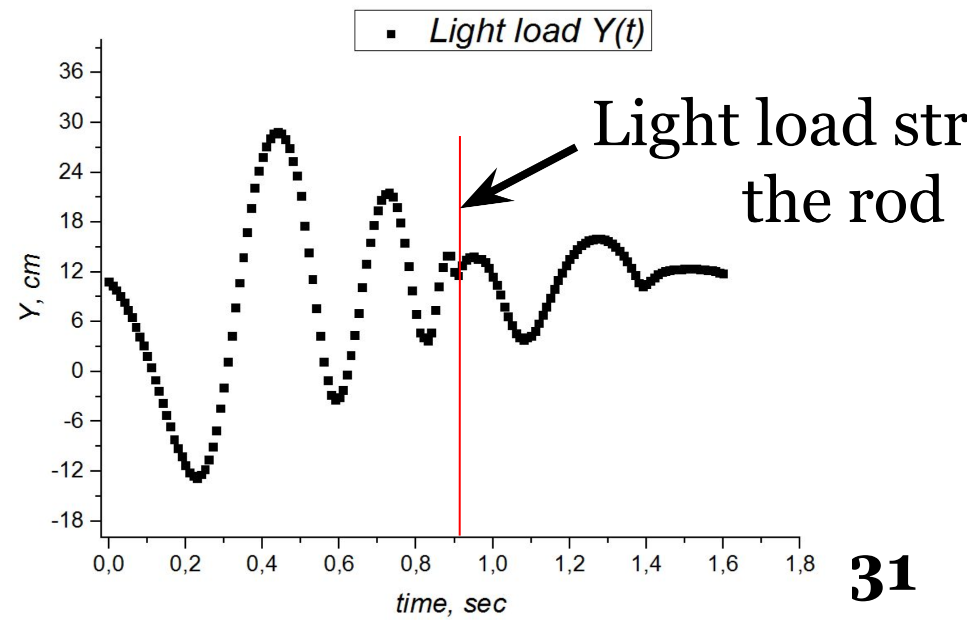
Dynamics of light load



Rod strike




Heavy load Y(t)
Y, cm
0
-4
-8
-12
-16
-20
-24
-28
-32
0,0
0,2
0,4
0,6
0,8
1,0
1,2
1,4
1,6
time, sec


Light load Y(t)
Y, cm
36
30
24
18
12
6
0
-6
-12
-18
0,0
0,2
0,4
0,6
0,8
1,0
1,2
1,4
1,6
1,8
time, sec
Light load str
the rod

