Image Warping / Morphing

[Wolberg 1996, Recent Advances in Image Morphing]

Computational Photography

Connelly Barnes

Some slides from Fredo Durand, Bill Freeman, James Hays

Morphing Video: Women in Art

- http://www.vimeo.com/1456037

Terminator 2 Morphing (1991)

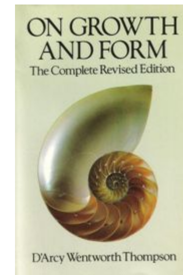
[Terminator 2 Clip (YouTube)]

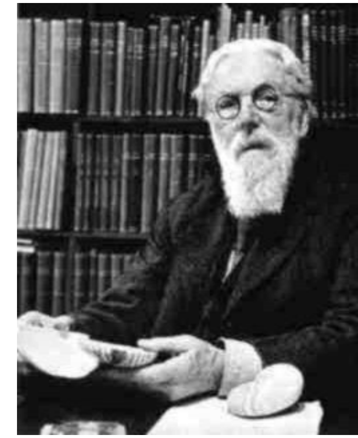
Image Warping in Biology

- D'Arcy Thompson

  http://en.wikipedia.org/wiki/D'Arcy_Thompson

- Importance of shape and structure in evolution





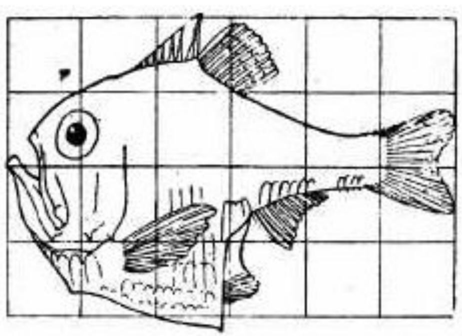


Fig. 517. *Argyropelecus Olfersi.*

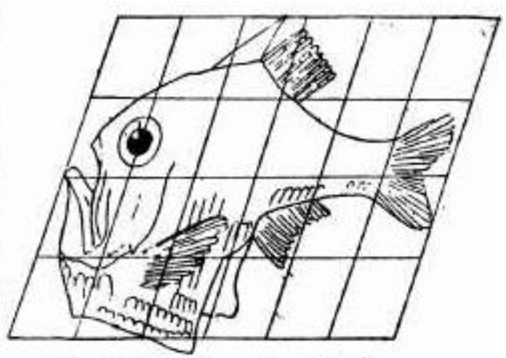
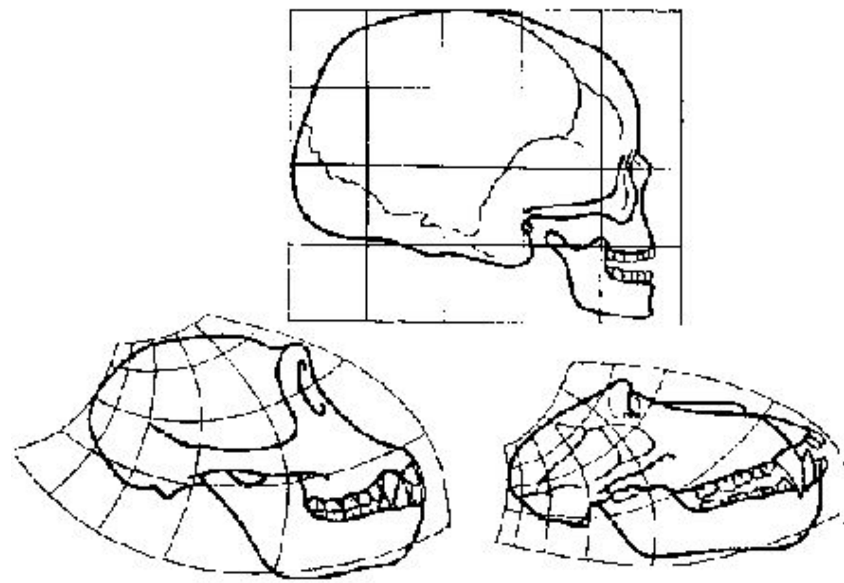


