KAZAKH BRITISH TECHNICAL UNIVERSITY

Physics 2

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Lecture 2

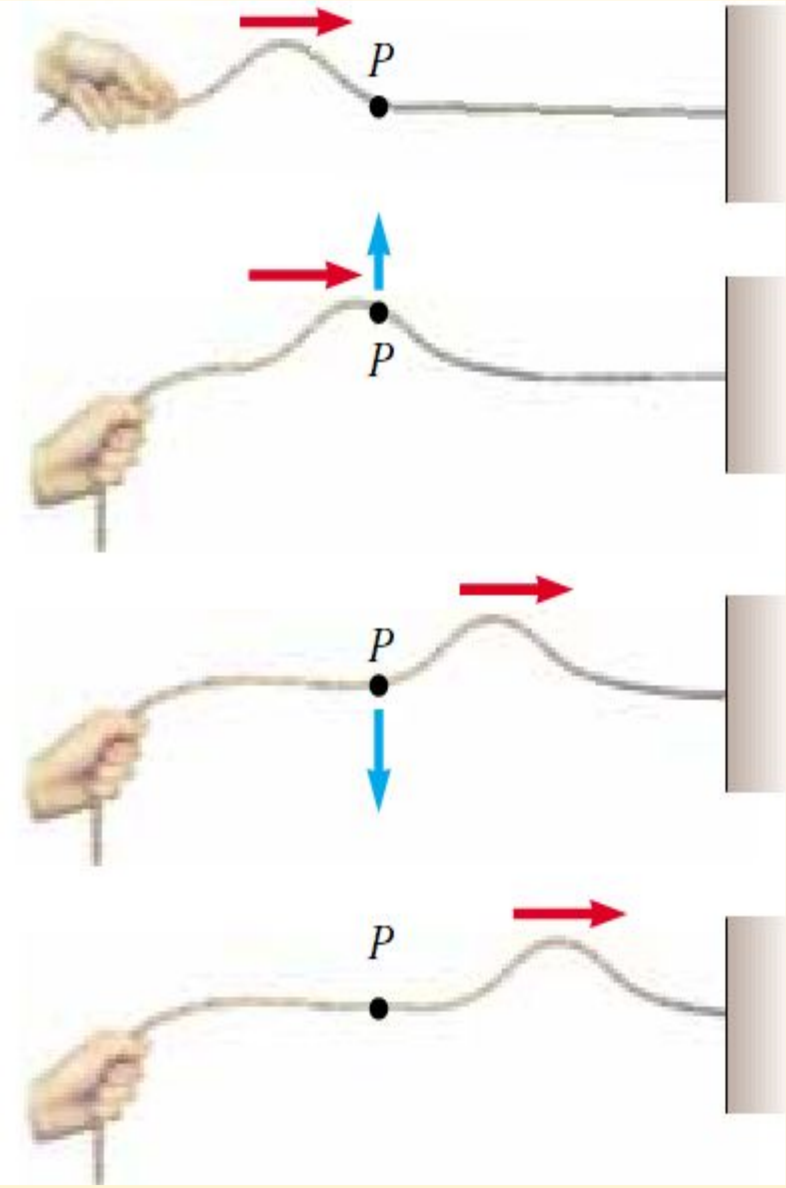
- Transverse Waves
- Longitudinal Waves
- Wave Function
- Sinusoidal Waves
- Wave Speed on a String
- Power of energy transfer
- The Doppler Effect
- Waves. The wave equation
- Electromagnetic waves. Maxwell's equations
- Poynting Vector
- Energy and Radiation Pressure