Numerical solution error

$$\Delta \approx \dot{h}\, dt$$

$$Total\ error = \sum \Delta_i$$

$$Iteration\ error - 3{,}4mm$$

$$Value - 350mm$$

$$Solution\ error \approx 1\%$$

$$\Delta \approx \dot{\alpha}\, dt$$

$$Total\ error = \sum \Delta_i$$

$$Iteration\ error - 0{,}03$$

$$Value - 3{,}33$$

$$Solution\ error \approx 1\%$$

Setup scheme

Massive string №1

$$\rho_1 = (0{,}80 \pm 0{,}02)\frac{g}{m} \qquad \mu_1 = 0{,}110 \pm 0{,}004$$

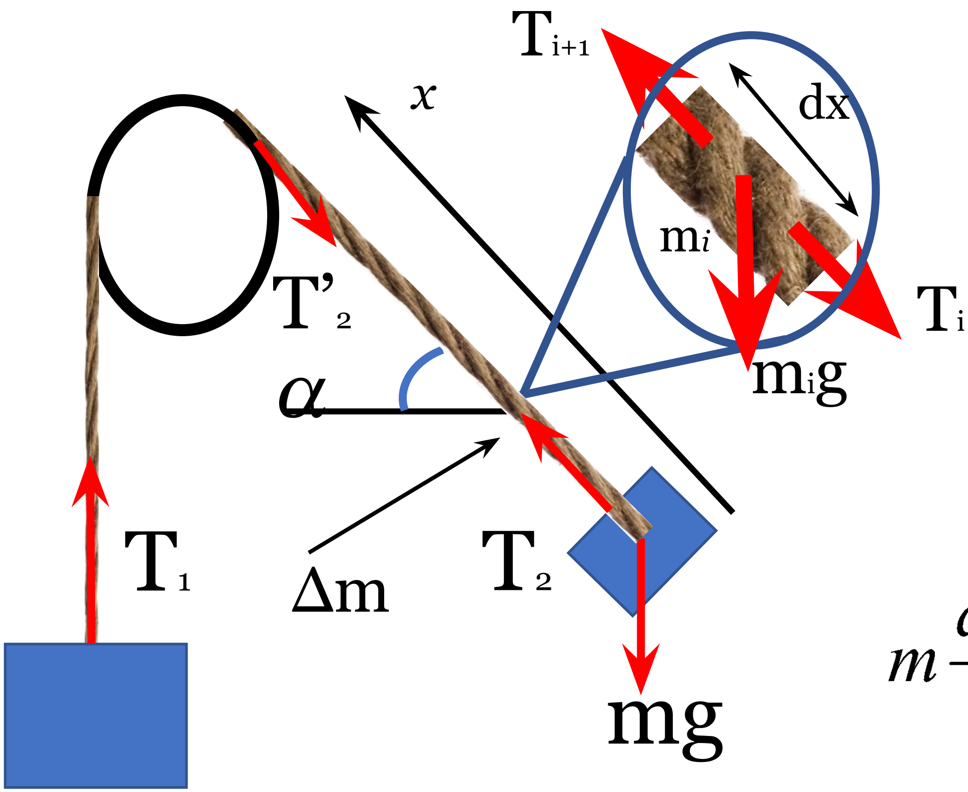
Massive string №2

$$\rho_2 = (1{,}80 \pm 0{,}02)\frac{g}{m} \qquad \mu_2 = 0{,}100 \pm 0{,}004$$

Electronic scale
measurements error = 0,01g

$\rho - linear\ density\ of\ string$

Corrections caused by massive string



$$m_i = m\frac{dx}{l}$$

$$2 - nd\ Newton\ law$$

$$m\frac{dx}{l}\omega^2 x = T_{i+1} - T_i - m\frac{dx}{l} g\sin\alpha$$

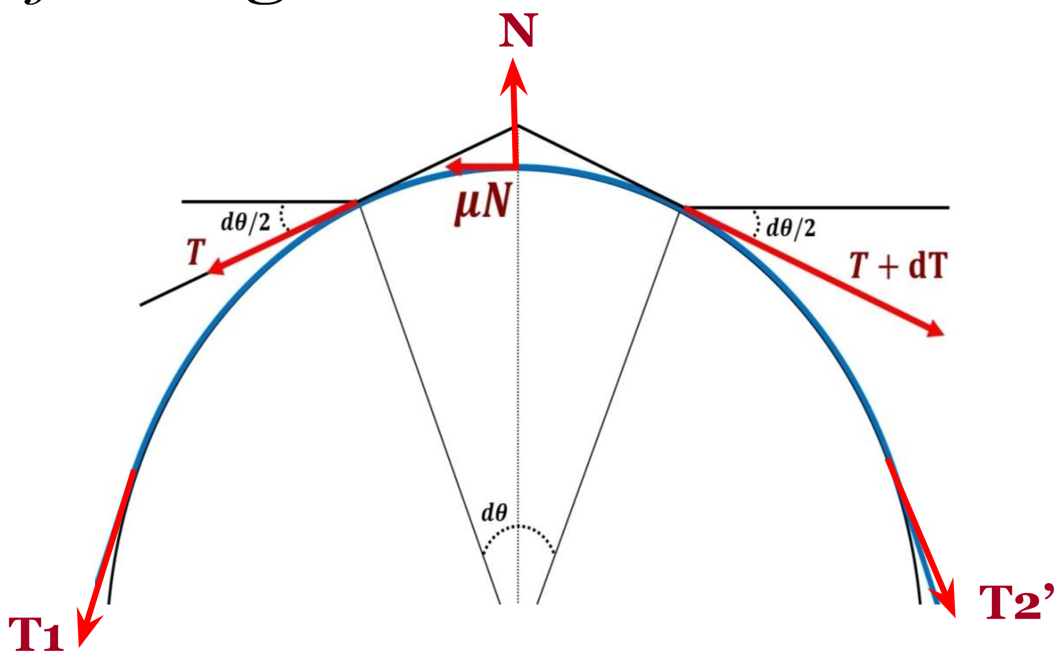
$$\boxed{\Delta T_2 = l\rho(\omega^2 l + \ddot{h} + R\ddot{\alpha} + g\sin\alpha)}$$