Fig. 518. *Sternoptyx diaphana.*



Skulls of a human, a chimpanzee and a baboon and transformations between them

Slide by Durand and Freeman

Cambrian Explosion

Opabinia

Hallucigenia

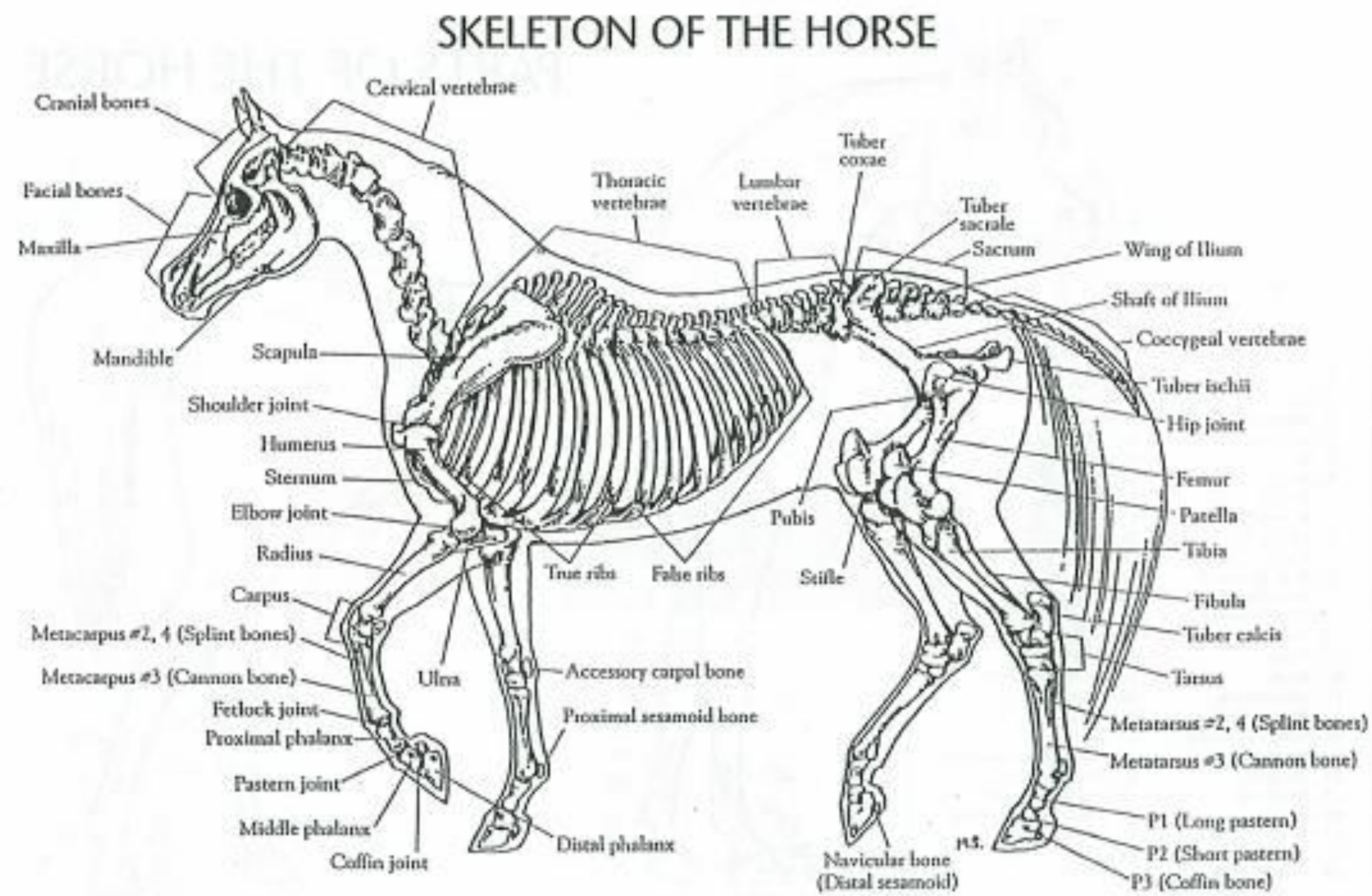
Pikaia

Marrella

Aysheaia

Source: http://www.earthlearningidea.com/

Skeletons



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10805 East Bethany Drive
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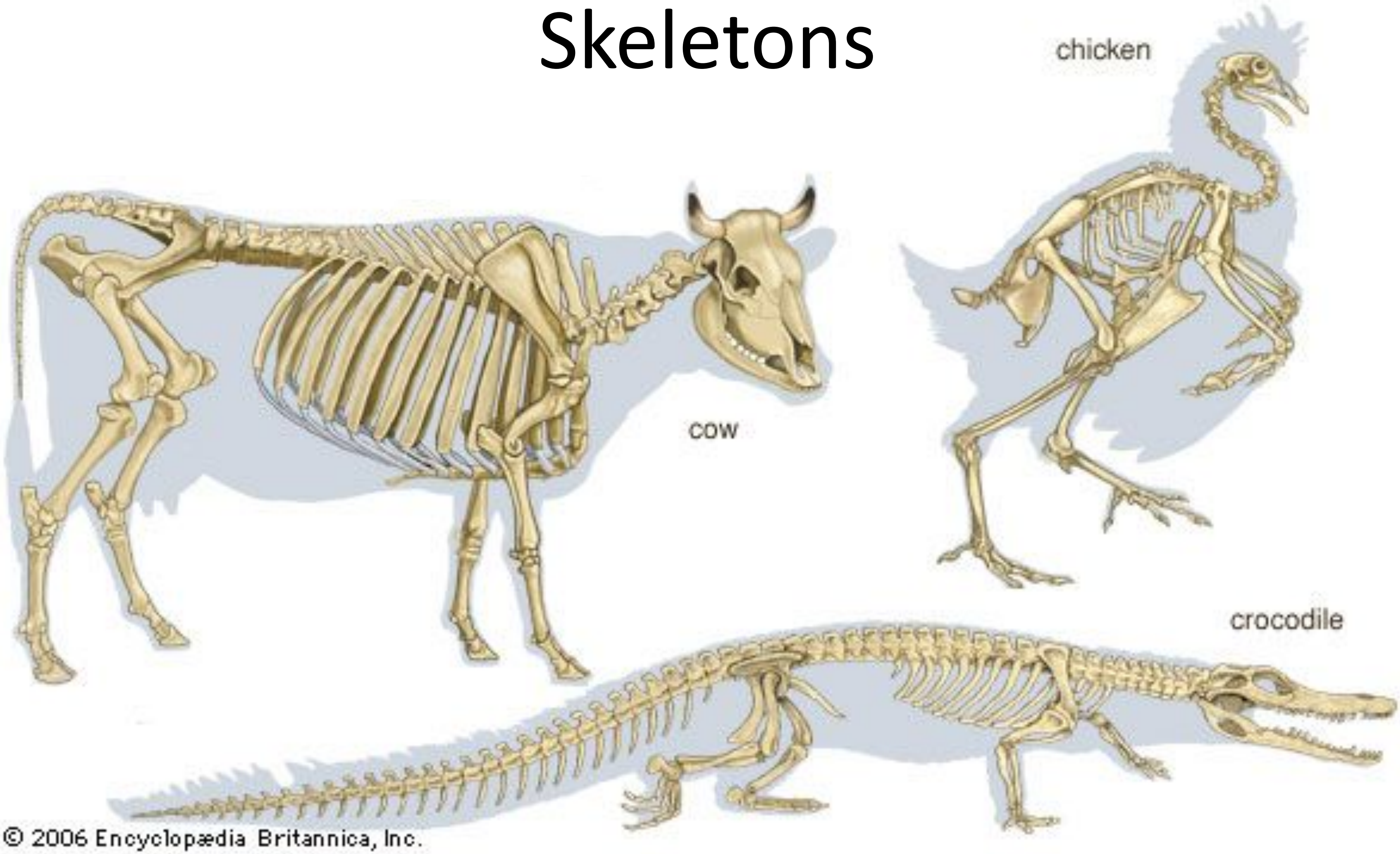
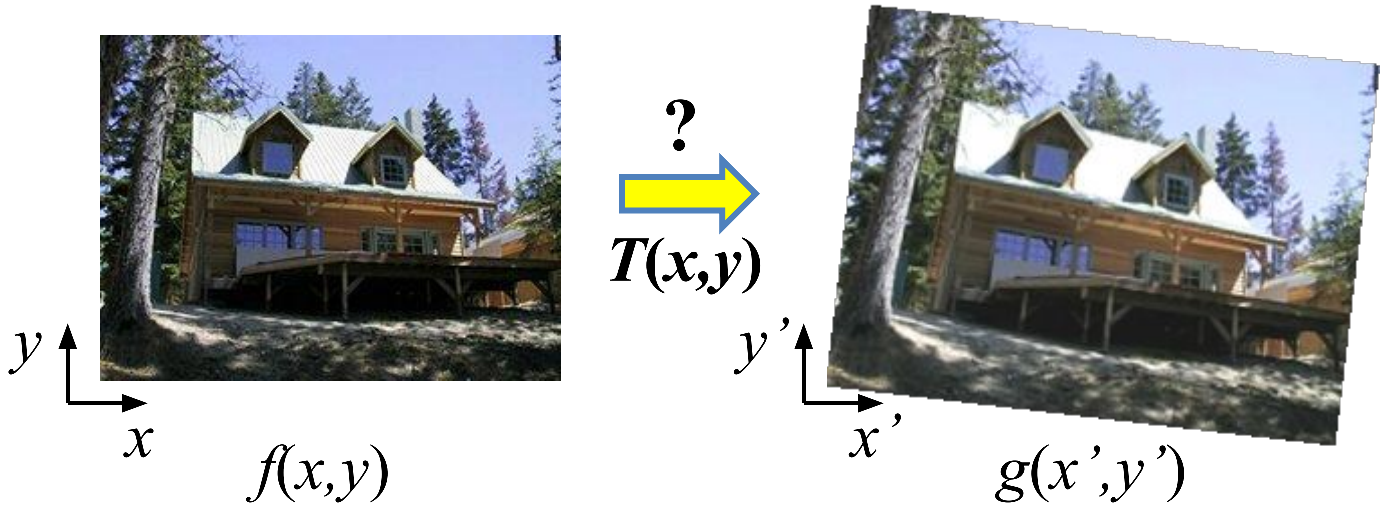
www.ArabianHorses.org

Skeletons

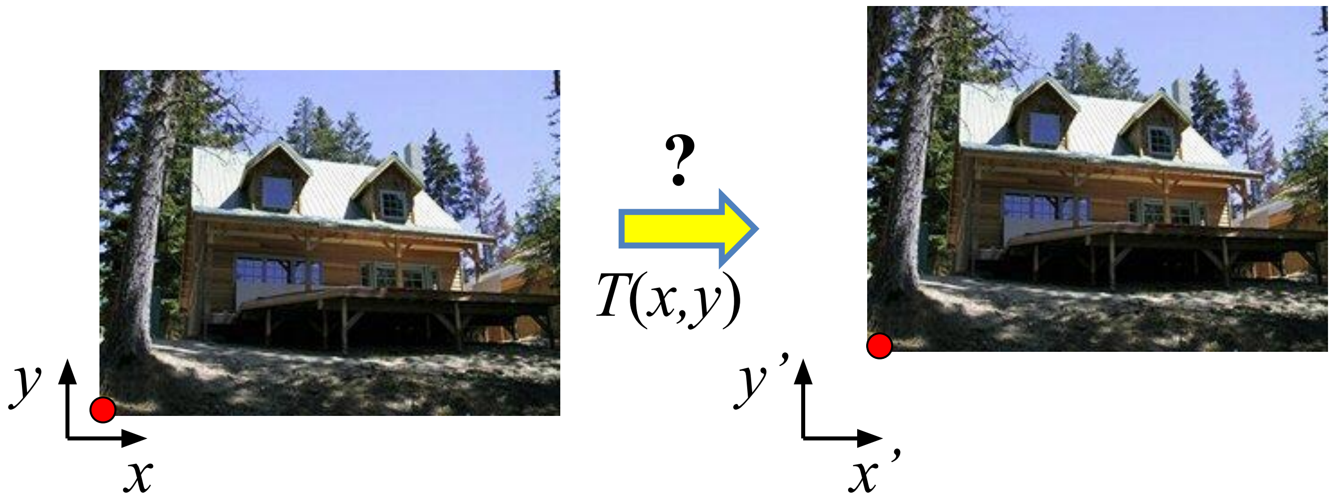


Recovering Transformations



- What if we know *f* and *g* and want to recover the transform T?
  - e.g. better align photographs you've taken
  - willing to let user provide correspondences
    - How many do we need?

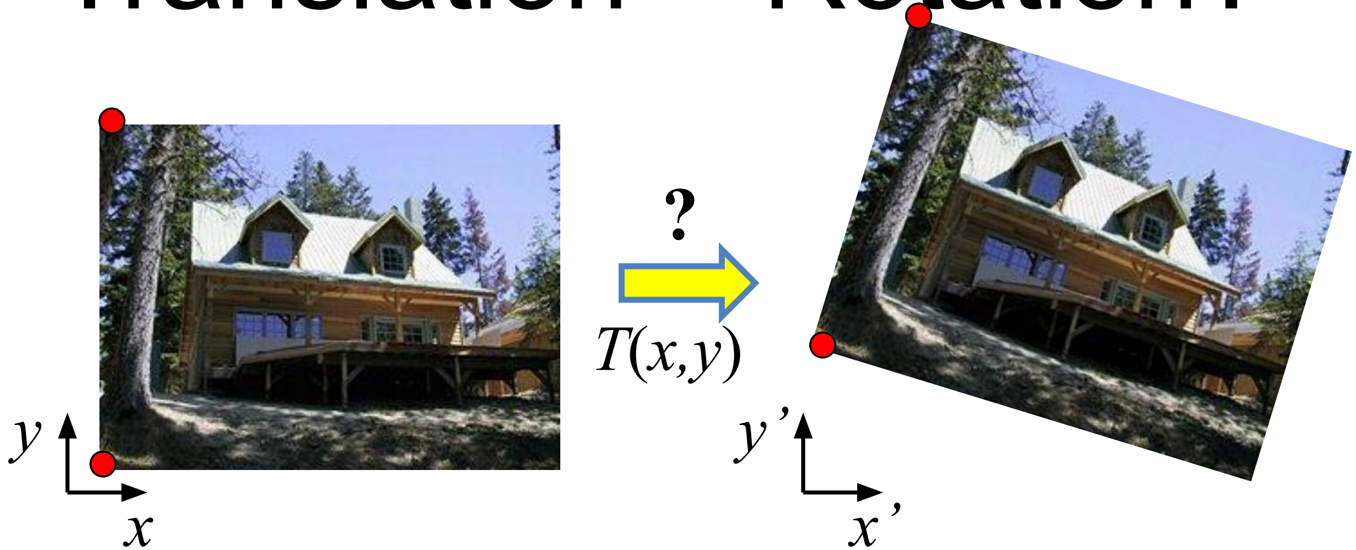
Translation: # correspondences?



- How many correspondences needed for translation?
- How many Degrees of Freedom?
- What is the transformation matrix?