Propagation of Disturbance

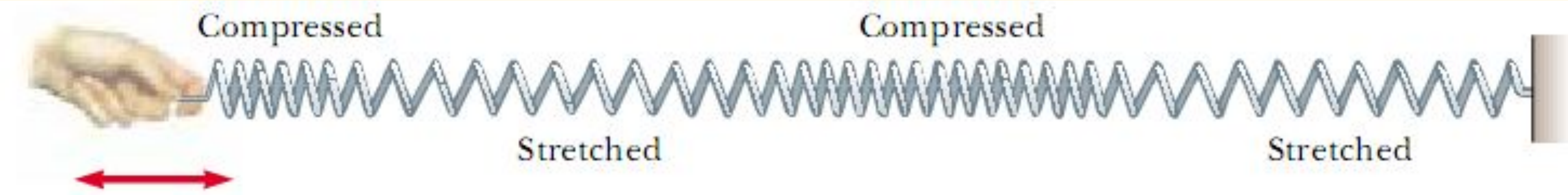
All mechanical waves require
(1) some source of disturbance,
(2) a medium that can be disturbed,
(3) some physical mechanism through which elements of the medium can influence each other.

In mechanical wave motion, energy is transferred by a physical disturbance in an elastic medium.

Transverse waves



Longitudinal Waves

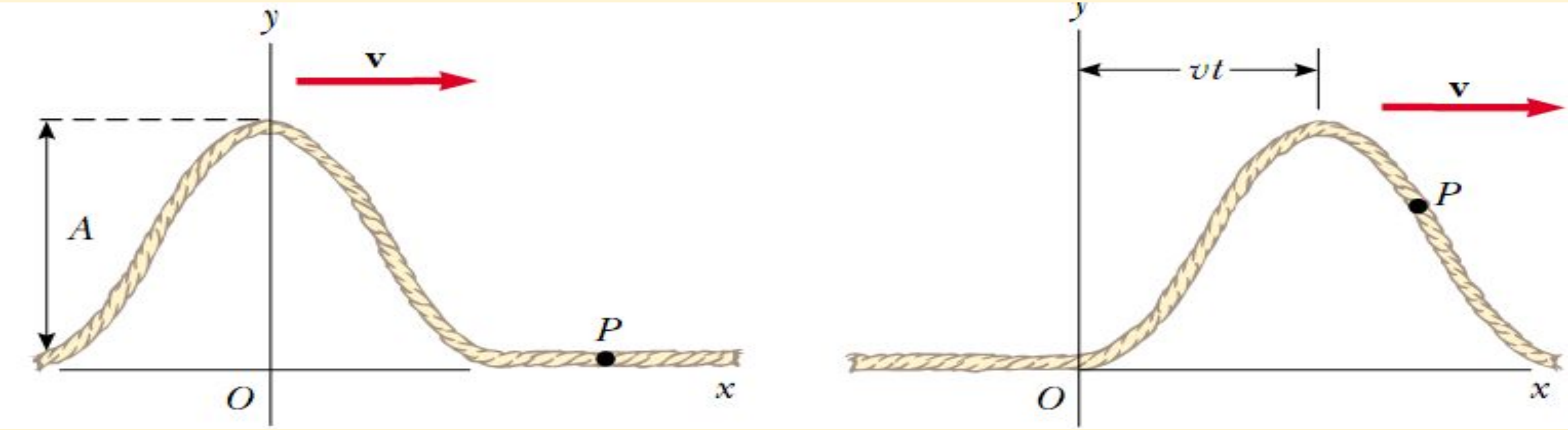


- A traveling wave or pulse that causes the elements of the medium to move parallel to the direction of propagation is called a longitudinal wave.

What Do Waves Transport?

- The disturbance travels or *propagates* with a definite speed through the medium. This speed is called the speed of propagation, or simply the **wave speed**.
- Mechanical waves transport **energy**, but not matter.