Corrections caused by massive string

ρ – *linear density of string*

~~ВЫВОД ФОРМУЛЫ~~

$$\boxed{\frac{T_1 - \rho v^2}{T_2' - \rho v^2} = e^{\mu\theta}}$$

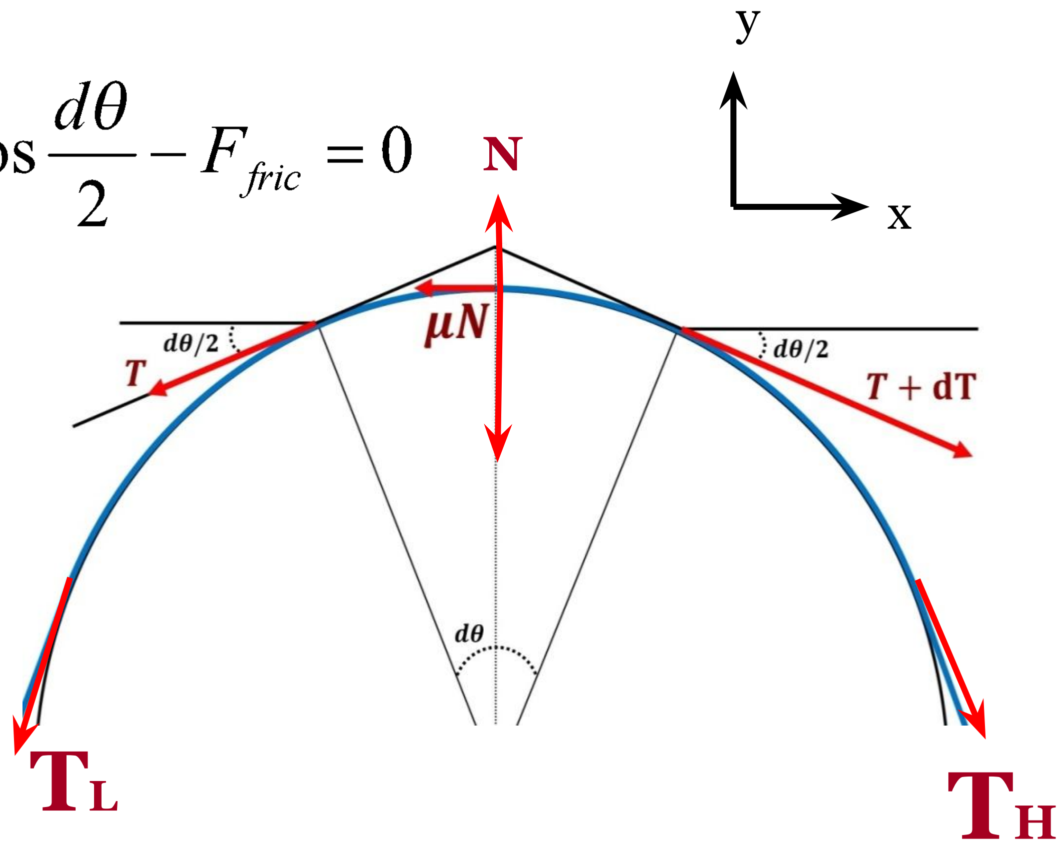


D. J. Dunn 2005
«Solid mechanics. Dynamics. Tutorial – pulley drive system»

Correction in Euler's formula caused by massive string

$$Y\ axis: N + \rho\, d\theta\, v^2 = T d\theta$$

$$X\ axis: (T + dT)\cos\frac{d\theta}{2} - T\cos\frac{d\theta}{2} - F_{fric} = 0$$