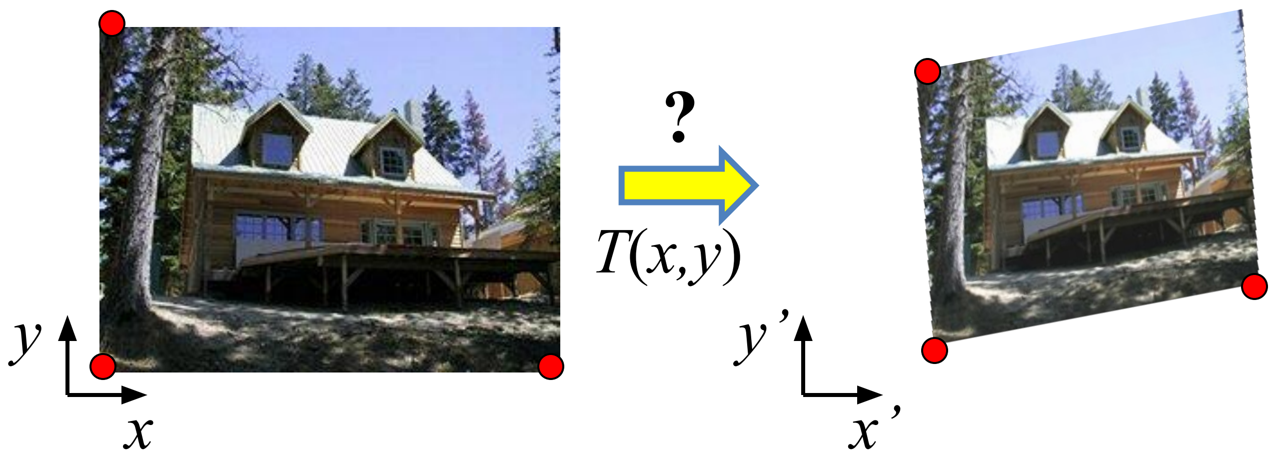
$$\mathbf{M} = \begin{bmatrix} 1 & 0 & p'_x - p_x \\ 0 & 1 & p'_y - p_y \\ 0 & 0 & 1 \end{bmatrix}$$

Translation + Rotation?



- How many correspondences needed for translation+rotation?
- How many DOF?

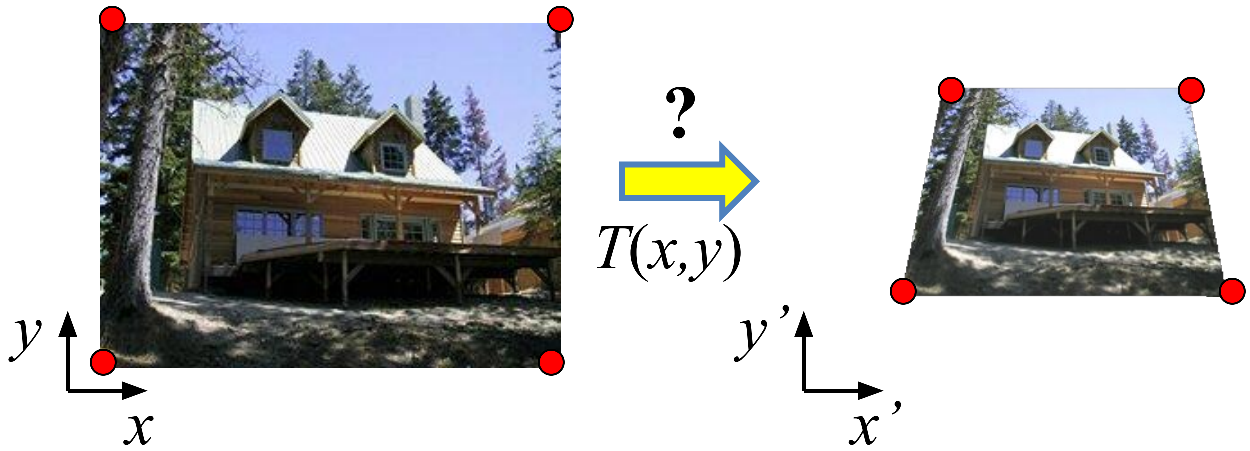
Affine: # correspondences?



- How many correspondences needed for affine transform?
- How many DOF?

$$T(x,y)=\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}+\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

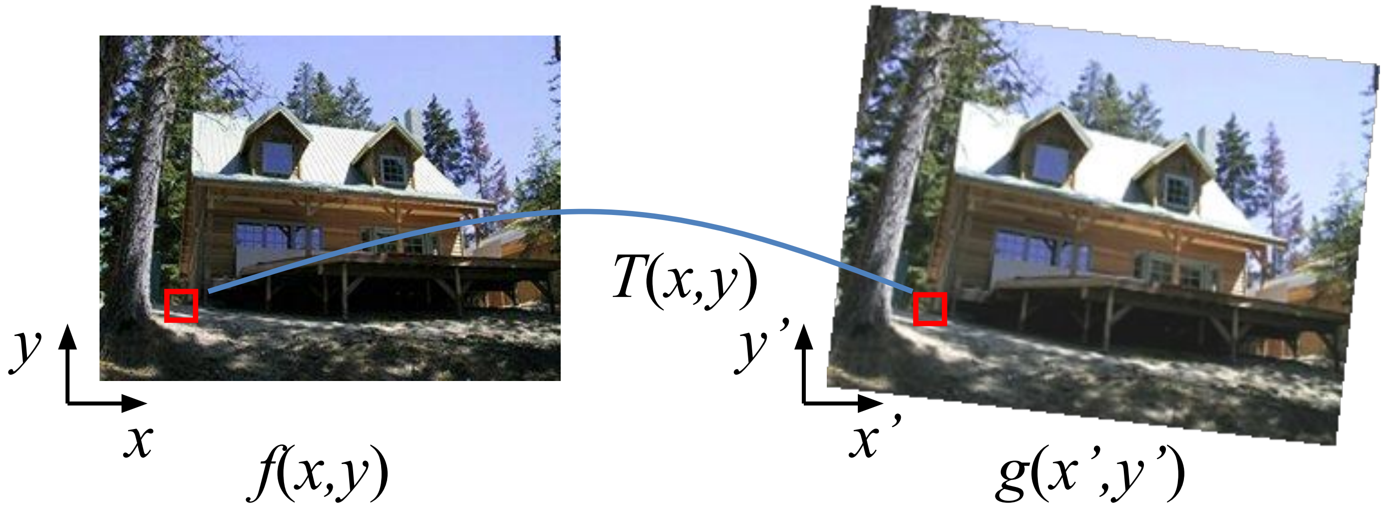
Projective / Homography



- How many correspondences needed for projective? How many DOF?

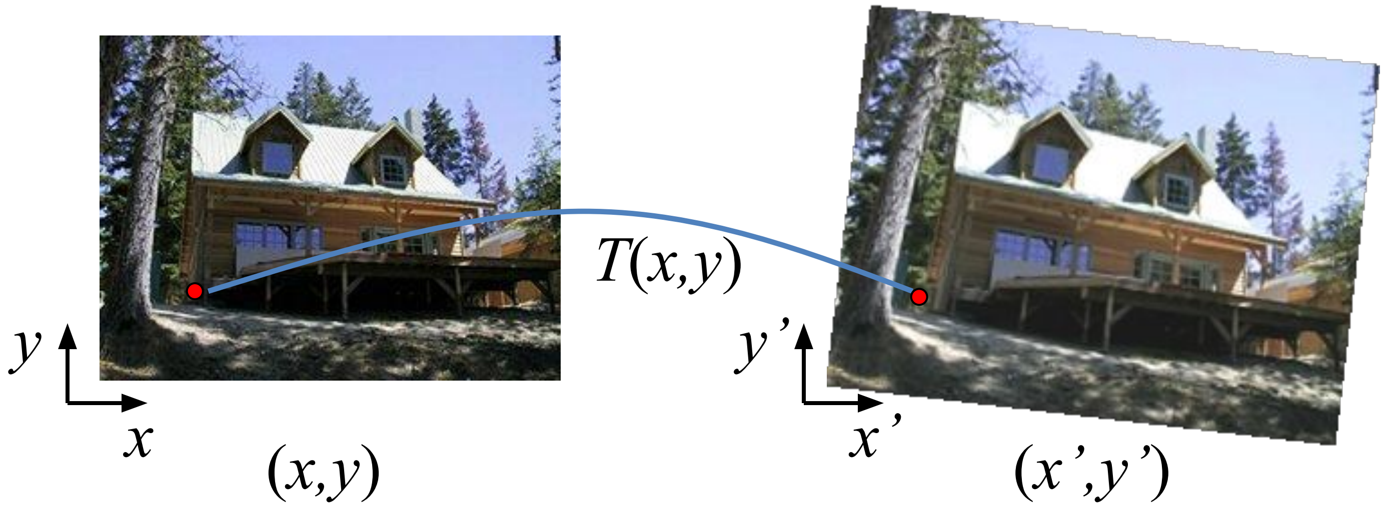
$$T(x,y) = h\left(\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}\right) \qquad h(x,y,z) = (x/z, y/z)$$

Image Warping



- Given a coordinate transform $(x',y') = T(x,y)$ and a source image $f(x,y)$, how do we compute a transformed image $g(x',y') = f(T(x,y))$?

