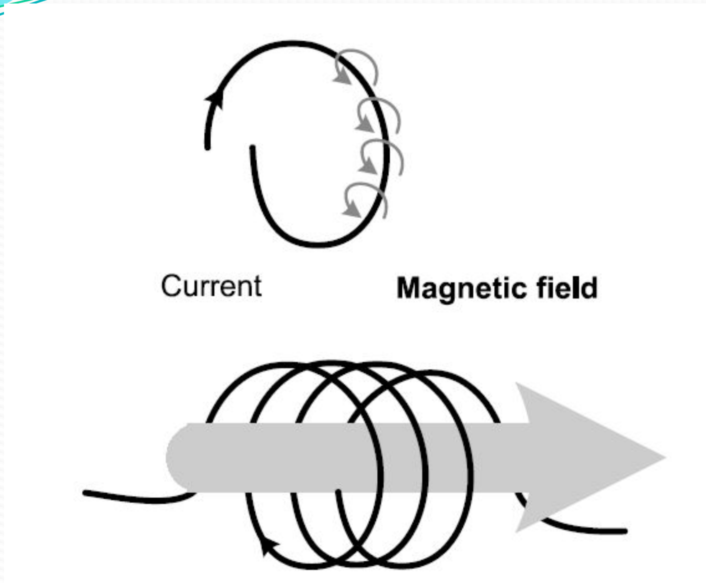
General terms of transmission lines performance and simulation

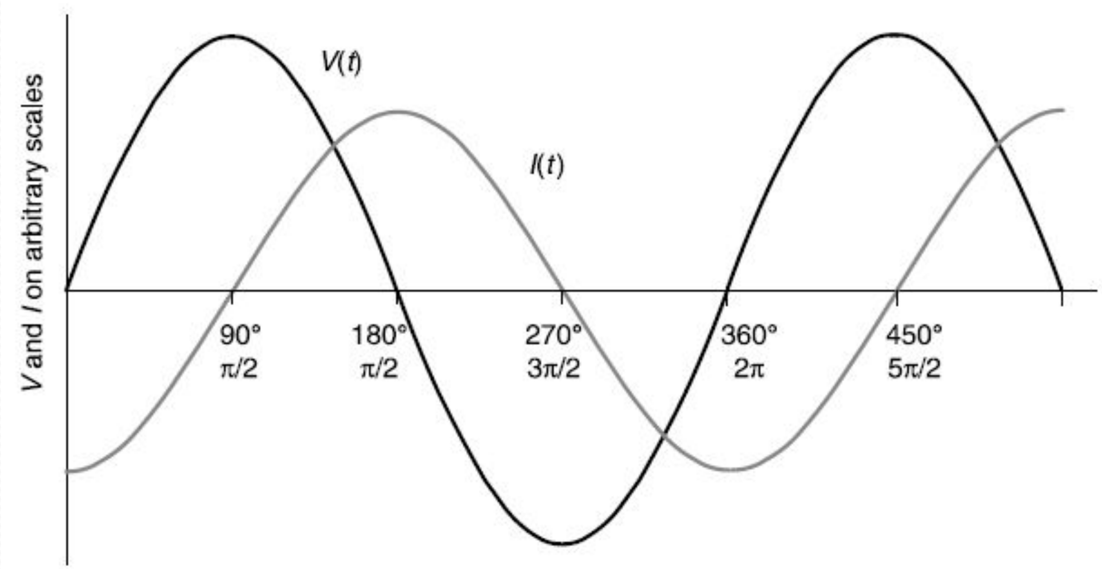
Prof. Evgeniy (Eugen) SHESKIN
SPbPU, Institute of Energy and Transport Systems

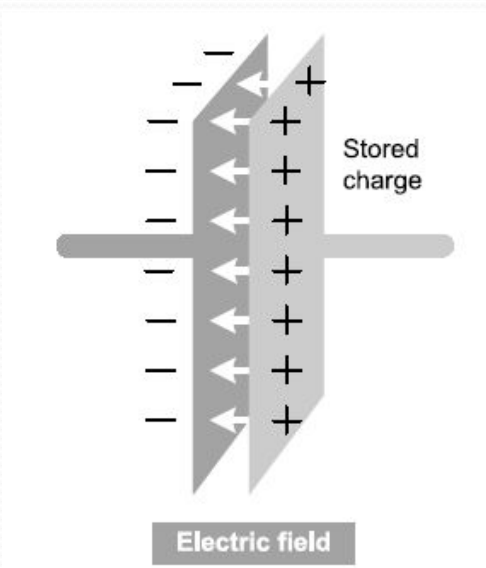
Topics

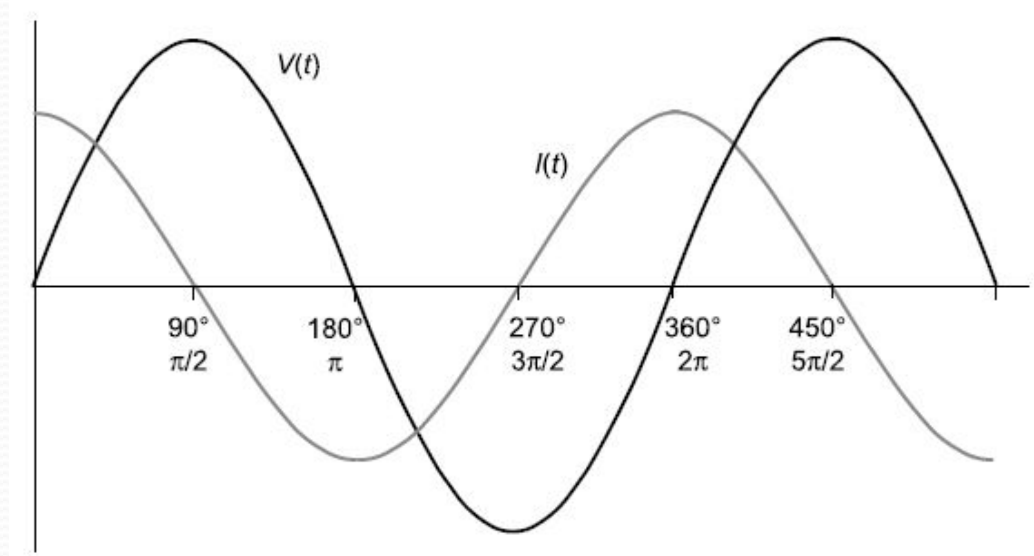
1. Power Systems Structure and Basic Elements
2. AC Transmission Lines Modeling
3. Classification Of Transmission Lines
4. Typical Parameters Of Transmission Lines
5. AC Transmission Lines Performance In No-Load Modes
6. AC Transmission Lines Performance Under Load Conditions
7. Power Transfer and Stability Considerations
8. Reactive Power Demand
9. Tasks