$$N = T d\theta - \rho\, d\theta\, v^2$$

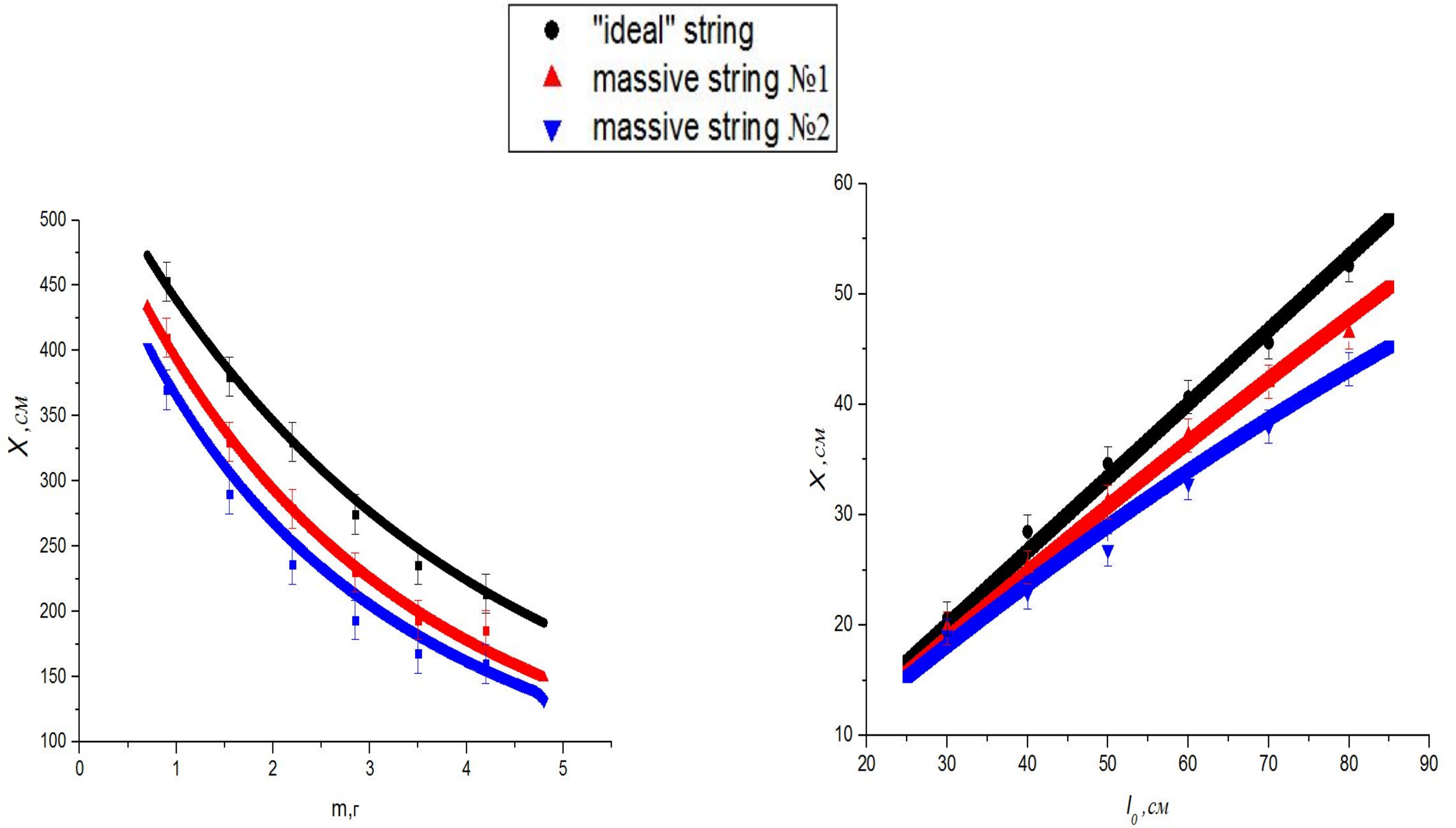
$$dT = F_{fric} = \mu N$$

$$dT = \mu d\theta \left(T - \rho\, v^2\right)$$

$$\boxed{\frac{T_1 - \rho v^2}{T_2^{'} - \rho v^2} = e^{\mu\theta}}$$

$\rho - linear\ density\ of\ string$

Comparing theory with experiment for massive string

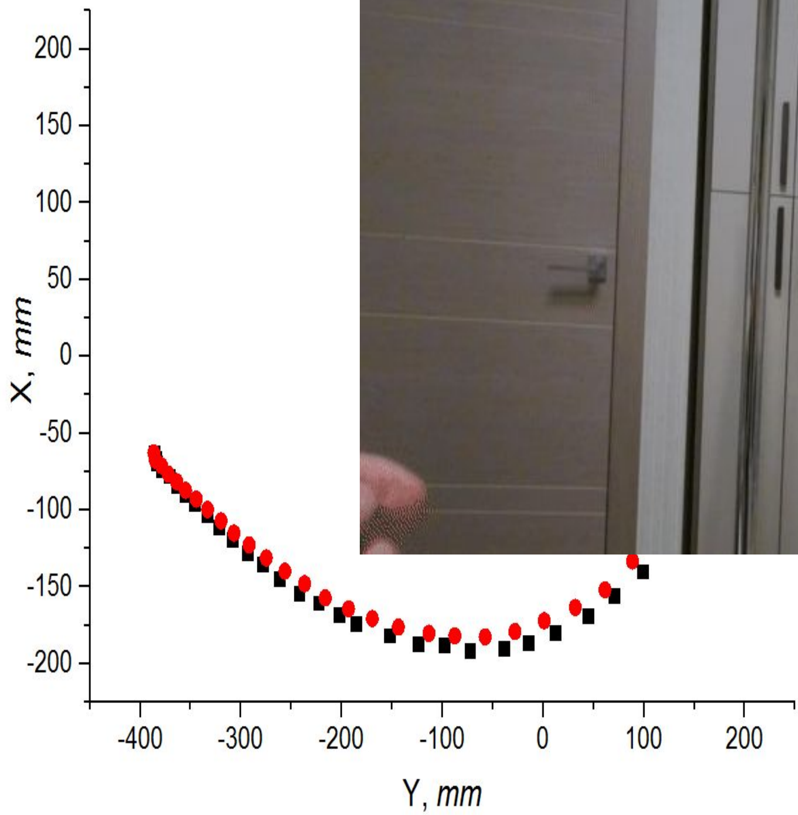


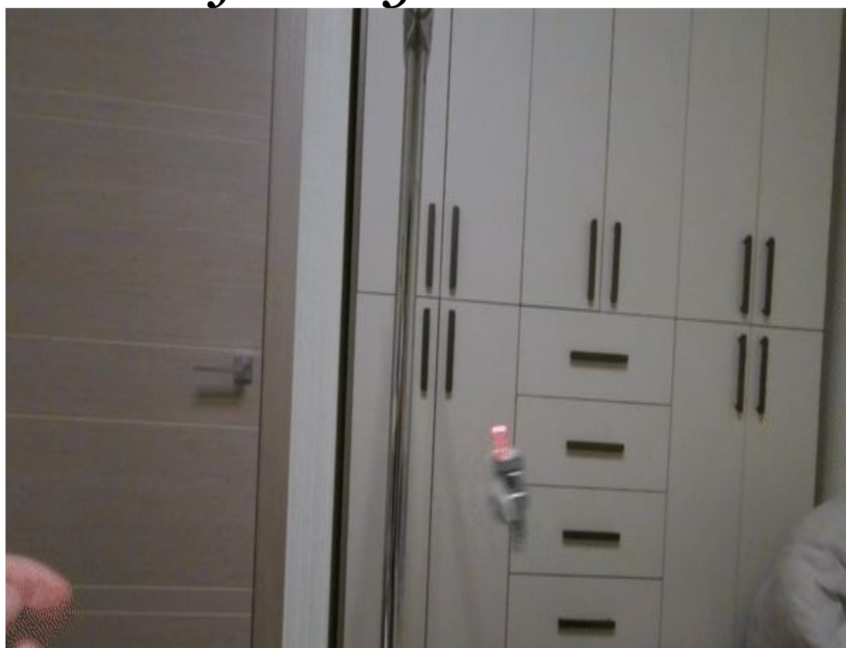
The theory agrees with the experiment!
The greater the mass of the thread, the smaller the value of X