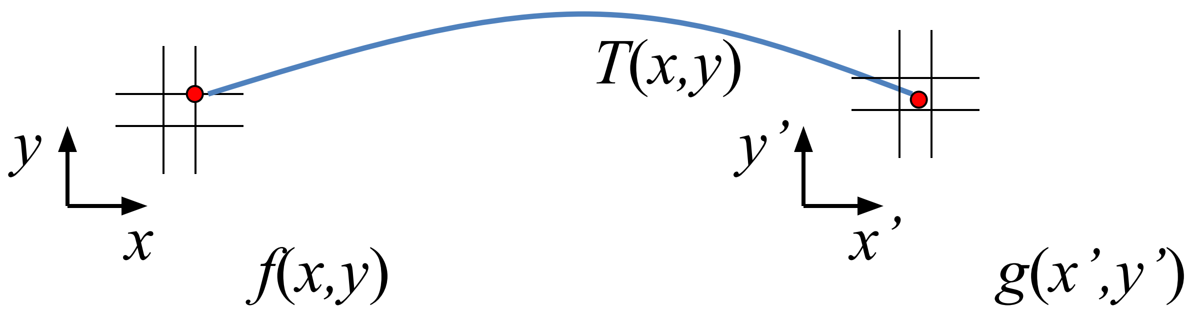
Forward warping



- Send each pixel (x,y) to its corresponding location

$(x',y') = T(x,y)$ in the second image

Forward warping

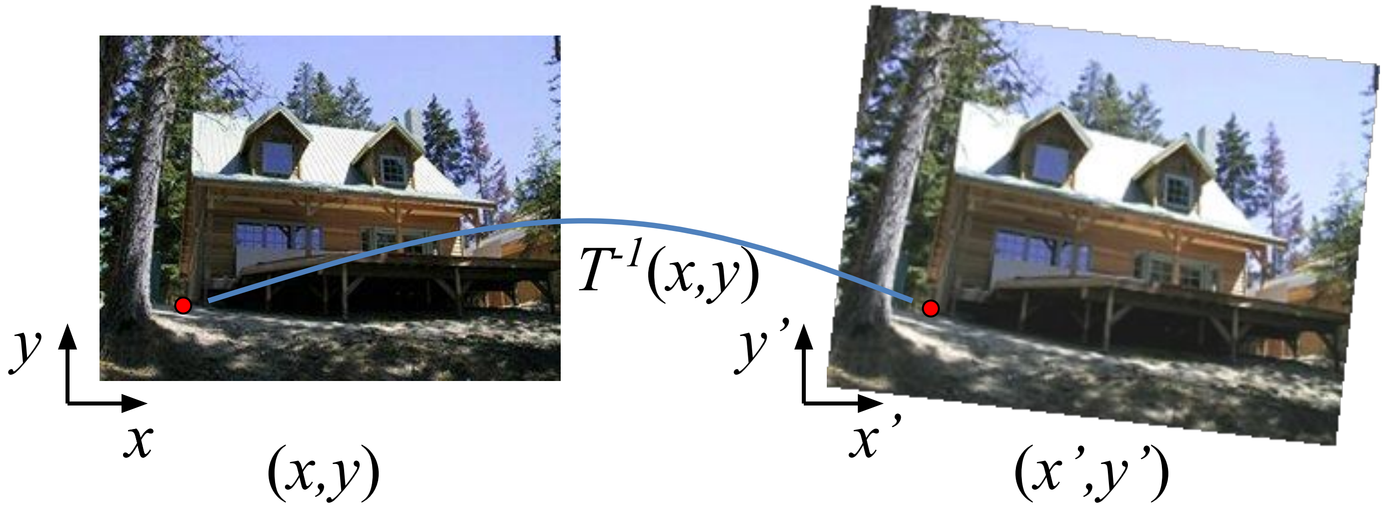


Q: what if pixel lands "between" two pixels?

A: distribute color among neighboring pixels (x',y')

- Known as "splatting"
- Can also interpolate points in target image: `griddata` (Matlab), `scipy.interpolate.griddata` (Python)

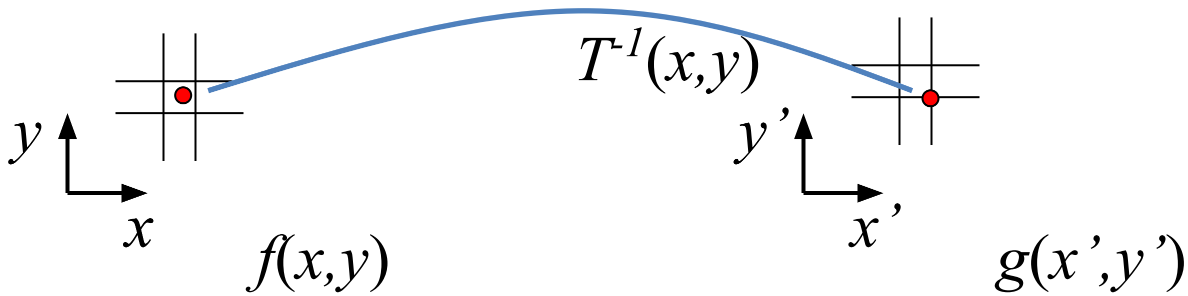
Inverse warping



- Get each pixel color $g(x',y')$ from its corresponding location

  $(x,y) = T^{-1}(x',y')$ in the first image

Inverse warping



Q: what if pixel comes from "between" two pixels?

A: *Interpolate* color value from neighbors

- nearest neighbor, bilinear, Gaussian, bicubic
- See [`interp2`] (Matlab), [`scipy.interpolate.interp2d`] (Python)

Forward vs. inverse warping

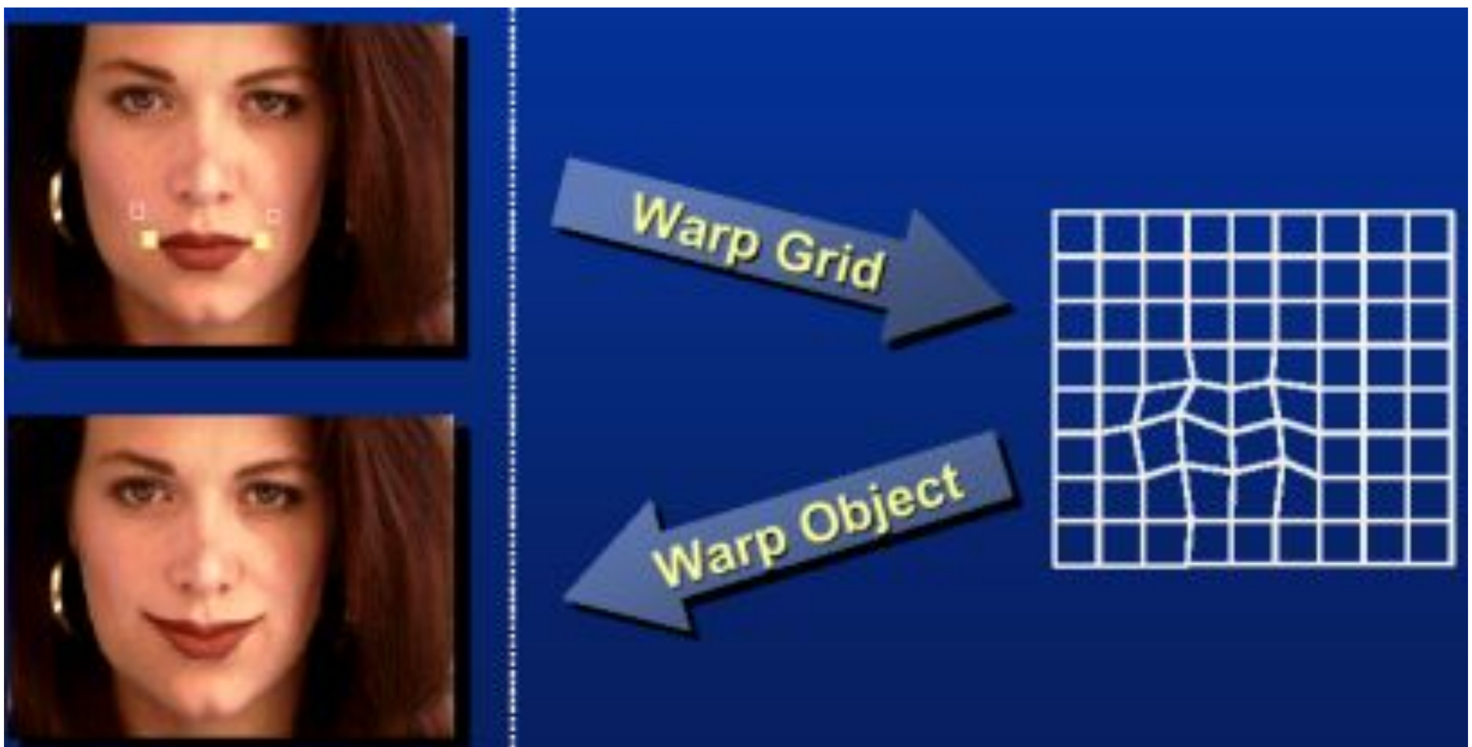
- Q: Which is better?

Forward vs. inverse warping

- Q: Which is better?
- A: Usually inverse – eliminates holes
  - However, it requires an invertible warp function
  - Not always possible

How to Obtain Warp Field?

- Move control points to specify a spline warp
- Spline produces a smooth vector field $T(x, y)$



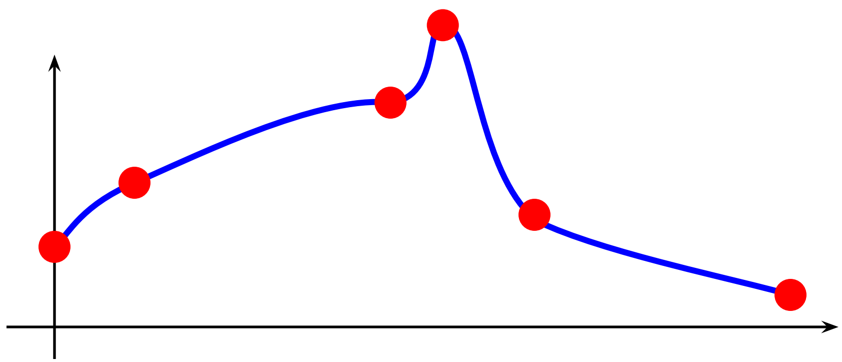
Slide Alyosha Efros

Warp as Interpolation

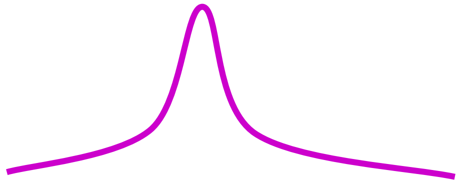
- **We are looking for a warping field**
  - A function that given a 2D point, returns a warped 2D point
- **We have a sparse number of correspondences**
  - These specify values of the warping field
- **This is an interpolation problem**
  - Given sparse data, find smooth function

Interpolation in 1D

- We are looking for a function f
- We have N data points: x_i, y_i
  - Scattered: spacing between x_i is non-uniform
- We want f so that
  - For each i, $f(x_i)=y_i$
  - f is smooth
- Depending on notion of smoothness, different f