Wave Function



The shape of the pulse traveling to the right does not change with time:

$$y(x,t)=y(x-vt,0)$$

We can define transverse, or y-positions of elements in the pulse traveling to the right using $f(x)$:

$$y(x,t)=f(x-vt)$$

And for a pulse traveling to the left:

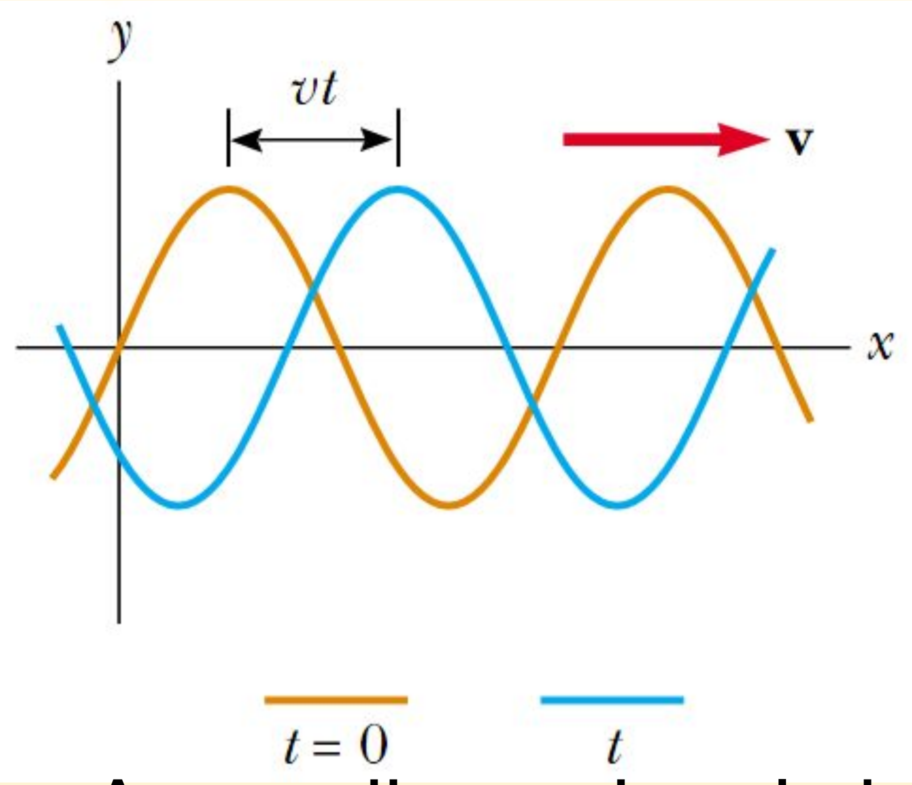
$$y(x,t)=f(x+vt)$$

The function $y(x,t)$ is called the wave function, v is the speed of wave propagation.

The wave function y(x,t) represents:

