BINOMIAL THEOREM

- **BINOMIAL THEOREM.** THIS IS THE FORMULA THAT REPRESENTS THE EXPRESSION $(a+b)^n$ FOR A POSITIVE INTEGER n AS A POLYNOMIAL:

$$(a+b)^n = a^n + C_n^1 a^{n-1} b + C_n^2 a^{n-2} b^2 + C_n^3 a^{n-3} b^3 + \ldots + C_n^{n-1} a b^{n-1} + b^n .$$

- NOTE THAT THE SUM OF THE EXPONENTS OF a AND b IS CONSTANT AND EQUAL TO n.

$$C_n^k = \binom{n}{k} = \frac{n!}{k!\,(n-k)!}$$ - BINOMIAL COEFFICIENTS, n - NON-NEGATIVE INTEGER

EXAMPLE:

FROM THE SET n $\{1,2,3,4\}$, SELECT ALL THE POSSIBLE COMBINATIONS OF TWO ELEMENTS, $k = \{1,2\}\ \{1,3\}\ \{1,4\}\ \{2,3\}\ \{2,4\}\ \{3,4\}$ IT TURNS OUT SIX OPTIONS. SUBSTITUTING VALUES INTO THE FORMULA, WE CHECK THE RESULT: BINOMIAL COEFFICIENT EXAMPLE

$$C_4^2 = \frac{4!}{2!\,(4-2)!} = 6$$

THE PROPERTIES OF BINOMIAL COEFFICIENTS

1. THE SUM OF THE COEFFICIENTS OF EXPANSION $(a + b)^n$ IS EQUAL TO 2^n.

IT IS SUFFICIENT TO PUT $a = b = 1$. THEN THE RIGHT SIDE OF NEWTON'S BINOMIAL EXPANSION WE WILL HAVE *a* SUM OF BINOMIAL COEFFICIENTS, AND ON THE LEFT:

$$(1+1)^n = 2^n.$$

2. THE COEFFICIENTS MEMBERS EQUIDISTANT FROM THE ENDS OF THE EXPANSION, ARE EQUAL.

THIS PROPERTY FOLLOWS FROM THE RELATION:

$$C_n^k = C_n^{n-k}.$$

3. THE AMOUNT OF THE EVEN TERMS IN THE EXPANSION COEFFICIENT EQUAL TO THE SUM OF THE ODD TERMS IN THE EXPANSION COEFFICIENTS; EACH OF THEM IS

$$2^{n-1}.$$

TO PROVE THIS WE USE THE BINOMIAL: $(1-1)^n = 0^n = 0.$
HERE THE EVEN MEMBERS ARE «+» SIGN, AND THE ODD - «−». AS A RESULT TURNS DECOMPOSITION 0, THEREFORE, THE AMOUNT OF BINOMIAL COEFFICIENTS ARE EQUAL TO EACH OTHER, SO EACH OF THEM IS: $2^n : 2 = 2^{n-1}$ GET TO PROVE.

- **EXAMPLES OF USING THE BINOMIAL OF NEWTON**

EXAMPLE 1. ARRANGE BEAN [illegible] IN POWERS OF x.

WE USE THE BINOMIAL THEOREM: [illegible]

THE VALUES OF THE BINOMIAL COEFFICIENTS CONSISTENTLY FIND THE FORMULA [illegible]

FOR EXAMPLE

[illegible]

[illegible]

EXAMPLE 2. PROVE FORMULA [illegible]

SUBSTITUTING IN THE FORMULA FOR EXPANSION [illegible] VALUE [illegible]

GET THE DESIRED RESULT.

EXAMPLE 3. PROVE RATIO [illegible]

WE USE THE RECURRENCE RELATION [illegible]

[illegible]

THANK YOU FOR ATTENTION