1. Basic Circuit Elements

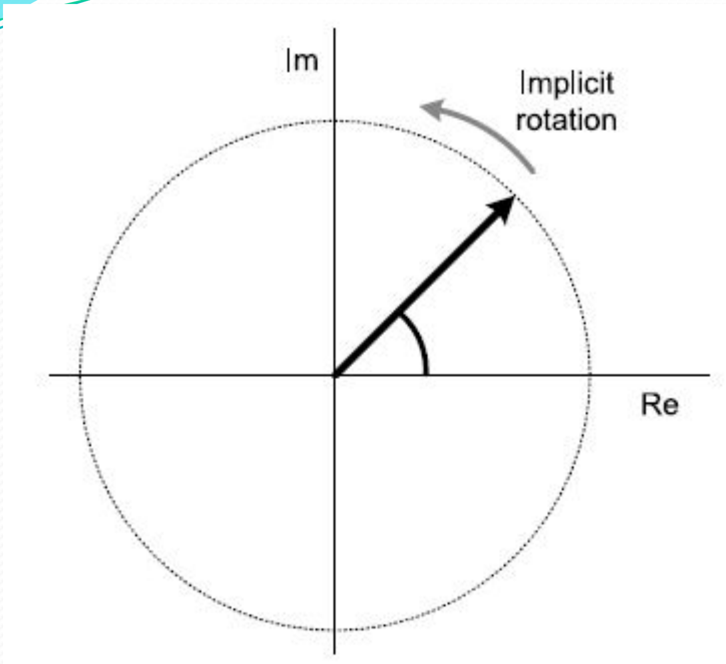




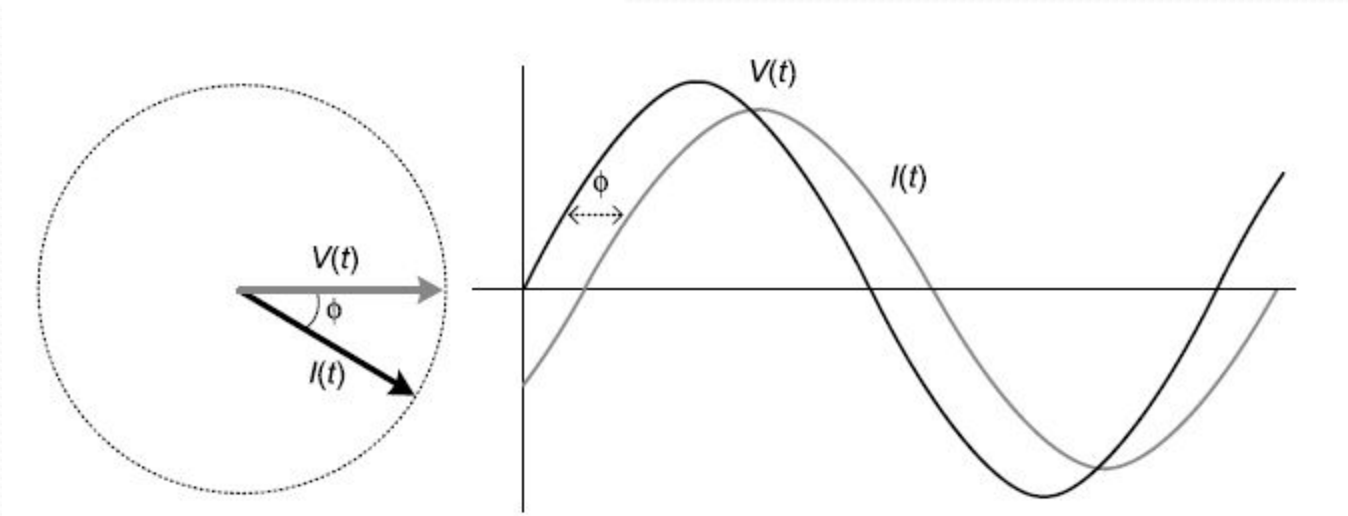




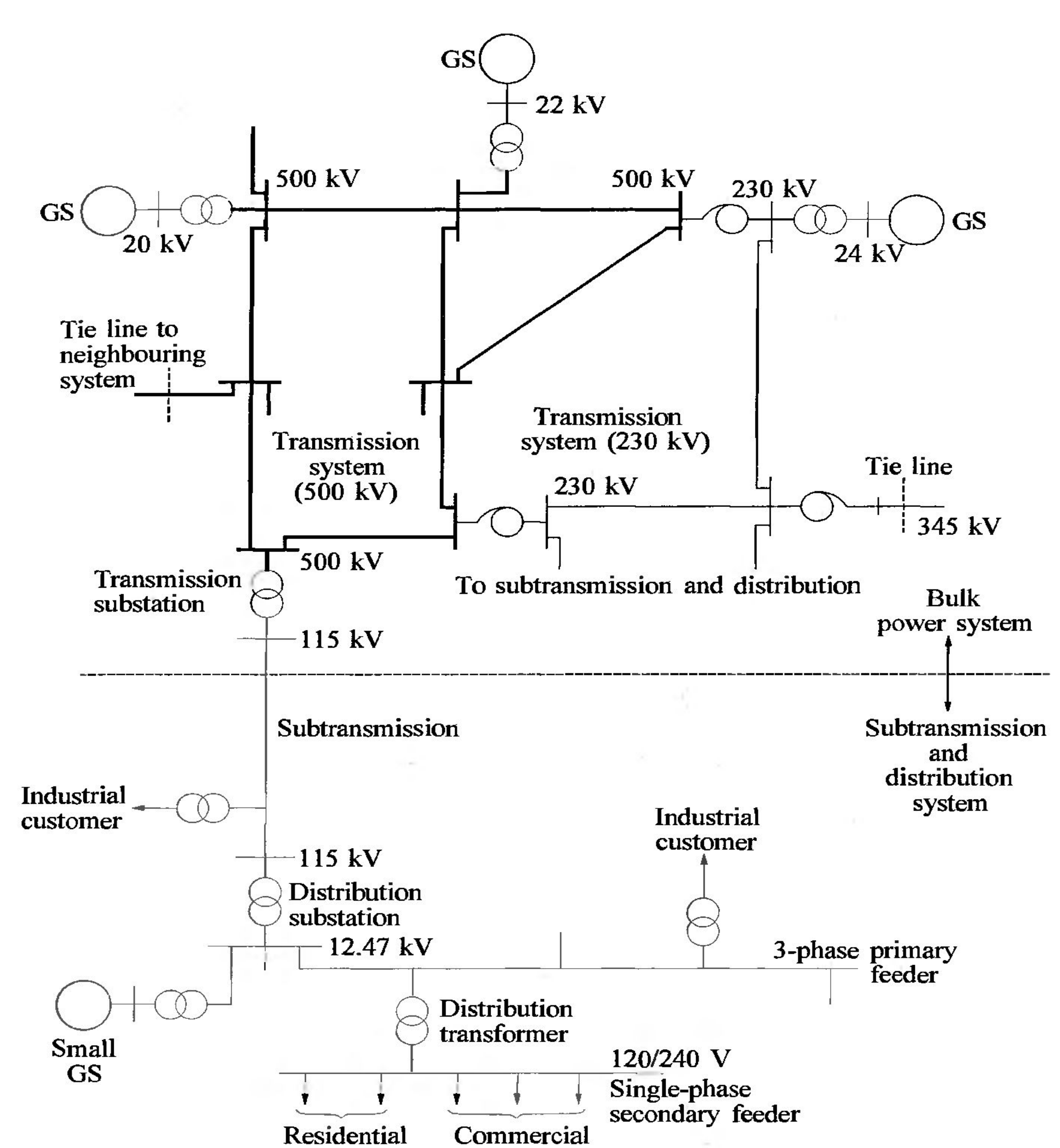
1. Phasor Notation



- sinusoidally varying voltage is represented as an arrow of constant length, spinning around at the constant frequency ω;
- we can ignore this circular spinning to the extent that it will be the same for all quantities, and <u>they are not spinning in relation to each other</u> (only when *f* = const!).