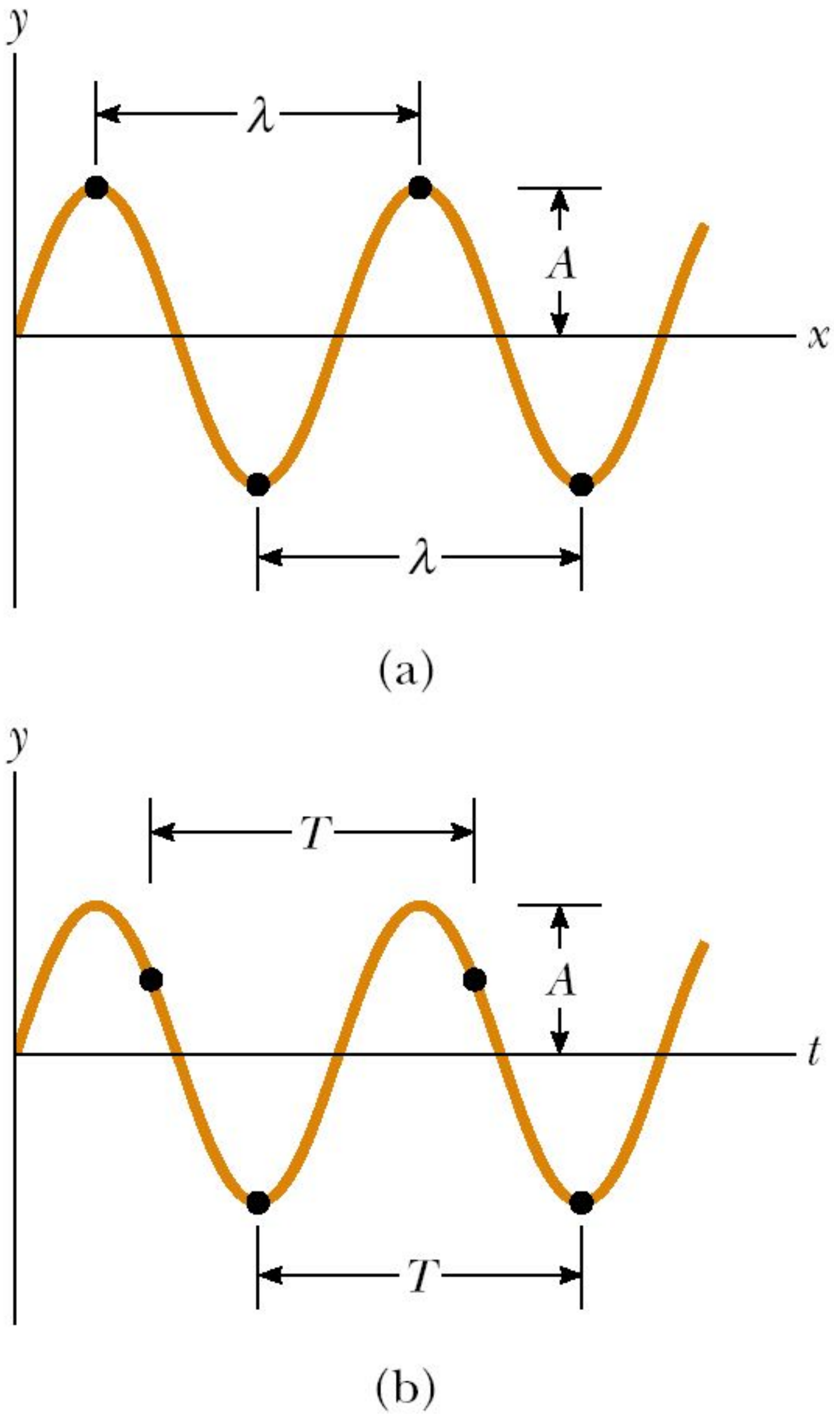
- in the case of transverse waves: the transverse position of any element located at position x at any time t
- in the case of longitudinal waves: the longitudinal displacement of a particle from the equilibrium position

Sinusoidal Waves



When the wave function is sinusoidal then we have sinusoidal wave.

A one-dimensional sinusoidal wave traveling to the right with a speed *V*. The brown curve represents a snapshot of the wave at $t = 0$, and the blue curve represents a snapshot at some later time *t*.


y
λ
A
x
(a)
y
T
A
t
(b)

The frequency of a periodic wave is the number of crests (or troughs, or any other point on the wave) that pass a given point in a unit time interval:

$$f = \frac{1}{T}$$

The sinusoidal wave function at $t=0$:

$$y(x, 0) = A \sin\left(\frac{2\pi}{\lambda} x\right)$$

The sinusoidal wave function at any t:

$$\boxed{y(x, t) = A \sin\left[\frac{2\pi}{\lambda}(x - vt)\right]}$$

If the wave travels to the left then x-vt must be replaced by $x+vt$.

$$v = \frac{\lambda}{T}$$

Then the wave function takes the form:

$$y = A \sin\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right]$$

Let's introduce new parameters:

Wave number: $$k \equiv \frac{2\pi}{\lambda}$$

Angular frequency: $$\omega \equiv \frac{2\pi}{T}$$

So the wave function is:

$$\boxed{y = A\sin(kx - \omega t)}$$

Connection of wave speed with other parameters:

$$v = \frac{\omega}{k} \qquad\qquad v = \lambda f$$

The foregoing wave function assumes that the vertical position *y* of an element of the medium is zero at *x=0* and *t=0*. This need not be the case. If it is not, we the wave function is expressed in the form:

$$y = A\sin(kx - \omega t + \phi)$$

φ is the **phase constant**.