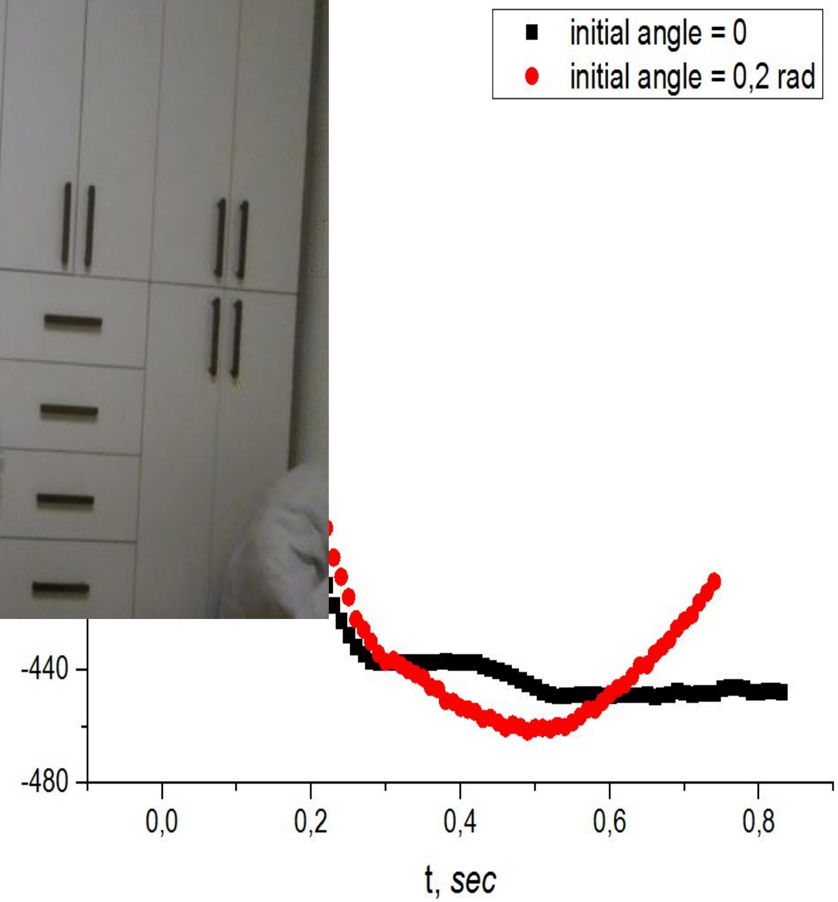
Swinging heavy load

Light load trajectory

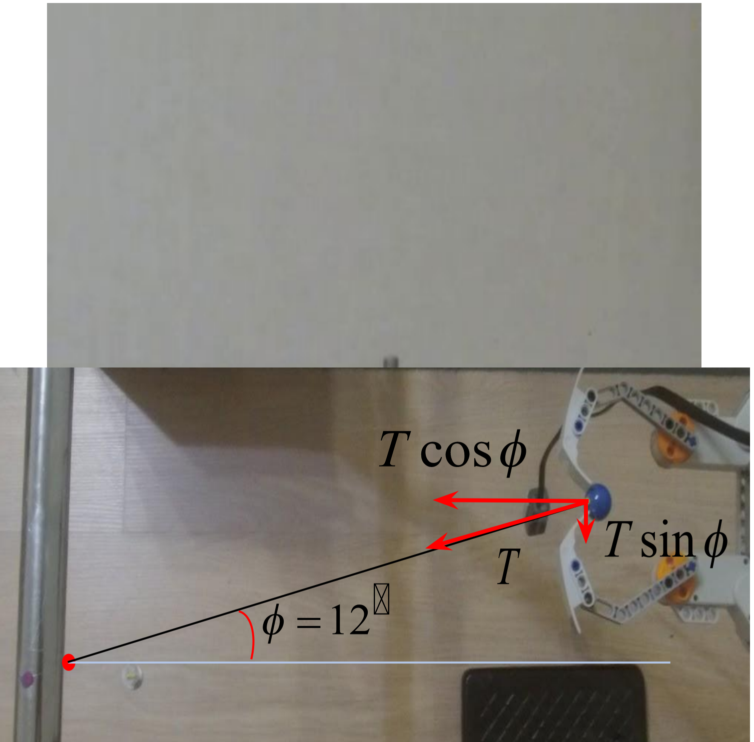
Heavy load Y(t)