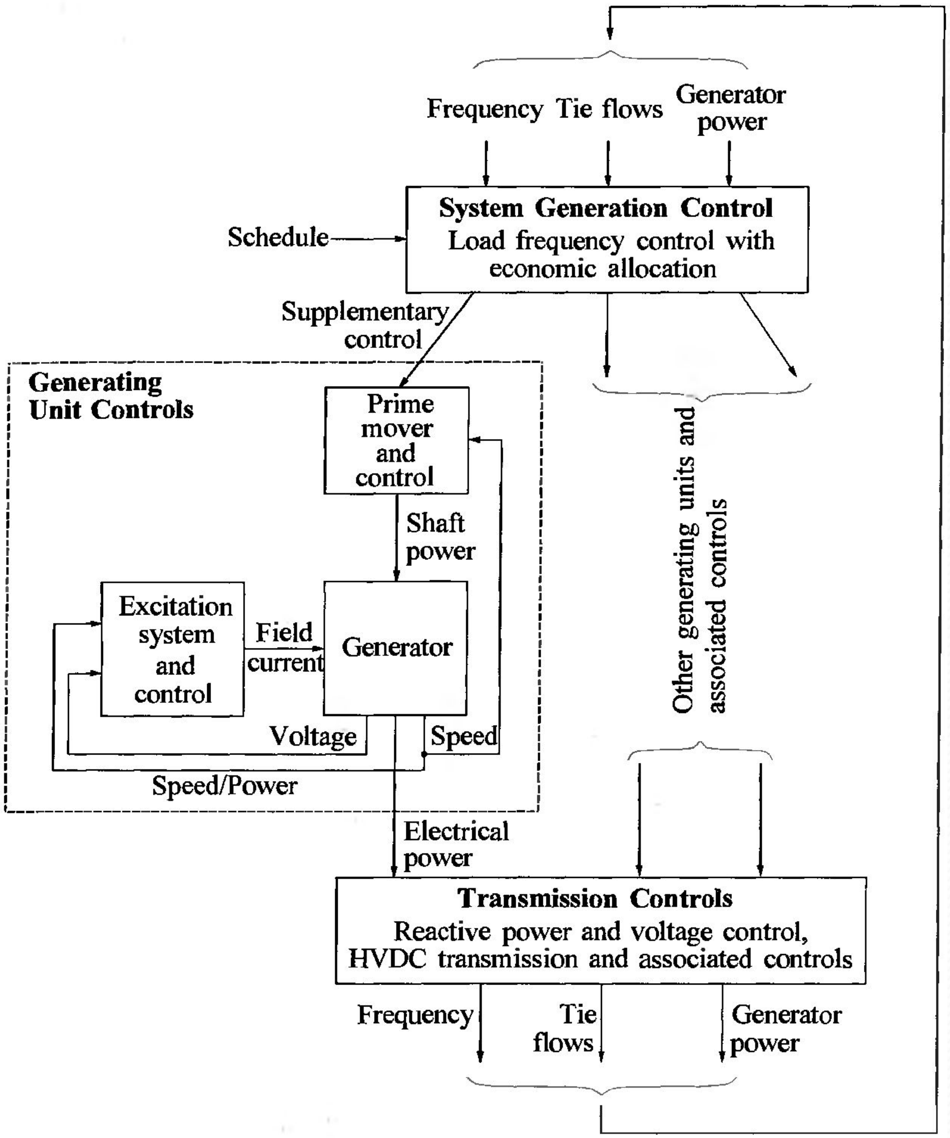


1. Power System Structure



Main elements:
1. Generators;
2. Transformers;
3. Transmission lines.

1. Control Structure



Things we can control:

- Power flows;
- System frequency;
- Node voltages.

2. AC Transmission Lines Modeling

- To develop performance equations and models for transmission lines;
- To examine the power transfer capabilities of transmission lines as influenced by voltage, reactive power, and system stability considerations;
- To examine factors influencing the flow of active power and reactive power through transmission networks;
- To describe analytical techniques for the analysis of power flow in transmission systems.

2. AC Transmission Lines Modeling

- *Series Resistance (R)*. The resistances of lines accounting for stranding and skin effect are determined from manufacturers' tables.
- *Shunt Conductance (G)*. The shunt conductance represents losses due to leakage currents along insulator strings and corona. In power lines, its effect is small and usually neglected.
- *Series Inductance (L)*. The line inductance depends on the partial flux linkages within the conductor cross section and external flux linkages

$$L = 2\times10^{-7}\ln\frac{D_{eq}}{D_s} \quad \text{H/m}$$

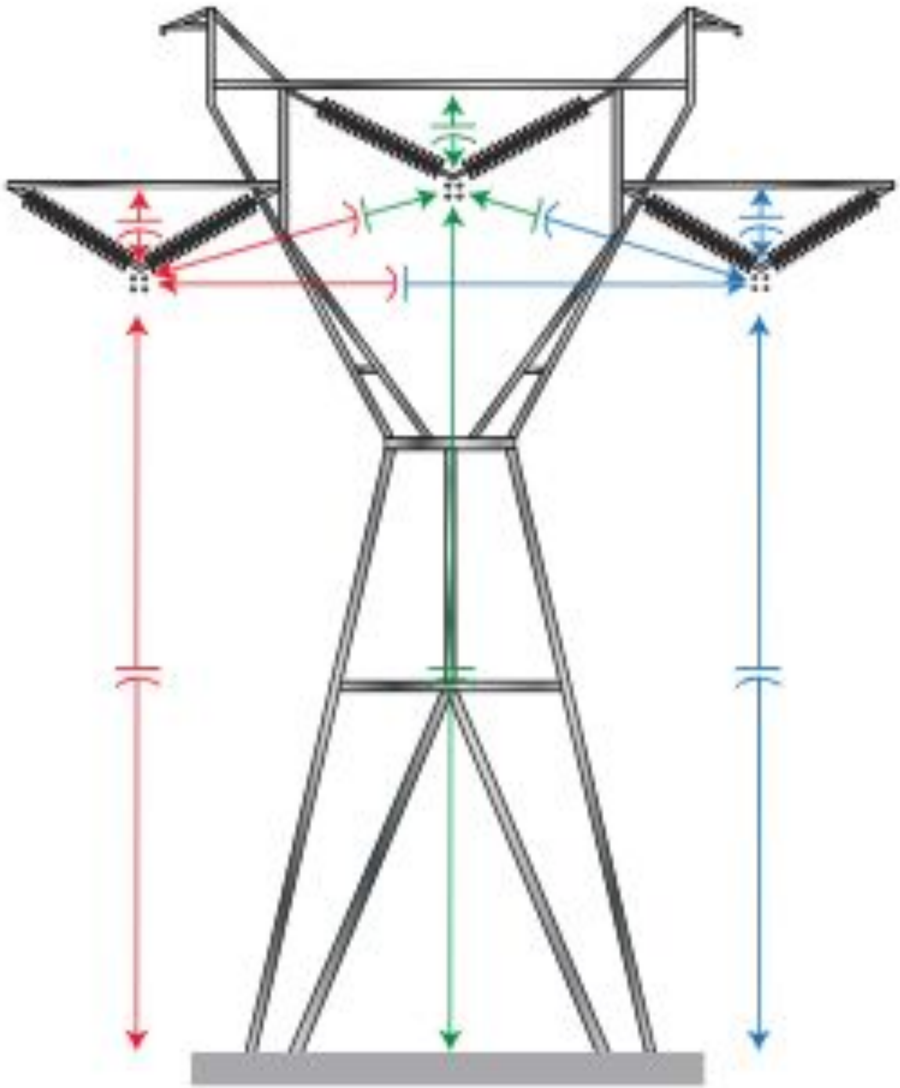
$$D_{eq} = (d_{ab}d_{bc}d_{ca})^{1/3}$$

- *Shunt Capacitance (C).* The potential difference between the conductors of a transmission line causes the conductors to be charged; the charge per unit of potential difference is the capacitance between conductors

