Arithmetic fundamentals of number systems

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Outline

- Different number systems
- Why use different ones?
- Binary / Octal / Hexadecimal
- Conversions
- Negative number representation
- Binary arithmetic
- Overflow / Underflow

Number Systems

Four number systems:

- Decimal (10)
- Binary (2)
- Octal (8)
- Hexadecimal (16)

Binary numbers

- Computers work only on two states
  - On
  - Off
- Basic memory elements hold only two states
  - Zero / One
- Thus a number system with two elements {0,1}
- A binary digit – bit !

Decimal numbers

$$1439 = 1 \times 10^{3} + 4 \times 10^{2} + 3 \times 10^{1} + 9 \times 10^{0}$$

Thousandsㅤ Hundreds ㅤTens ㅤOnes

- Radix = 10

Binary ➔ Decimal

$$1101 = 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$$

$$= 1 \times 8 + 1 \times 4 + 0 \times 2 + 1 \times 1$$

$$= 8 + 4 + 0 + 1$$

$$\boxed{(1101)_2 = (13)_{10}}$$

1, 2, 4, 8, 16, 32, 64, 128, 256, 512, …

Decimal ➔ Binary

2	13	1	LSB
2	6	0	
2	3	1	
2	1	1	MSB
	0		

$$\boxed{(13)_{10} = (1101)_2}$$

Octal ➔ Decimal

$137 = 1 \times 8^2 + 3 \times 8^1 + 7 \times 8^0$

$= 1 \times 64 + 3 \times 8 + 7 \times 1$

$= 64 + 24 + 7$

$\boxed{(137)_8 = (95)_{10}}$

- Digits used in Octal number system – 0 to 7

Decimal ➔ Octal

8	95	7	LSP
8	11	3	
8	1	1	
	0		MSP

$$\boxed{(95)_{10} = (137)_8}$$

Hex ➔ Decimal

$$BAD = 11 \times 16^2 + 10 \times 16^1 + 13 \times 16^0$$

$$= 11 \times 256 + 10 \times 16 + 13 \times 1$$

$$= 2816 + 160 + 13$$

$$\boxed{(BAD)_{16} = (2989)_{10}}$$

A = 10, B = 11, C = 12, D = 13, E = 14, F = 15

Decimal ➔ Hex

16	2989	13	LSP
16	186	10	
16	11	11	
	0		MSP

$$\boxed{(2989)_{10} = (BAD)_{16}}$$

Why octal or hex?

- Ease of use and conversion
- Three bits make one octal digit

  111 010 110 101

  7 2 6 5 => 7265 in octal

- Four bits make one hexadecimal digit

  1110 1011 0101 → 4 bits = nibble

  E B 5 => EB5 in hex

Roman Numerals

A Brief History of Roman Numerals

- Roman numerals originated in ancient Rome. This ancient counting system is believed to have started with the ancient Etruscans.
- The symbol for one in the roman numeral system probably represented a single tally mark which people would notch into wood or dirt to keep track of items or events they were counting. It would also be easy to write on a wax tablet.

Arabic numbers $\longrightarrow$ Roman numerals conversion

- Roman numerals are written as combinations of seven letters.

  I = 1 V = 5 X = 10 L = 50

  C = 100 D =500 M = 1000
- The letters can be written as capital (XVI) or lower-case letters (xvi).

As a general guide

- Roman Numerals are made up by adding or subtracting numbers like this:-
- 11=10+1 = XI 9 = 10 – 1 = IX
- 40 = 50 –10 = XL
- If you want to say 1,100 in Roman Numerals, you would say M for 1000 and then put a C after it for 100; 1,100 = MC
- 900 =1000 – 100 so the C comes before M = CM

Some more examples:

- VIII = 5+3 = 8
- XIX = 10+ 9 = 19
- (Remember 9 is always = IX (1 less than 10)
- XL = 50-10 = 40
- XC = 100-10 = 90
- Try these on whiteboards:

  7 =   12 =   15 =   20 =

Check your answers.

7 = VII

12 = XII

15 = XV

20 = XX

Can you convert these numbers to Roman Numerals?

- 17 = 22 = 26 = 29 = 30 =
- 32 = 35 = 50 = 40 =
- 44 = 49 = 58 = 60 =

Were you correct?

- 17=XVII 22=XXII 26=XXVI 29=XXIX
- 30=XXX 32=XXXII 35=XXXV
- 50=L 40=XL
- 44=XLIV 49=XLIX 58=LVIII 60=LX

Some more large numbers to try:

- 600 = 700 = 800 =
- 1000 = 900 =
- 1600 = 1700 = 1900 =
- 2000 =

Check your answers.

- 600 = DC 700 = DCC 800 = DCCC
- 1000 = M 900 = CM
- 1600 =MDC 1700 =MDCC
- 1900 = MCM 2000 =MM

The last one

- Can you convert 2017?
- MMXVII

Now try to write today's date.

Day / Month / Year

- Well done. You are a Roman Numeral Converter!

Binary Arithmetic

- Addition
- Subtraction

Addition

Like normal decimal addition

B

A

+	0	1
0	0	1
1	1	10

0101 (5)

+ 1001 (9)

1110 (14)

The carry out of the MSB is neglected

Subtraction

Like normal decimal subtraction

B

A

-	0	1
0	0	11
1	1	0

 1001 (9)

\- 0101 (5)

 0100 (4)

A borrow (shown in red) from the MSB implies a negative