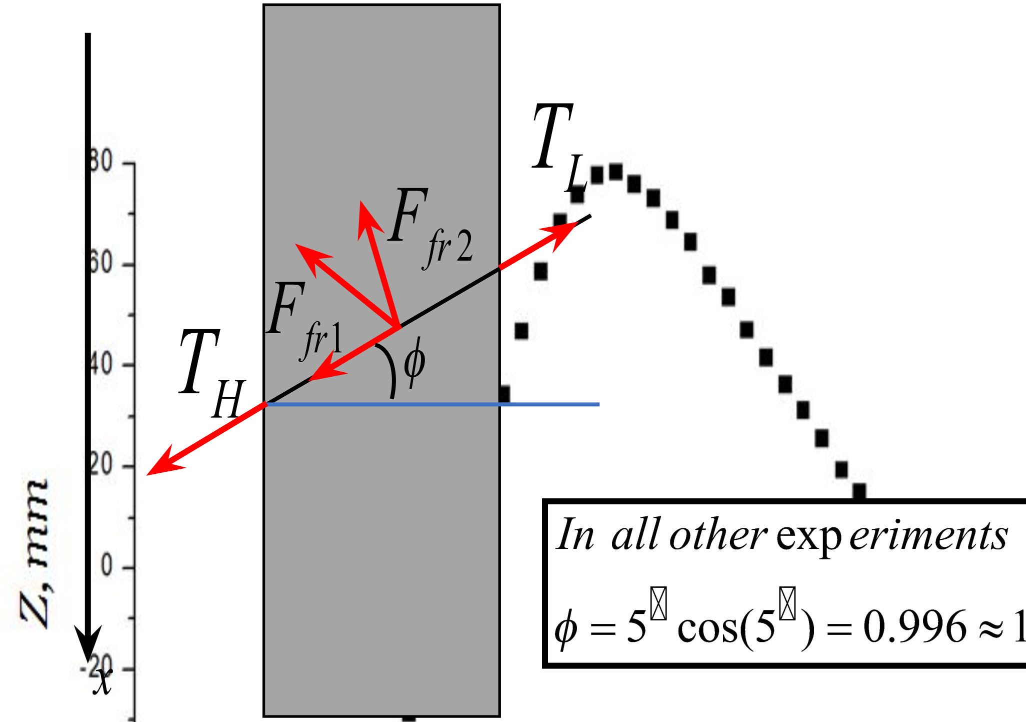






3 - dimensional movement





2 – nd Newton law in projection on x axe :

$$T_h\left(1-e^{-\mu\frac{\pi}{2}}\right)\cos^2\left(2\phi_{\max}\right) = T_h e^{-\mu\frac{\pi}{2}}\sin\phi_{\max}$$

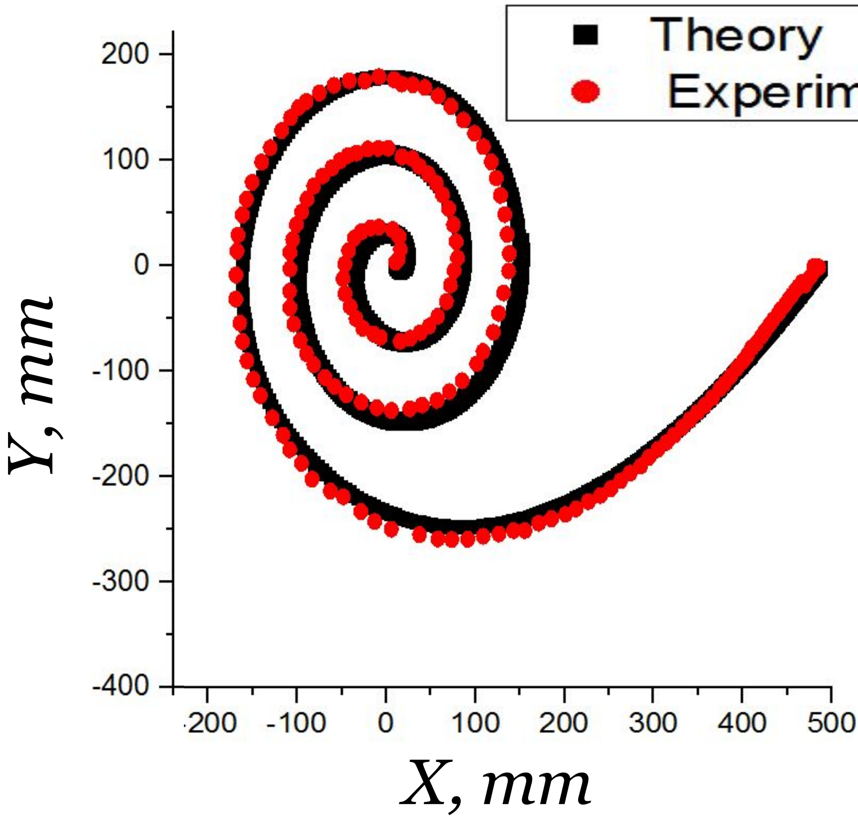
$$\sin\phi_{\max} = 0.17 \quad \phi_{\max} \approx 11^\circ$$

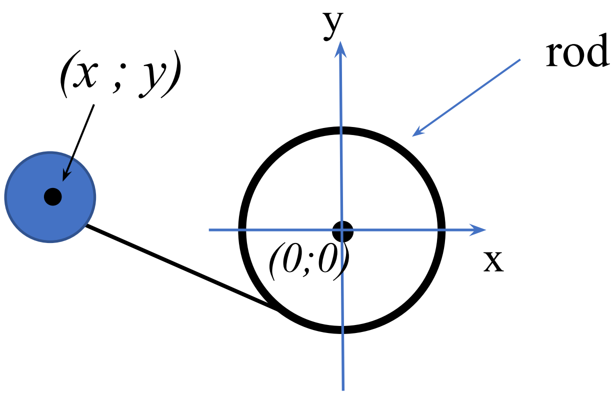
Maximal angle φ can be predicted very well