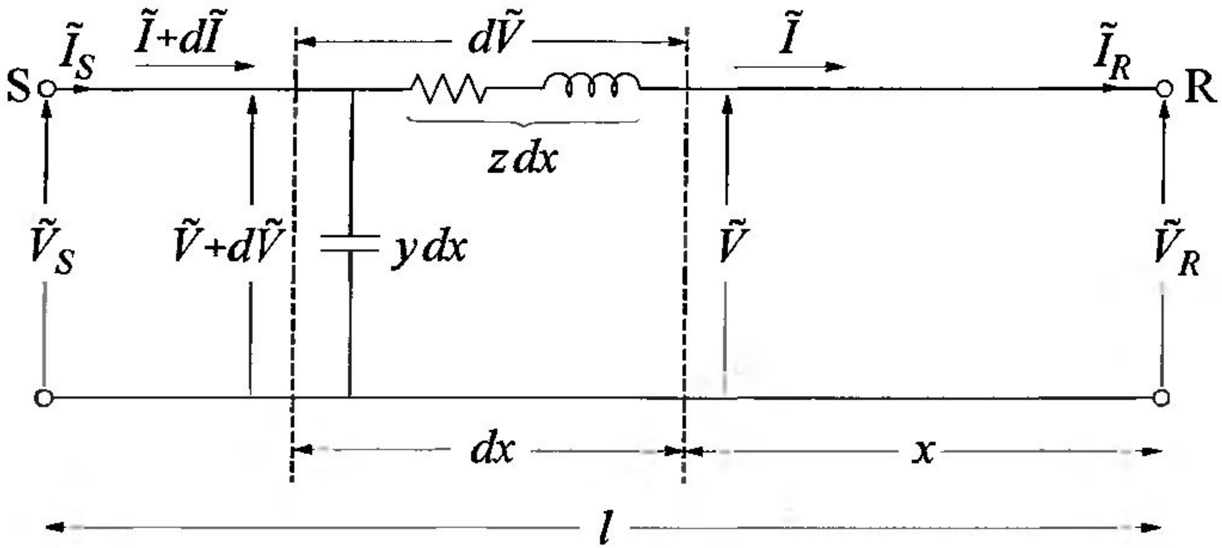
$$C = \frac{2\pi k}{\ln(D_{eq}/r)} \quad \text{F/m}$$

2. AC Transmission Lines Modeling

- AC transmission line tower construction defines its electric parameters.
- AC transmission line electric parameters define its performance in various under-voltage conditions.



2. AC Transmission Lines Modeling



$$d\tilde{V} = \tilde{I}(z\,dx)$$

$$\frac{d\tilde{V}}{dx} = \tilde{I}z$$

$$d\tilde{I} = \tilde{V}(y\,dx)$$

$$\frac{d\tilde{I}}{dx} = \tilde{V}y$$

$z = R + j\omega L$ = series impedance per unit length/phase
$y = G + j\omega C$ = shunt admittance per unit length/phase
l = length of the line

$$\frac{d^2\tilde{V}}{dx^2} = z\frac{d\tilde{I}}{dx} = yz\tilde{V}$$

$$\frac{d^2\tilde{I}}{dx^2} = y\frac{d\tilde{V}}{dx} = yz\tilde{I}$$

$$\tilde{V} = \frac{\tilde{V}_R+Z_C\tilde{I}_R}{2}e^{\gamma x} + \frac{\tilde{V}_R-Z_C\tilde{I}_R}{2}e^{-\gamma x}$$

$$\tilde{I} = \frac{\tilde{V}_R/Z_C+\tilde{I}_R}{2}e^{\gamma x} - \frac{\tilde{V}_R/Z_C-\tilde{I}_R}{2}e^{-\gamma x}$$

2. AC Transmission Lines Modeling

- The constant Z_c is called the *characteristic impedance* and γ is called the *propagation constant*.
- The constants γ and Z_c are complex quantities. The real part of the propagation constant γ is called the *attenuation constant* α, and the imaginary part the *phase constant* β.

$$Z_C = \sqrt{z/y}$$

$$\gamma = \sqrt{yz} = \alpha + j\beta$$

$$e^{\gamma x} = e^{(\alpha + j\beta)x} = e^{\alpha x}(\cos\beta x + j\sin\beta x)$$

$$e^{-\gamma x} = e^{-\alpha x}(\cos\beta x - j\sin\beta x)$$

$$\tilde{V} = \tilde{V}_R\cos\beta x + jZ_C\tilde{I}_R\sin\beta x$$

$$\tilde{I} = \tilde{I}_R\cos\beta x + j(\tilde{V}_R/Z_C)\sin\beta x$$

2. AC Transmission Lines Modeling

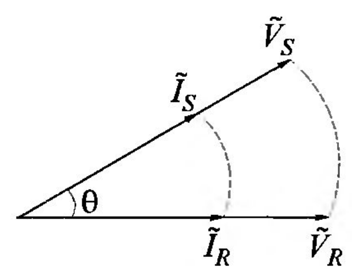
- The power delivered by a transmission line when it is terminated by its surge impedance is known as the *natural load* or *surge impedance load* (SIL).

$$\mathbf{SIL} = \frac{V_0^2}{Z_C} \quad \mathrm{W}$$

$$\tilde{V} = \tilde{V}_R e^{\gamma x}$$

$$\tilde{I} = \tilde{I}_R e^{\gamma x}$$

- *V* and *I* have constant amplitude along the line.
- *V* and *I* are in phase throughout the length of the line.
- The phase angle between the sending end and receiving end voltages (currents) is equal to θ.