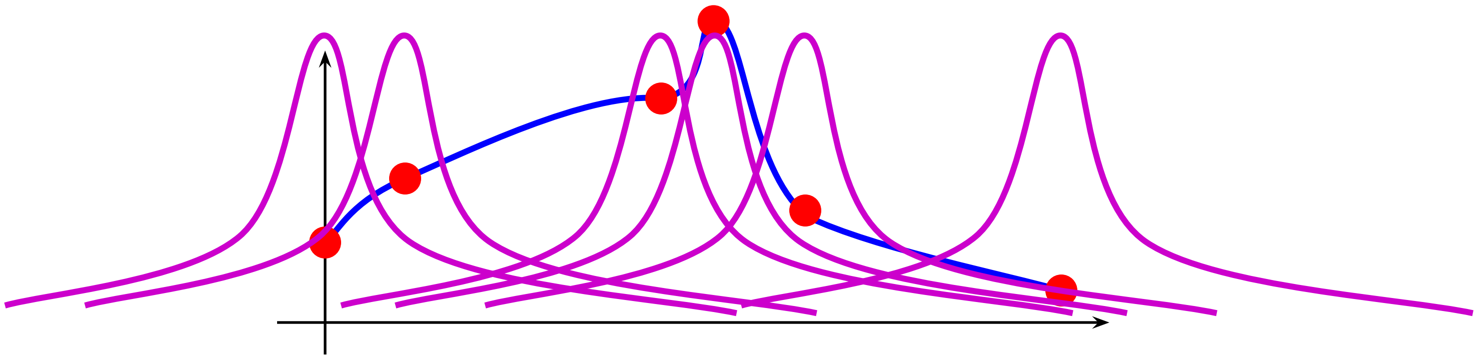


Radial Basis Functions (RBF)

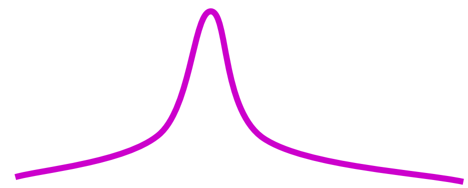


- Place a smooth kernel *R* centered on each data point x_i

$$f(z) = \sum \alpha_i R\left(\left\|z - x_i\right\|\right)$$



Radial Basis Functions (RBF)

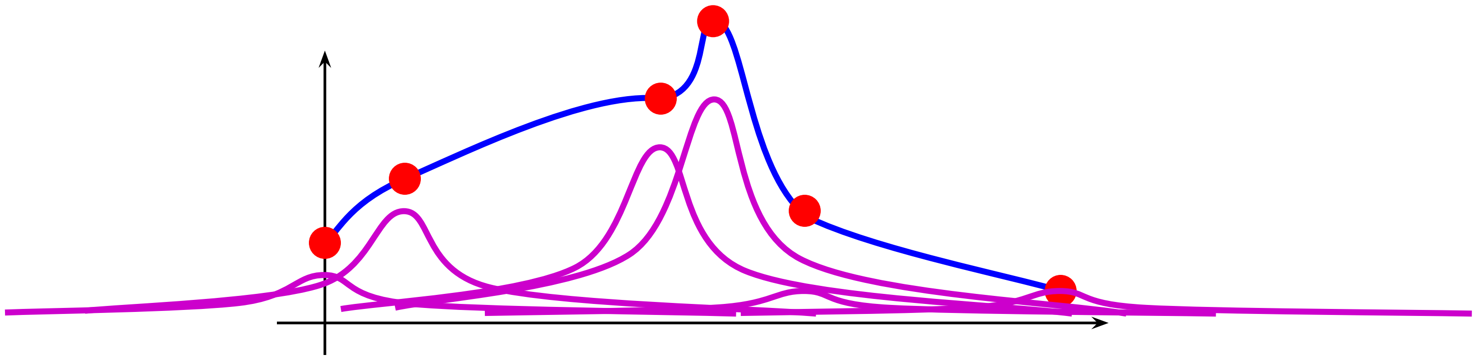


- Place a smooth kernel *R* centered on each data point x_i

$$f(z) = \sum \alpha_i R\left(\left\| z - x_i \right\|\right)$$

- Find weights α_i to make sure we interpolate the data

  for each i, $f(x_i)=y_i$



Radial Basis Function Kernels

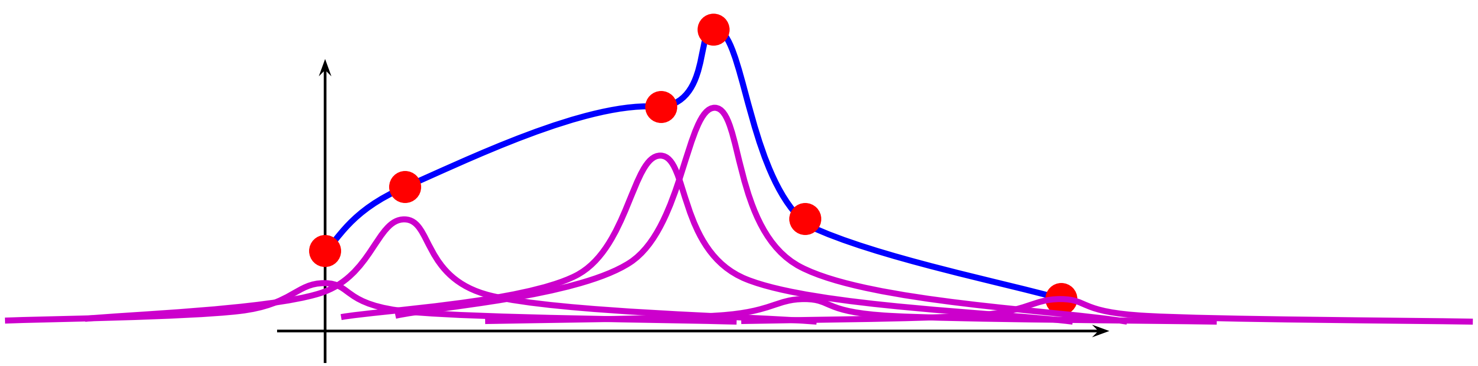
Linear	$R(r) = r$
Cubic	$R(r) = r^3$
Quintic	$R(r) = r^5$
Thin plate	$R(r) = r^2 \log r$
Inverse	$R(r) = 1/\sqrt{(r/w)^2 + 1}$
Multiquadratic	$R(r) = \sqrt{(r/w)^2 + 1}$

Solve RBF Interpolation Problem

$$f(z) = \sum \alpha_i R\left(\left\| z - x_i \right\|\right)$$

For each *j*, $\sum \alpha_i R\left(\left\| x_j - x_i \right\|\right) = y_j$

- In 1D: *N* equations, *N* unknowns, linear solver.
- In *n*-D: Denote $\boldsymbol{\alpha}_i$, $\mathbf{x}_i$, $\mathbf{y}_i \in \mathbf{R}^m$
  Solve *Nm* equations in *Nm* unknowns $\boldsymbol{\alpha}_i$.



RBF Summary

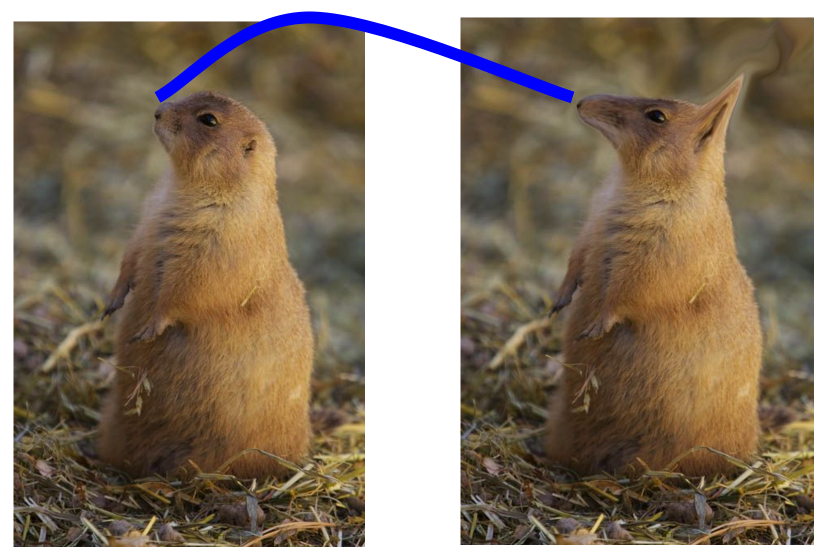
- Interpolates “scattered data”, or data defined only at a few sparse locations.
- Basis functions have infinite extent…

- Python: scipy.interpolate.Rbf

- MATLAB: Google “matlab rbf interpolation” (3rd party code)

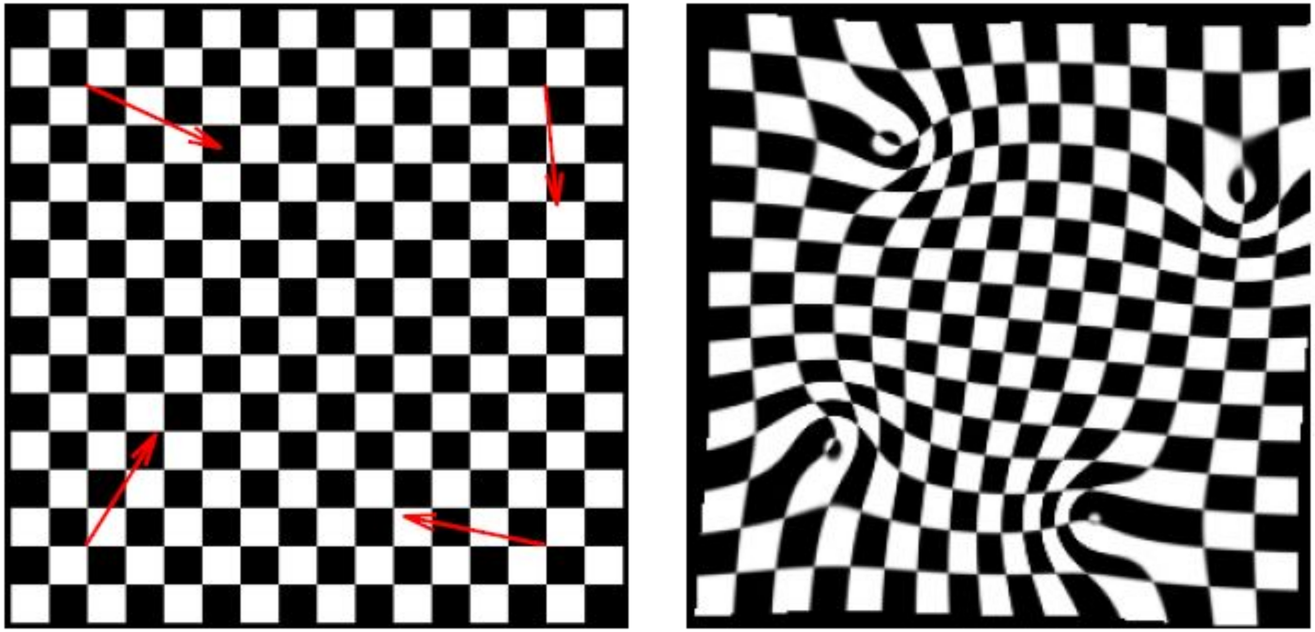
Applying a warp: use inverse

- Forward warp:
  - For each pixel in **input** image
    - Paste color **to warped** location in output
  - Problem: gaps