Mainly problem can be solved as 2-dimensional

Qualitative explanation ✔ | Quantitative model ● | Parametric investigation

Light load trajectory





$$\max\left(\frac{X_{teor}}{X_{prac}}\right) = 5.5\%$$

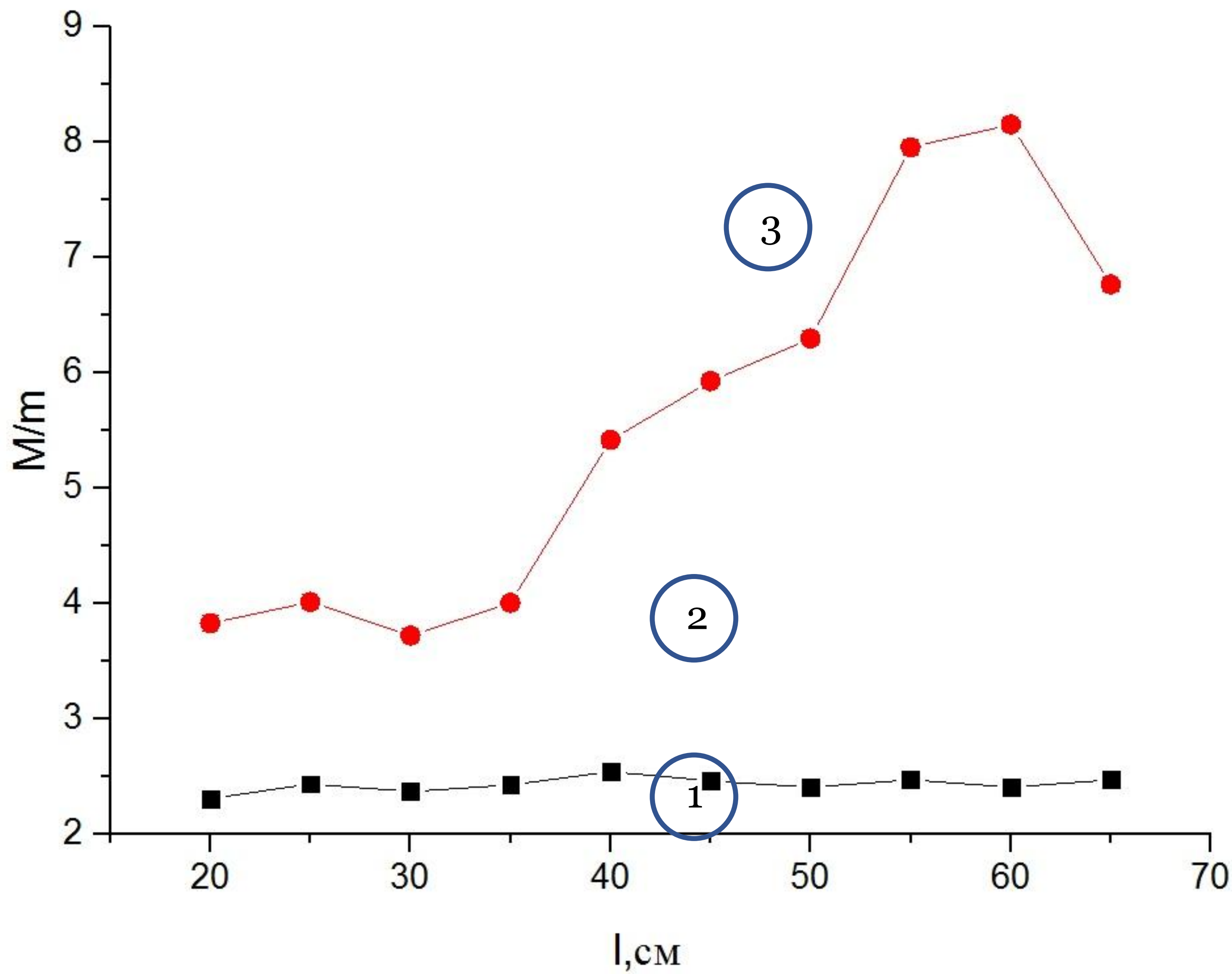
$$\max\left(\frac{Y_{teor}}{Y_{prac}}\right) = 4.5\%$$

M=18 g
m=3 g
l_0=50 cm
α=0

It's good agreement between theory and experiment

Qualitative explanation ✓ | Quantitative model ✓ | Parametric investigation •

Режимы запусков



Setup scheme (переделать)