Wave Speed on String

- If a string under tension is pulled sideways and then released, the **tension** is responsible for accelerating a particular element of the string back toward its equilibrium position. The acceleration of the element in *y*-direction increases with increasing tension, and the wave speed is greater. Thus, **the wave speed increases with increasing tension**.
- Likewise, the **wave speed should decrease as the mass per unit length of the string increases**. This is because it is more difficult to accelerate a massive element of the string than a light element.

- *T* is the tension in the string
- μ is mass per unit length of the string
- Then the wave speed on the string is

$$\boxed{v = \sqrt{\frac{T}{\mu}}}$$

- Do not confuse the *T* in this equation for the tension with the symbol *T* used for the period of a wave.

Rate of Energy Transfer by Sinusoidal Waves on Strings

Waves transport energy when they propagate through a medium.

$$\boxed{\mathcal{P} = \frac{1}{2}\mu\omega^2 A^2 v}$$

- *P* is the power or rate of energy transfer
- *m* is mass per unit length of the string
- ω is the wave angular frequency
- A is the wave amplitude
- V is the wave speed
- **In general, the rate of energy transfer in any sinusoidal wave is proportional to the square of the angular frequency and to the square of the amplitude.**

The Doppler Effect

- Doppler effect is the shift in frequency and wavelength of waves that results from a relative motion of the source, observer and medium.
- If the source of sound moves relative to the observer, then the frequency of the heard sound differs to the frequency of the source:

$$\boxed{f' = \left(\frac{v + v_O}{v - v_S}\right) f}$$

- f is the frequency of the source
- V is the speed of sound in the media
- V_S is the speed of the source relative to the media, positive direction is toward the observer
- V_O is the speed of the observer, relative to the media, positive direction is toward the source
- $f`'$ is the frequency heard by the observer
- The Doppler Effect is common for all types of waves: mechanical, sound, electromagnetic waves.

- When the source is stationary with respect to the medium the wavelength does not change.

$$\lambda` = \lambda$$

- When the source moves with respect to the medium the wavelength changes:

$$\lambda` = \lambda - v_s/f$$

- So when the observer is stationary with respect to the medium and the source approaches the observer the wavelength decreases and vice versa.

Wave Equation

- From the wave function we can get an expression for the transverse velocity $\partial y/\partial t$ of any particle in a transverse wave:

$$\boxed{\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}}$$

- $\partial y/\partial t$ means partial derivative of function $y(x,t)$ by t, keeping x constant.
- $\partial^2 y/\partial x^2$ is the second partial derivative of y with respect to x at t constant.
- y is:
  - the transverse displacement of a media particle in the case of transverse waves
  - the longitudinal displacement of a media particle from the equilibrium position in the case of longitudinal waves (or variations in either the pressure or the density of the gas through which the sound waves are propagating)
  - In the case of electromagnetic waves, y corresponds to electric or magnetic field components.
- x is the displacement of the traveling wave
- V is the wave speed: $V=dx/dt$

Electromagnetic Waves

The properties of electromagnetic waves can be deduced from Maxwell's equations:

$$\oint_S \mathbf{E} \cdot d\mathbf{A} = \frac{q}{\epsilon_0} \tag{1}$$

$$\oint_S \mathbf{B} \cdot d\mathbf{A} = 0 \tag{2}$$

$$\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt} \tag{3}$$

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I + \epsilon_0 \mu_0 \frac{d\Phi_E}{dt} \tag{4}$$

- Equation (1):

$$\oint_S \mathbf{E} \cdot d\mathbf{A} = \frac{q}{\epsilon_0}$$

- Here integration goes across an enclosed surface, q is the charge inside it.
- This is the Gauss's law: the total electric flux through any closed surface equals the net charge inside that surface divided by $\boldsymbol{\varepsilon}_0$.
- This law relates an electric field to the charge distribution that creates it.

- Equation (2):

$$\oint_S \mathbf{B} \cdot d\mathbf{A} = 0$$

- Here integration goes across an enclosed surface.