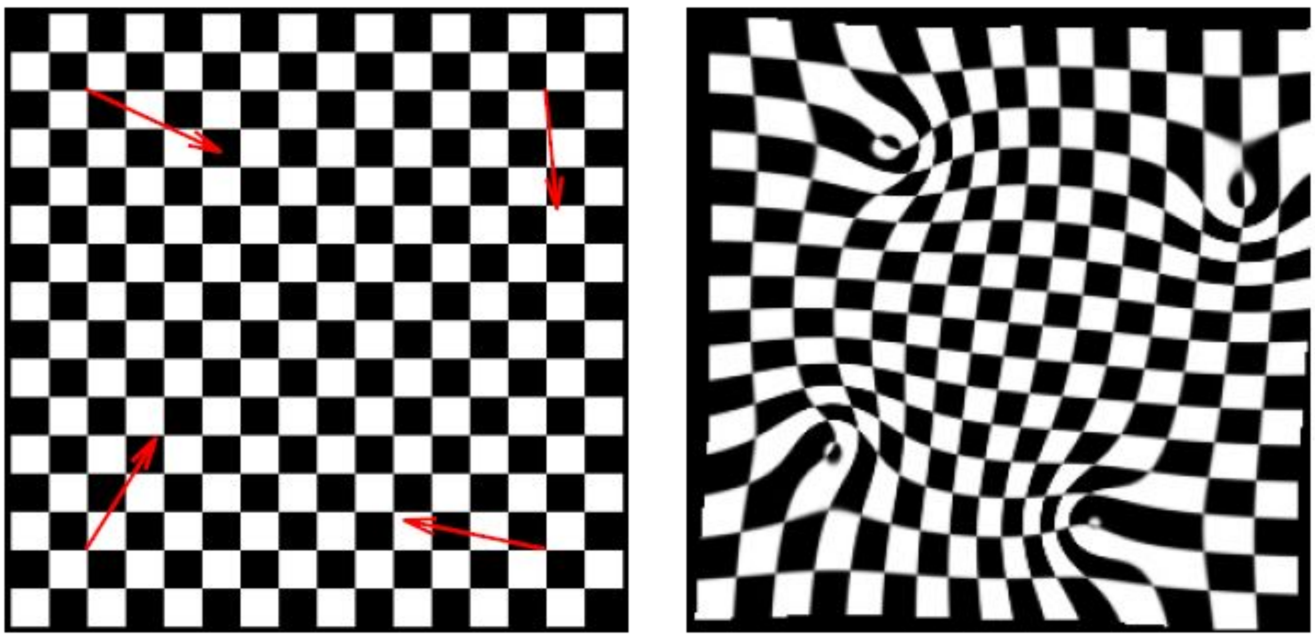
- Inverse warp
  - For each pixel in **output** image
    - Lookup color **from** **inverse-warped** location

Example



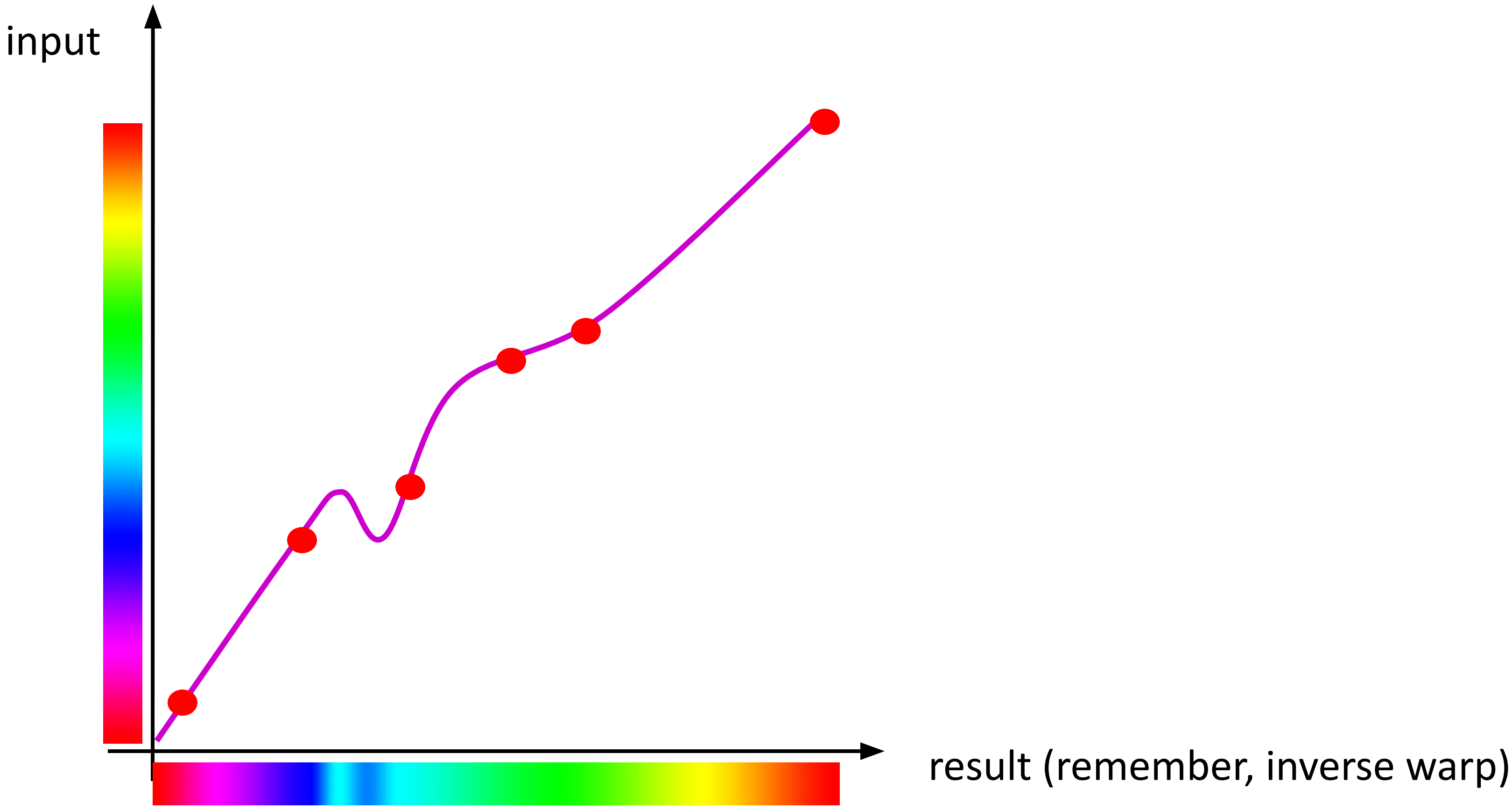
Example

- Fold problems
  - Oh well…



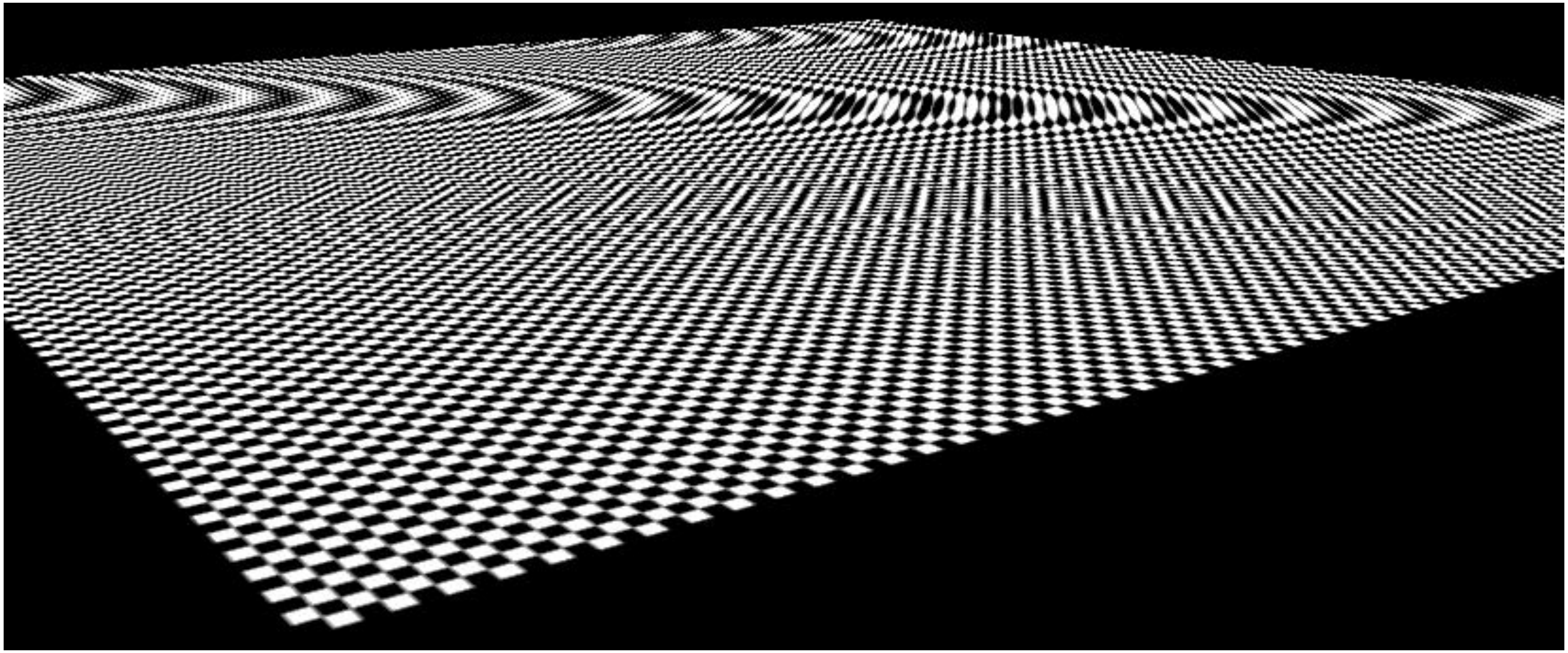
1D equivalent of folds

- No guarantee that our 1D RBF is monotonic



Aliasing Issues with Warping

- *Aliasing* can happen if warps are extreme. This is especially noticeable during animation.