$\theta = \beta l$
l = line length

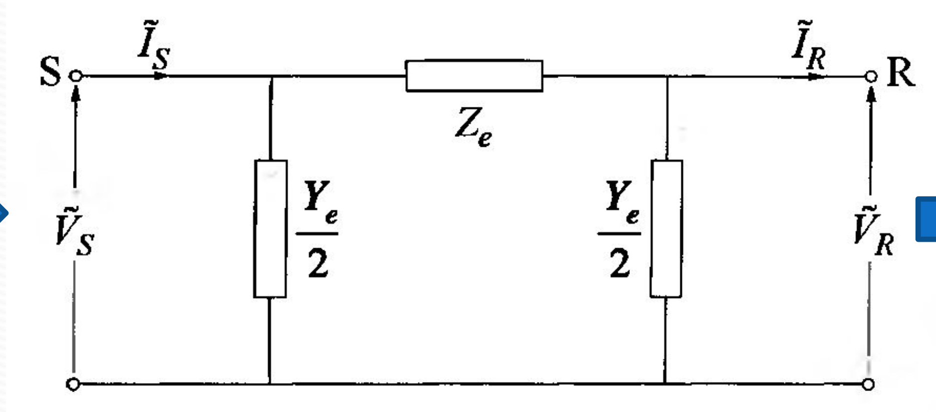
2. AC Transmission Lines Modeling

We are letting $x = l$

$$\tilde{V}_S = \tilde{V}_R \frac{e^{\gamma l} + e^{-\gamma l}}{2} + Z_C \tilde{I}_R \frac{e^{\gamma l} - e^{-\gamma l}}{2}$$

$$= \tilde{V}_R \cosh(\gamma l) + Z_C \tilde{I}_R \sinh(\gamma l)$$

$$\tilde{I}_S = \tilde{I}_R \cosh(\gamma l) + \frac{\tilde{V}_R}{Z_C} \sinh(\gamma l)$$



$$\tilde{V}_s = Z_e \left(\tilde{I}_R + \frac{Y_e}{2} \tilde{V}_R \right) + \tilde{V}_R$$

$$= \left(\frac{Z_e Y_e}{2} + 1 \right) \tilde{V}_R + Z_e \tilde{I}_R$$

$$Z_e = Z_C \sinh(\gamma l)$$

$$\frac{Z_e Y_e}{2} + 1 = \cosh(\gamma l)$$

$$\frac{Y_e}{2} = \frac{1}{Z_C} \frac{\cosh(\gamma l) - 1}{\sinh(\gamma l)}$$

$$= \frac{1}{Z_C} \tanh\left(\frac{\gamma l}{2} \right)$$

3. Classification of TL by Length

- Short lines: lines shorter than about 100 km (60 mi). They have negligible shunt capacitance, and may be represented by their series impedance.
- Medium-length lines: lines with lengths in the range of 100 km to about 300 km (190 mi). They may be represented by the nominal π equivalent circuit.
- Long lines: lines longer than about 300 km. For such lines the distributed effects of the parameters are significant. They need to be represented by the equivalent π circuit. Alternatively, they may be represented by cascaded sections of shorter lengths, with each section represented by a nominal π equivalent.

4. Typical Parameters

Nominal Voltage	230 kV	345 kV	500 kV	765 kV	1,100 kV
R (Ω/km) $x_L=\omega L$ (Ω/km) $b_C=\omega C$ (μs/km)	0.050 0.488 3.371	0.037 0.367 4.518	0.028 0.325 5.200	0.012 0.329 4.978	0.005 0.292 5.544
α (nepers/km) β (rad/km)	0.000067 0.00128	0.000066 0.00129	0.000057 0.00130	0.000025 0.00128	0.000012 0.00127
Z_C (Ω) SIL (MW)	380 140	285 420	250 1000	257 2280	230 5260
Charging MVA/km $= V_0^2 b_C$	0.18	0.54	1.30	2.92	6.71

5. Performance of a TL (no-load)

(a) Receiving end is opened (I_R=0)

$$\tilde{V} = \frac{\tilde{V}_R}{2} e^{\gamma x} + \frac{\tilde{V}_R}{2} e^{-\gamma x} \qquad \tilde{I} = \frac{\tilde{V}_R}{2Z_C} e^{\gamma x} - \frac{\tilde{V}_R}{2Z_C} e^{-\gamma x}$$

Neglecting losses

$$\tilde{V} = \tilde{V}_R \cos(\beta x)$$
$$\tilde{I} = j(\tilde{V}_R / Z_C)\sin(\beta x)$$

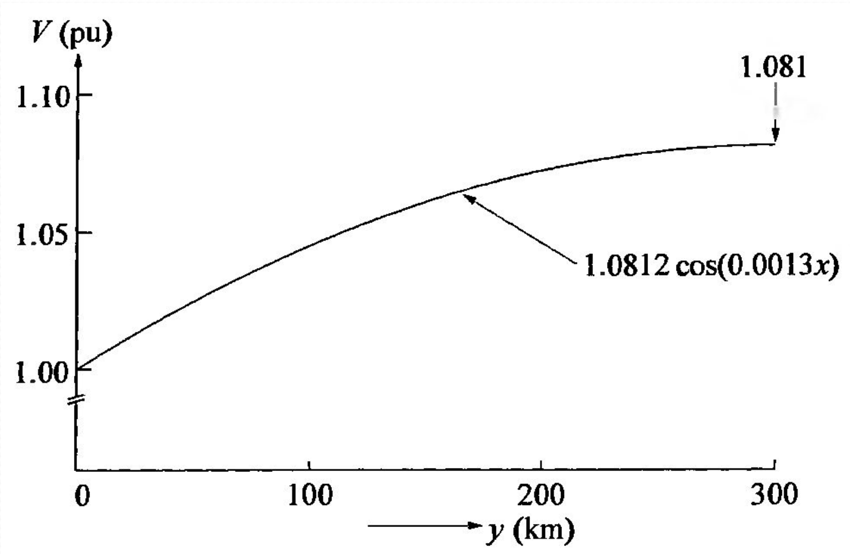
$$\Rightarrow$$

$$\boxed{\begin{aligned} \tilde{E}_S &= \tilde{V}_R \cos\beta l & \tilde{I}_S &= j(\tilde{V}_R/Z_C)\sin\theta \\ &= \tilde{V}_R \cos\theta & &= j(\tilde{E}_S/Z_C)\tan\theta \end{aligned}}$$

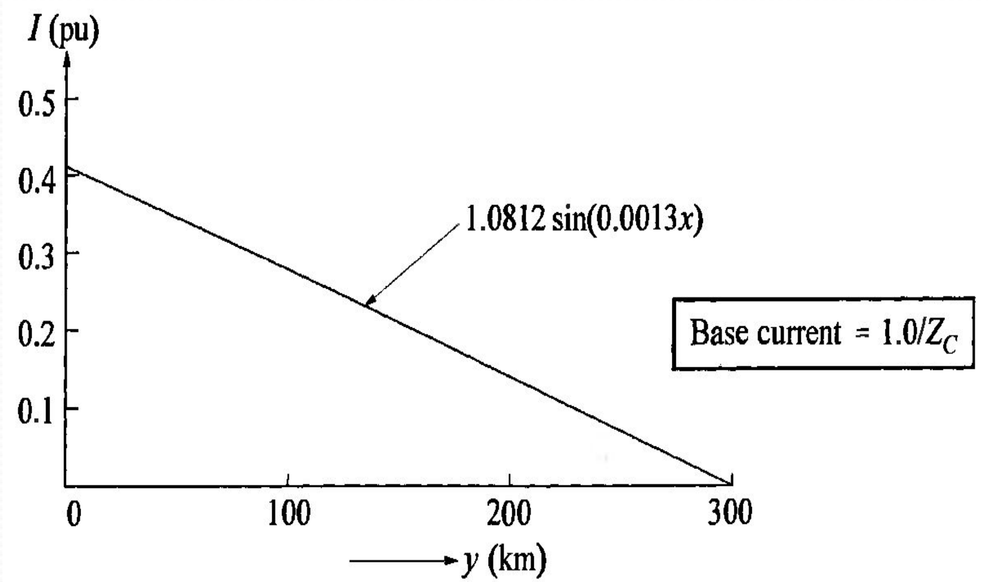
<u>**Example:**</u> 300 km, 500 kV overhead line, sending end at rated voltage (1 p.u.). Voltage and current profiles?