  It can be considered as Gauss's law in magnetism, states that the net magnetic flux through a closed surface is zero.
- That is, the number of magnetic field lines that enter a closed volume must equal the number that leave that volume. This implies that magnetic field lines cannot begin or end at any point. It means that there is no isolated magnetic monopoles exist in nature.

- Equation (3):

$$\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$$

- Here integration goes along an enclosed path, Φ_B is a magnetic flux through that enclosed path.
- This equation is Faraday's law of induction, which describes the creation of an electric field by a changing magnetic flux.
- This law states that the emf, which is the line integral of the electric field around any closed path, equals the rate of change of magnetic flux through any surface area bounded by that path.

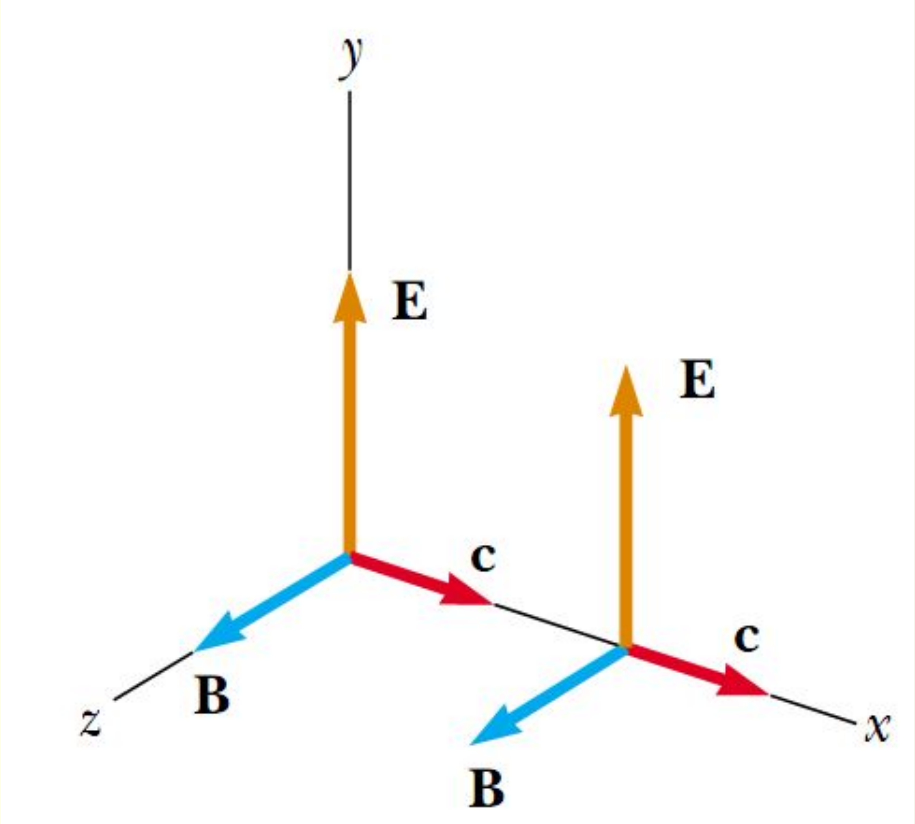
- Equation (4):

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I + \epsilon_0 \mu_0 \frac{d\Phi_E}{dt}$$

- This is Ampère–Maxwell law, or the generalized form of Ampère's law. It describes the creation of a magnetic field by an electric field and electric currents. the line integral of the magnetic field around any closed path is the sum of μ_0 times the net current through that path and $\varepsilon_0\mu_0$ times the rate of change of electric flux through any surface bounded by that path.

Plane-Wave Assumption

We assume that an electromagnetic wave travels in the x-direction. In this wave, the electric field E is in the y-direction, and the magnetic field B is in the z-direction. Waves such as this one, in which the electric and magnetic fields are restricted to being parallel to a pair of perpendicular axes, are said to be linearly polarized waves. Furthermore, we assume that at any point in space, the magnitudes E and B of the fields depend upon x and t only, and not upon the y or z coordinate.