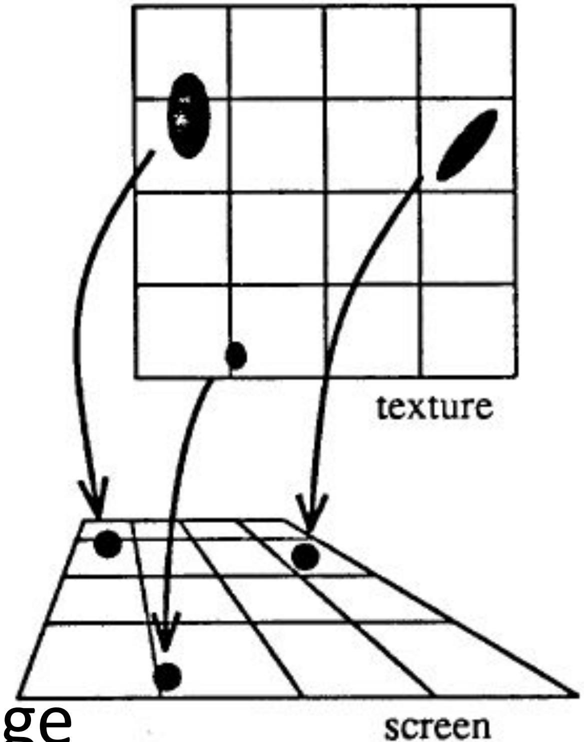


Aliasing Solution

- Use an ellipsoidal Gaussian:

$$G(x,y) = G_{\sigma_1}(x)G_{\sigma_2}(y)$$

- "Elliptical Weighted Average" (EWA)
- Filter is deformed based on warping.
- For inverse warping, each output (warped) pixel does a weighted average of nearby pixels against the filter.
- Can approximate with circular Gaussian.



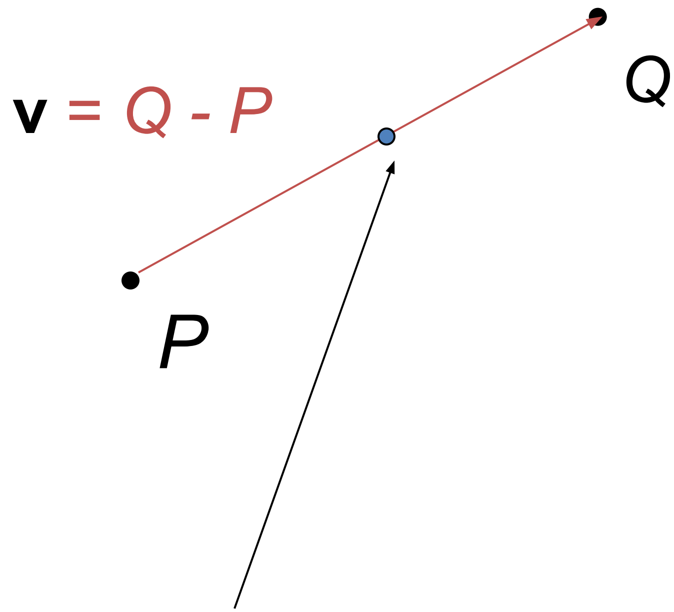
[Paul Heckbert Master's Thesis]

Morphing = Object Averaging

- The aim is to find "an average" between two objects
  - Not an average of two images of objects…
  - …but an image of the average object!
  - How can we make a smooth transition in time?
    - Do a "weighted average" over time t
- How do we know what the average object looks like?
  - We haven't a clue!
  - But we can often fake something reasonable
    - Usually required user/artist input

Linear Interpolation

How can we linearly transition between point *P* and point *Q*?



$P + t\,\mathbf{v}$

$= (1-t)P + tQ,$ e.g. $t = 0.5$

- P and Q can be anything:
  - points on a plane (2D) or in space (3D)
  - Colors in RGB or HSV (3D)
  - Whole images (m-by-n D)… etc.

Idea #1: Cross-Dissolve

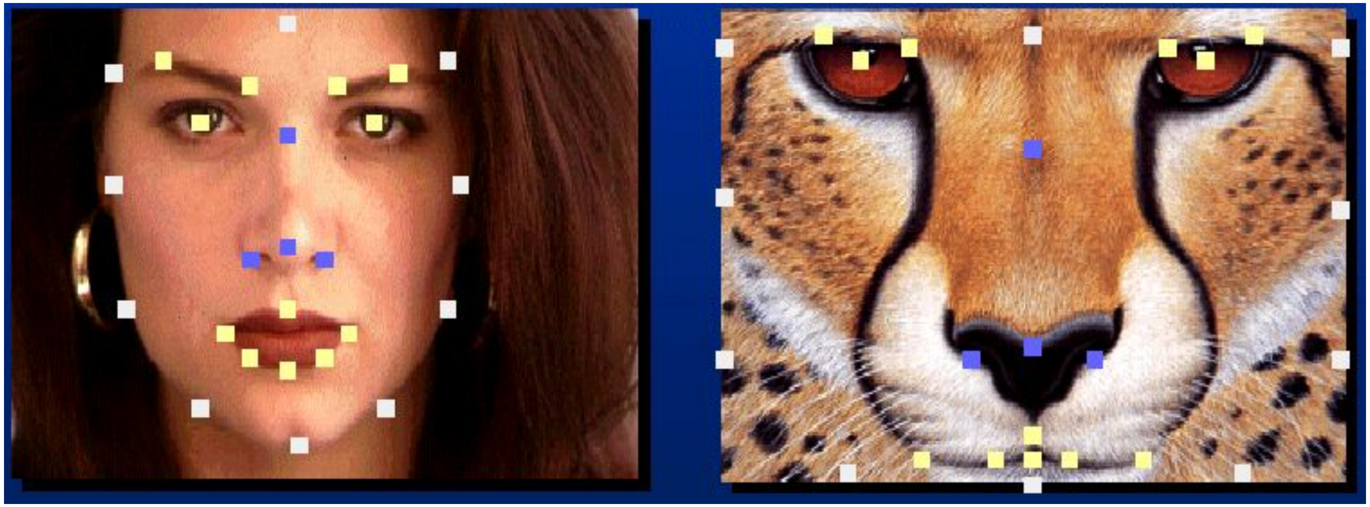
- Interpolate whole images:
- $\text{Image}_{\text{halfway}} = (1\text{-}t)^*\text{Image}_1 + t^*\text{image}_2$
- This is called **cross-dissolve** in film industry

- But what if the images are not aligned?

Idea #2: Align, then cross-disolve

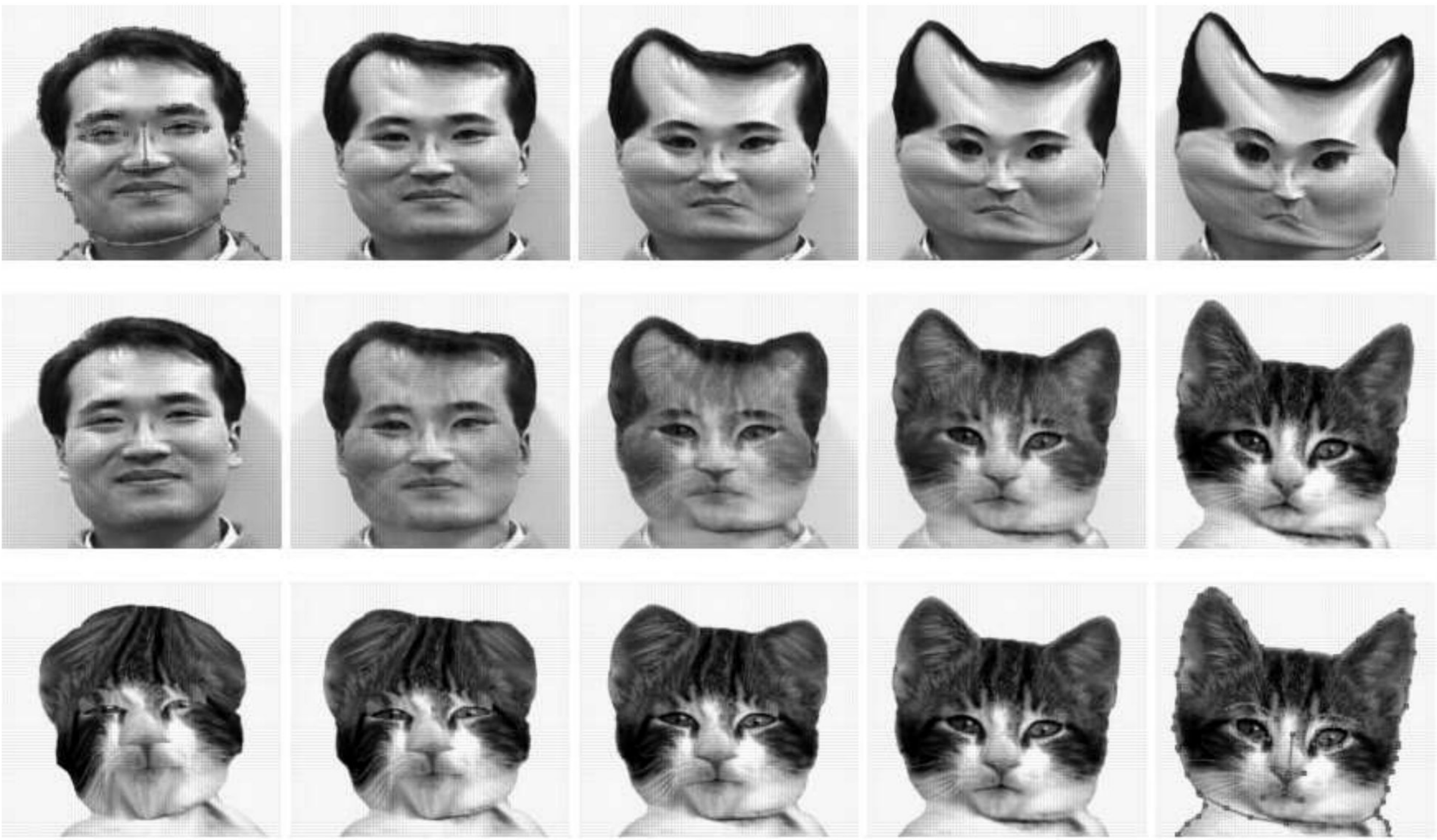
- Align first, then cross-dissolve
  - Alignment using global warp – picture still valid