Горизонтальный стержень

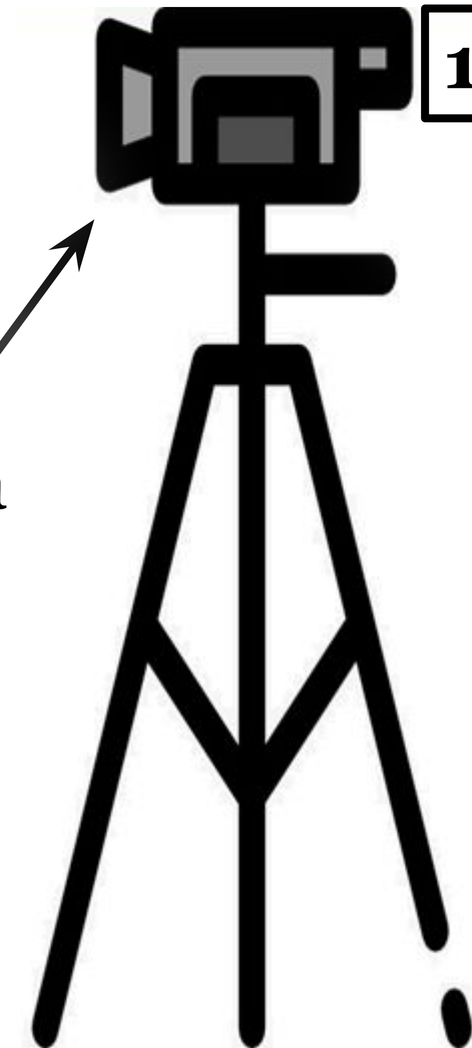
Лёгкий грузик

Рыболовная леска

Опора

Тяжёлый груз

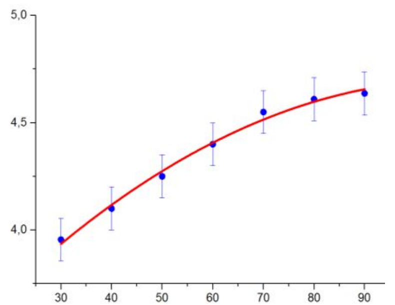
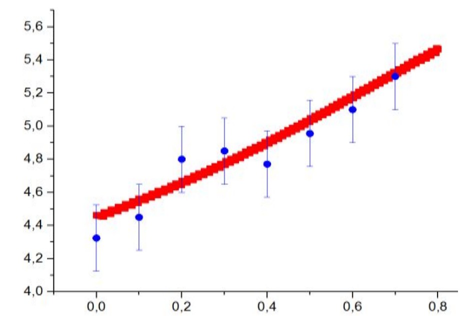
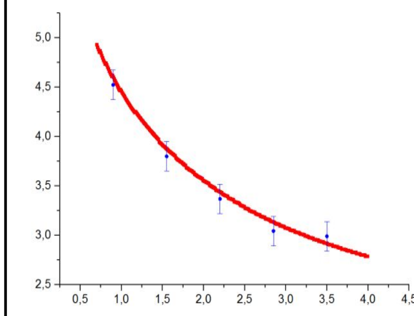
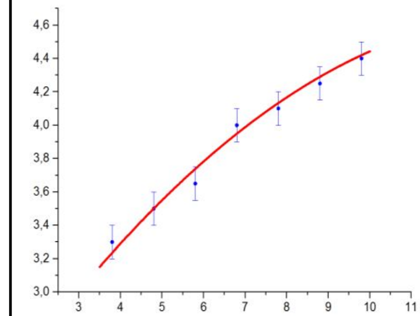
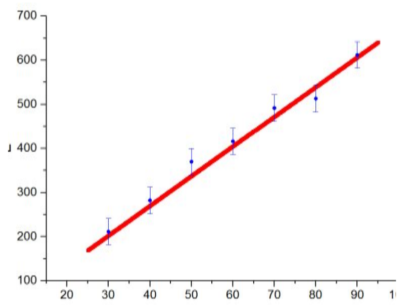
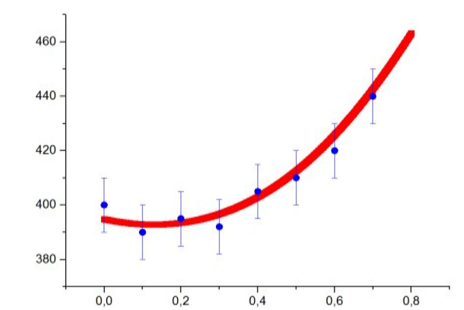
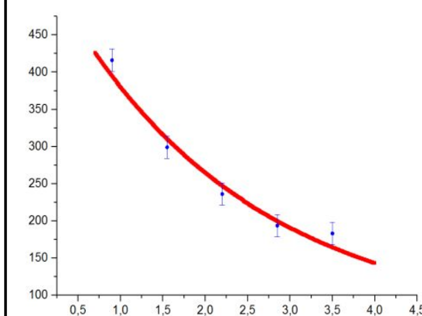
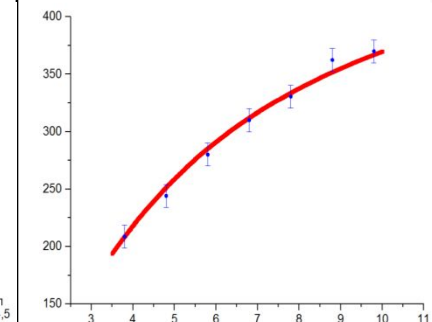
Камера



100 fps

Светодиод

Качественное объяснение | Мат. модель | **Параметрическое исследование**

	l_{0}, *cm*	α_{0}, *rad*	m, *g*	M, *g*
ϕ, *rad*				
H, *mm*				
t, *sec*