An electromagnetic wave traveling at velocity c in the positive x-direction. The electric field is along the y-direction, and the magnetic field is along the z-direction. These fields depend only on x and t.

- In empty space there is no currents and free charges: *I=0, q=0,* then the 4-th Maxwell's equation turns into:

$$\oint \mathbf{B} \cdot d\mathbf{s} = \epsilon_0 \mu_0 \frac{d\Phi_E}{dt}$$

- Using it with the 3-d Maxwell's equation

$$\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$$

and the plane-wave assumption, we obtain the following differential equations relating *E* and *B:*

$$\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$$

$$\frac{\partial B}{\partial x} = -\mu_0 \epsilon_0 \frac{\partial E}{\partial t}$$

And eventually we obtain:

$$\boxed{\frac{\partial^2 E}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2}}$$

$$\boxed{\frac{\partial^2 B}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 B}{\partial t^2}}$$

These two equations both have the form of the general wave equation with the wave speed v replaced by c, the speed of light:

$$\boxed{c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}}$$

- μ_0 is the free space magnetic permeability:

$$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}$$

- ε_0 is the free space electric permeability:

$$\epsilon_0 = 8.854\,19 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2}$$

- c is the speed of light in vacuum:

$$c = 2.997\,92 \times 10^{8}\ \mathrm{m/s}$$

$$\frac{\partial^2 E}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2} \qquad \frac{\partial^2 B}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 B}{\partial t^2}$$

$$E = E_{\max} \cos(kx - \omega t)$$

$$B = B_{\max} \cos(kx - \omega t)$$

$$\boxed{\frac{E_{\max}}{B_{\max}} = \frac{E}{B} = c}$$

Electromagnetic Waves Properties (Summary)

- The solutions of Maxwell's third and fourth equations are wave-like, with both E and B satisfying a wave equation.
- Electromagnetic waves travel through empty space at the speed of light c.
- The components of the electric and magnetic fields of plane electromagnetic waves are perpendicular to each other and perpendicular to the direction of wave propagation. So, electromagnetic waves are transverse waves.
- The magnitudes of E and B in empty space are related by the expression $E/B = c$.
- Electromagnetic waves obey the principle of superposition.

Poynting Vector

The rate of flow of energy in electromagnetic waves is

$$\boxed{\mathbf{S} \equiv \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B}}$$

- ***S*** is called the **Poynting vector.** The magnitude of the Poynting vector represents the rate at which energy flows through a unit surface area perpendicular to the direction of wave propagation. Thus, the magnitude of the Poynting vector represents power per unit area. The direction of the vector is along the direction of wave propagation.

Energy of Electromagnetic Waves

Electromagnetic waves carry energy with total instantaneous energy density:

$$u = u_E + u_B = \epsilon_0 E^2 = \frac{B^2}{\mu_0}$$

This instantaneous energy is carried in equal amounts by the electric and magnetic fields:

$$u_E = \frac{1}{2}\epsilon_0 E^2 \qquad u_B = \frac{B^2}{2\mu_0}$$

$$u_B = u_E = \frac{1}{2}\epsilon_0 E^2 = \frac{B^2}{2\mu_0}$$

When this total instantaneous energy density is averaged over one or more cycles of an electromagnetic wave, we obtain a factor of 1/2. Hence, for any electromagnetic wave, the total average energy per unit volume is

$$\boxed{u_{av} = \epsilon_0 (E^2)_{av} = \frac{1}{2}\epsilon_0 E_{max}^2 = \frac{B_{max}^2}{2\mu_0}}$$

Pressure of Electromagnetic Waves

Electromagnetic waves exert pressure on the surface. If the surface is absolutely absorbing, then the pressure per unit area of the surface is

$$P = \frac{S}{c}$$

In the case of absolutely reflecting surface, the pressure per unit area of the surface doubles:

$$P = \frac{2S}{c}$$

Units in Si

- Wavenumber k rad/m
- Phase constant φ rad
- Poynting vector S W/m^2