5. Performance of a TL (no-load)

Voltage profile



Current profile



5. Performance of a TL (no-load)

(b) Line connected to sources at both ends

$$\tilde{E}_S = \frac{\tilde{E}_R + Z_C \tilde{I}_R}{2} e^{\gamma l} + \frac{\tilde{E}_R - Z_C \tilde{I}_R}{2} e^{-\gamma l}$$

$$\tilde{I}_R = \frac{2\tilde{E}_S - \tilde{E}_R(e^{\gamma l} + e^{-\gamma l})}{Z_C(e^{\gamma l} - e^{-\gamma l})}$$

$$\tilde{V} = \frac{\tilde{E}_S - \tilde{E}_R e^{-\gamma l}}{e^{\gamma l} - e^{-\gamma l}} e^{\gamma x} + \frac{\tilde{E}_R e^{\gamma l} - \tilde{E}_S}{e^{\gamma l} - e^{-\gamma l}} e^{-\gamma x}$$

$$\tilde{I} = \frac{\tilde{E}_S - \tilde{E}_R e^{-\gamma l}}{Z_C(e^{\gamma l} - e^{-\gamma l})} e^{\gamma x} - \frac{\tilde{E}_R e^{\gamma l} - \tilde{E}_S}{Z_C(e^{\gamma l} - e^{-\gamma l})} e^{-\gamma x}$$

Assuming $E_S = E_R$

$$\tilde{V} = \frac{\tilde{E}_S - \tilde{E}_S e^{-\gamma l}}{e^{\gamma l} - e^{-\gamma l}} e^{\gamma x} + \frac{\tilde{E}_S e^{\gamma l} - \tilde{E}_S}{e^{\gamma l} - e^{-\gamma l}} e^{-\gamma x}$$

$$\tilde{I} = \frac{\tilde{E}_S - \tilde{E}_S e^{-\gamma l}}{Z_C(e^{\gamma l} - e^{-\gamma l})} e^{\gamma x} - \frac{\tilde{E}_S e^{\gamma l} - \tilde{E}_S}{Z_C(e^{\gamma l} - e^{-\gamma l})} e^{-\gamma x}$$

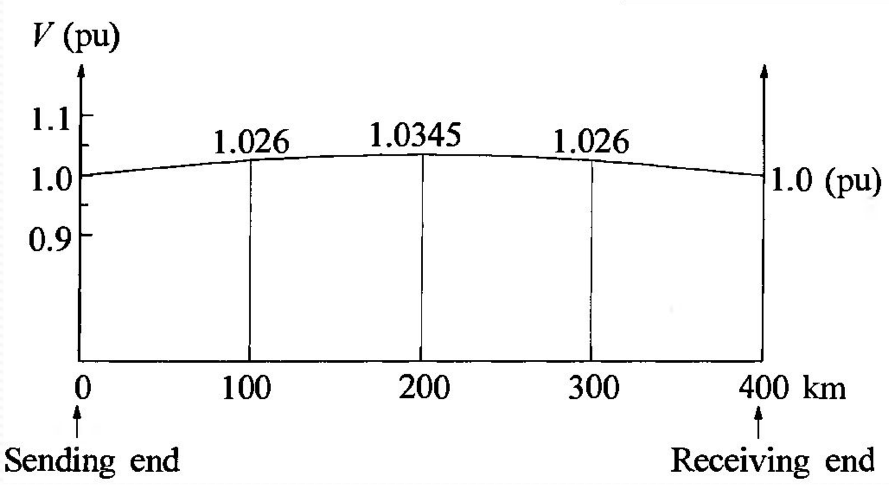
For a lossless line, $\gamma = j\beta$. With $\theta = \beta l$, we have

$$\tilde{V} = \tilde{E}_S \frac{\cos\beta(l/2 - x)}{\cos(\theta/2)}$$

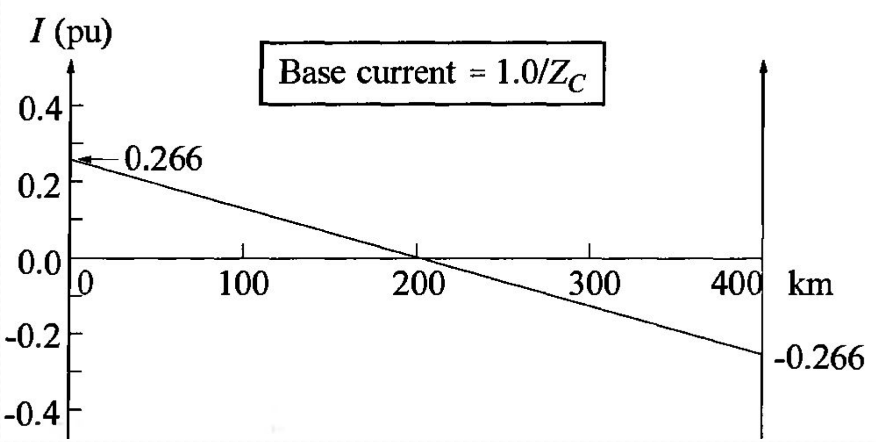
$$\tilde{I} = -j\frac{\tilde{E}_S}{Z_C} \frac{\sin\beta(l/2 - x)}{\cos(\theta/2)}$$

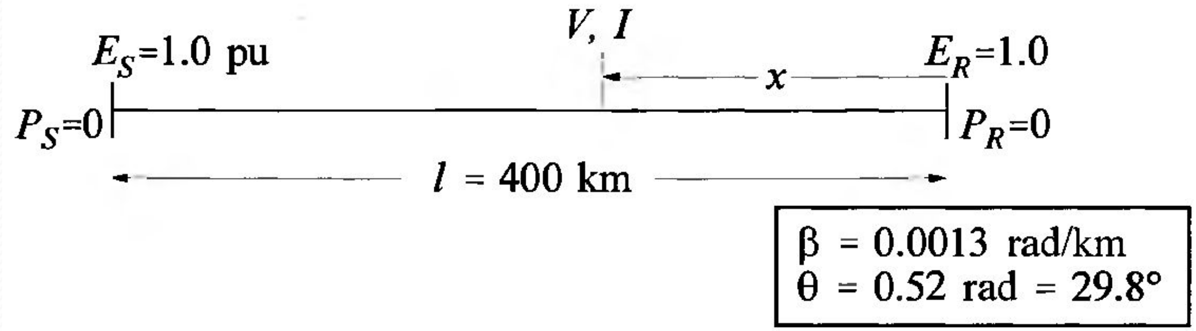
5. Performance of a TL (no-load)

Voltage profile



Current profile





6. Performance of a TL (under load)

(a) Radial line with fixed sending end voltage; load $P_R + jQ_R$.