Full Morphing

A  B

- What if there is no simple global function that aligns two images?
- User specifies corresponding feature points
- Construct warp animations A -> B and B -> A
- Cross dissolve these

Full Morphing



Full Morphing

Image A

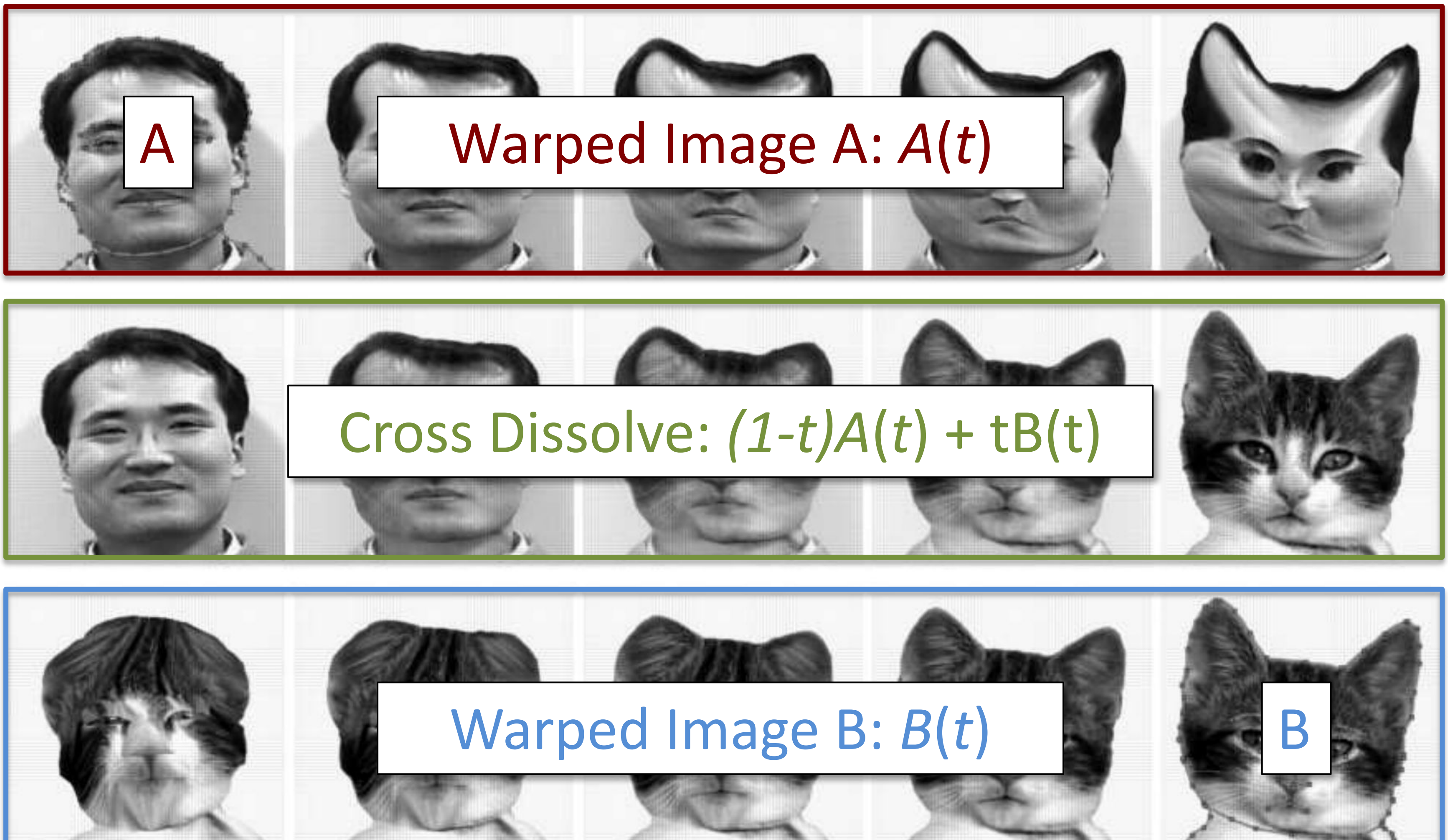
Image B

1. Find warping fields from user constraints (points or lines):
   Warp field $T_{AB}(x, y)$ that maps A pixel to B pixel
   Warp field $T_{BA}(x, y)$ that maps B pixel to A pixel

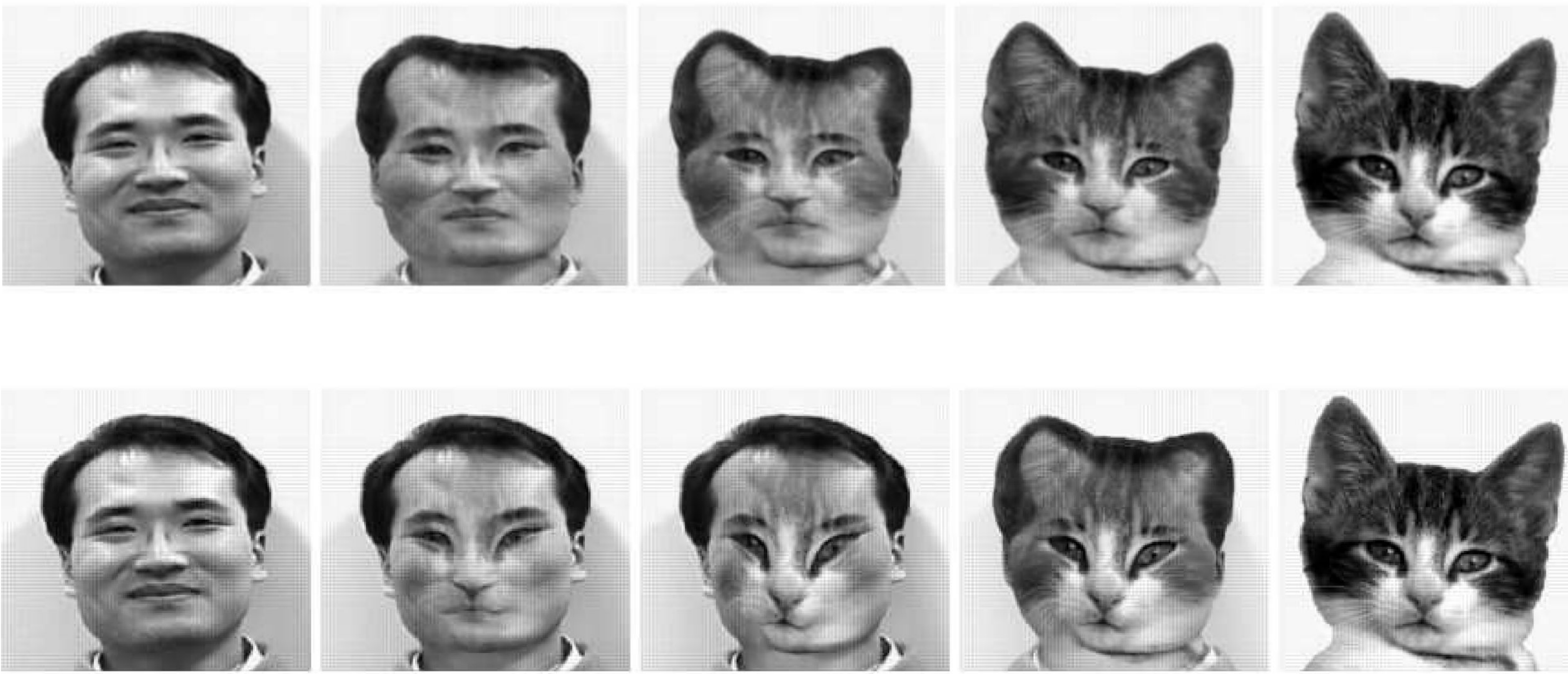
2. Make video $A(t)$ that warps A over time to the shape of B
   Start warp field at identity and linearly interpolate to T_{BA}
   Construct video $B(t)$ that warps B over time to shape of A

3. Cross dissolve these two videos.

Full Morphing



Catman!



Conclusion

- Illustrates general principle in graphics:
  - First register, then blend
- Avoids ghosting

[Michael Jackson - Black or White]