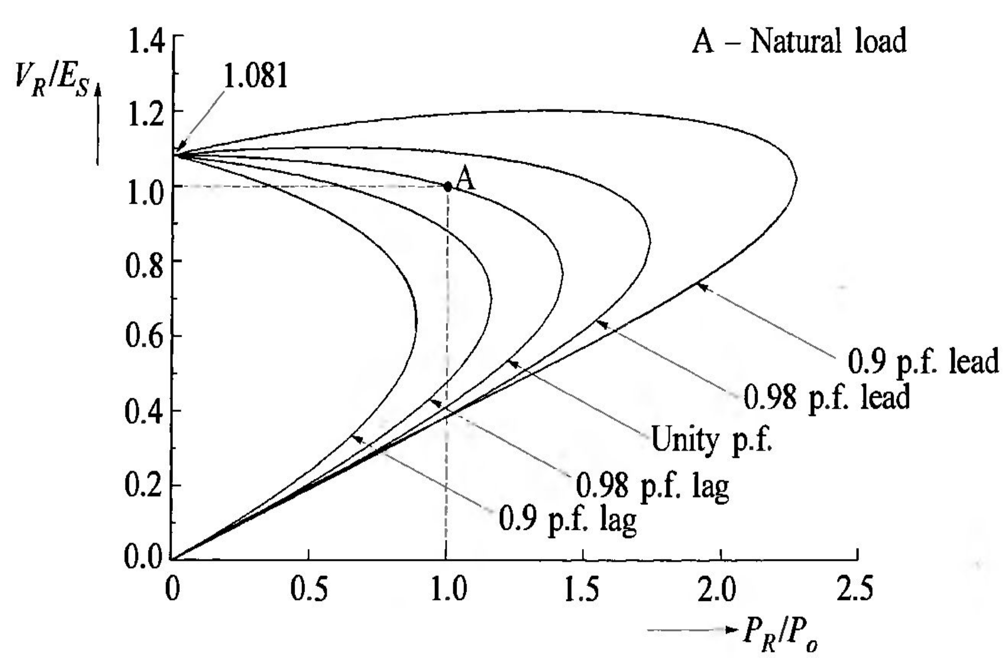
$$\tilde{I}_R = \frac{P_R - jQ_R}{\tilde{V}_R^*} \quad \Rightarrow \quad \tilde{E}_S = \frac{\tilde{V}_R + Z_C(P_R - jQ_R)/\tilde{V}_R^*}{2} e^{\gamma l} + \frac{\tilde{V}_R - Z_C(P_R - jQ_R)/\tilde{V}_R^*}{2} e^{-\gamma l}$$

For a lossless line

$$\Rightarrow \quad \tilde{E}_S = \tilde{V}_R \cos\theta + jZ_C \sin\theta \left(\frac{P_R - jQ_R}{\tilde{V}_R^*} \right)$$

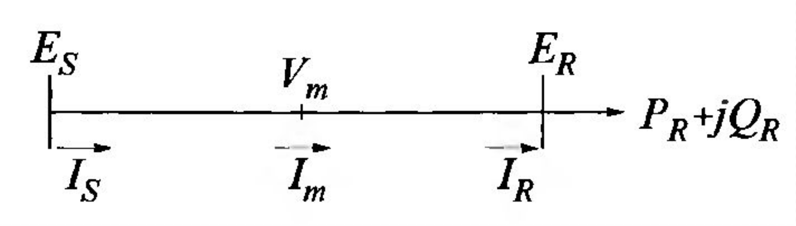
Several fundamental properties of AC transmission:

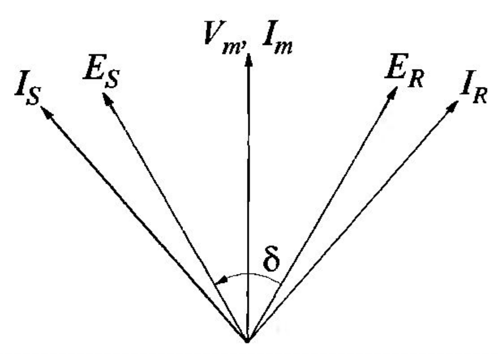
- There is an inherent maximum limit of power that can be transmitted at any load power factor;
- Any value of power below the maximum can be transmitted at two different values of V_R. The normal operation is at the upper value, within narrow limits around 1.0 pu;
- The load power factor has a significant influence on V_R and the maximum power that can be transmitted.



6. Performance of a TL (under load)

(b) Line connected to sources at both ends





As in the no-load case, assume the magnitudes of the source voltages at the two ends to be equal.

Under load, E_S leads E_R in phase:

- The midpoint voltage is midway in phase between E_S and E_R;
- The power factor at midpoint is unity;
- With $P_R > P_{NAT}$ both ends supply reactive power to the line; with $P_R < P_{NAT}$, both ends absorb reactive power from the line.

7. Power Transfer and Stability Considerations

$$\tilde{E}_S = \tilde{E}_R\cos\theta + jZ_C\sin\theta\left(\frac{P_R - jQ_R}{\tilde{E}_R^*}\right)$$

Let δ be the angle by which E_S leads E_R

$$\tilde{E}_S = E_S e^{j\delta} = E_S(\cos\delta + j\sin\delta)$$

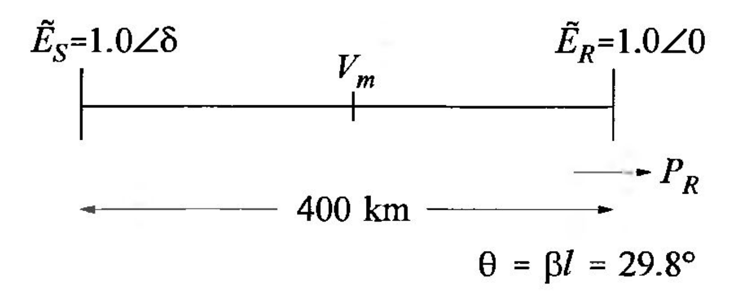
Equating real and imaginary parts

$$E_S\cos\delta = E_R\cos\theta + Z_C(Q_R/E_R)\sin\theta$$

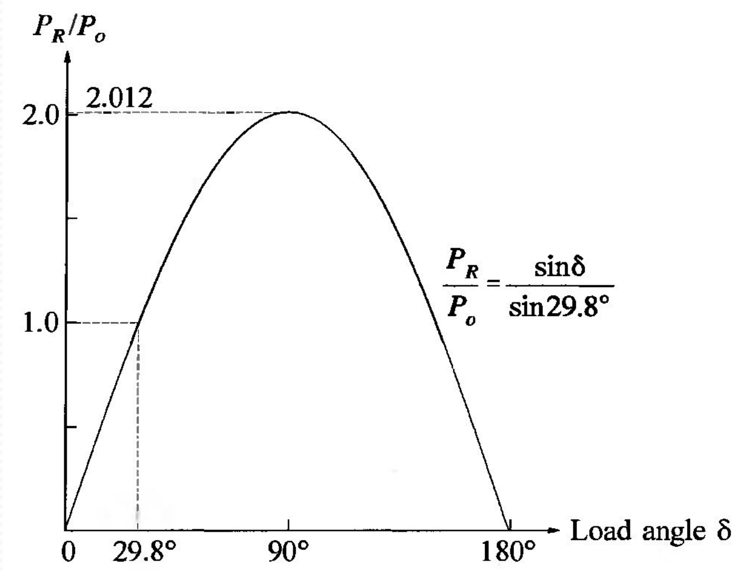
$$E_S\sin\delta = Z_C(P_R/E_R)\sin\theta$$

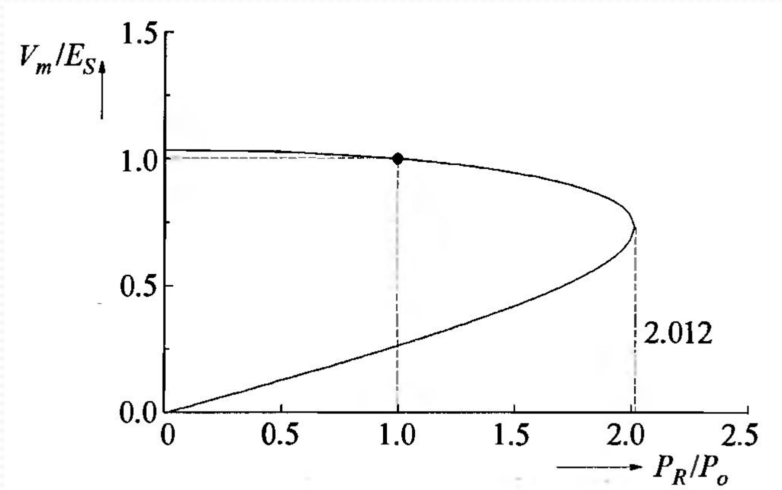
$$\boxed{P_R = \frac{E_S E_R}{Z_C\sin\theta}\sin\delta}$$

7. Power Transfer and Stability Considerations



As the load angle is increased, the transmitted power increases. This is accompanied by a reduction in the midpoint voltage V_m and an increase in the midpoint current I_m so that there is an increase in power. Up to a certain point the increase in l_m dominates over the decrease of V_m. When the load angle reaches 90°, the transmitted power reaches its maximum value. Beyond this, the decrease in V_m is greater than the accompanying increase in I_m, hence, their product decreases with any further increase in transmission angle.



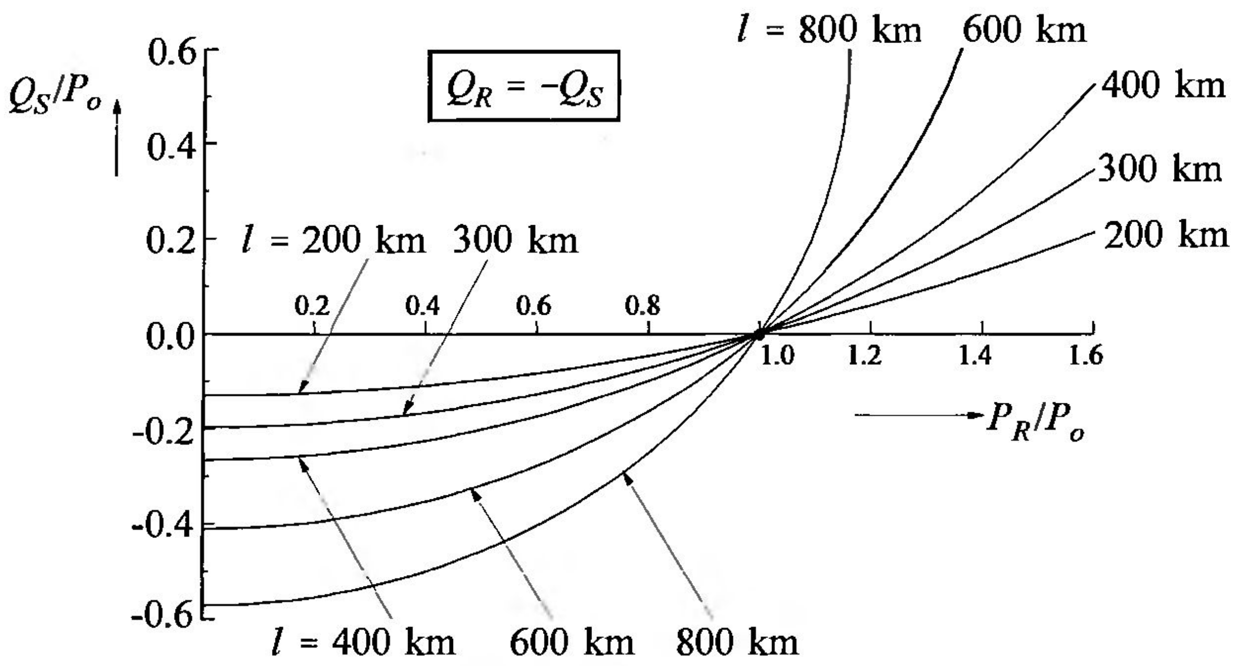


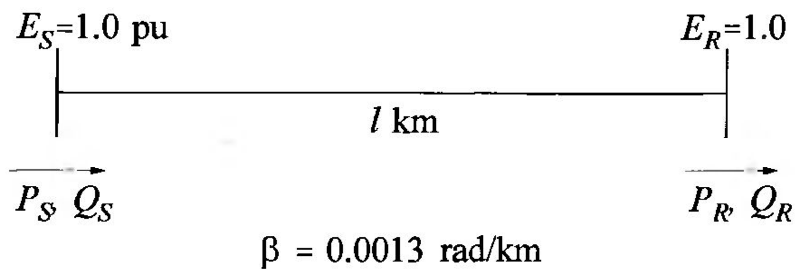
8. Reactive Power Demand

$$E_S\cos\delta = E_R\cos\theta + Z_C(Q_R/E_R)\sin\theta$$

$$Q_R = \frac{E_R(E_S\cos\delta - E_R\cos\theta)}{Z_C\sin\theta}$$

$$Q_S = \frac{-E_S(E_R\cos\delta - E_S\cos\theta)}{Z_C\sin\theta}$$





9. Tasks

1. Using lossless line equations, solve the case for the line with fixed (known) sending end voltage and shunt reactor with X_R impedance installed at receiving end.
2. Using lossless line equations, solve the case for the line with fixed (known) sending end voltage and impedance X_S and shunt reactor with X_R impedance installed at receiving end.
3. Determine the maximum voltage at line with fixed sending end voltage, l = 500 km, $X_S/Z_c = 0.3$.
4. Determine the necessary ratings of a shunt reactor installed at the receiving end of a 750 kV, l = 500 km, $X_S/Z_c = 0.3$, P_{SIL}= 2000 MW line to ensure U_R=1.05$Umax$ (maximum allowable voltage).
5. Using data from the 4th task (assuming that you have chosen the reactor), find maximum voltage on the line (value and coordinate).

10. Answers

1. $$\dot{K} = K = \frac{\dot{U}_{\text{K}}}{\dot{U}_{\text{H}}} = \frac{1}{\cos\lambda + \sin\lambda\cdot\left(Z_{\text{B}}/x_p\right)} = \frac{\sin\alpha_p}{\sin\left(\lambda+\alpha_p\right)},$$
2. $$U_{\text{H}} = \frac{E\cdot\sin\left(\lambda+\alpha_p\right)\cdot\cos\alpha_c}{\sin\left(\lambda+\alpha_c+\alpha_p\right)}.$$
3. 1.4 pu.
4. 0.31 pu, 600 Mvar
5. Xmax = 214 km (from sending end), Umax = 1,1 pu.

Thank you for your attention!