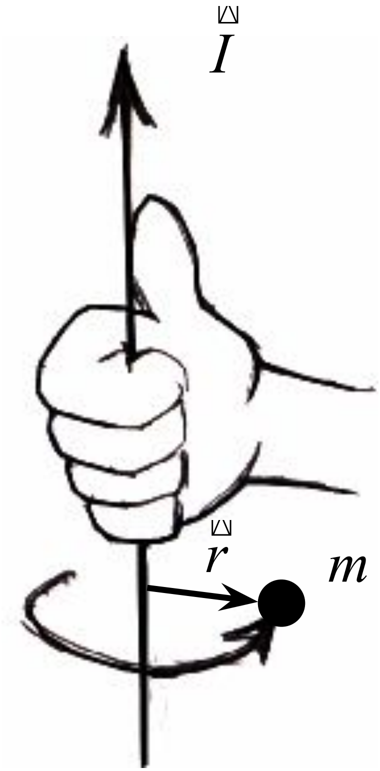
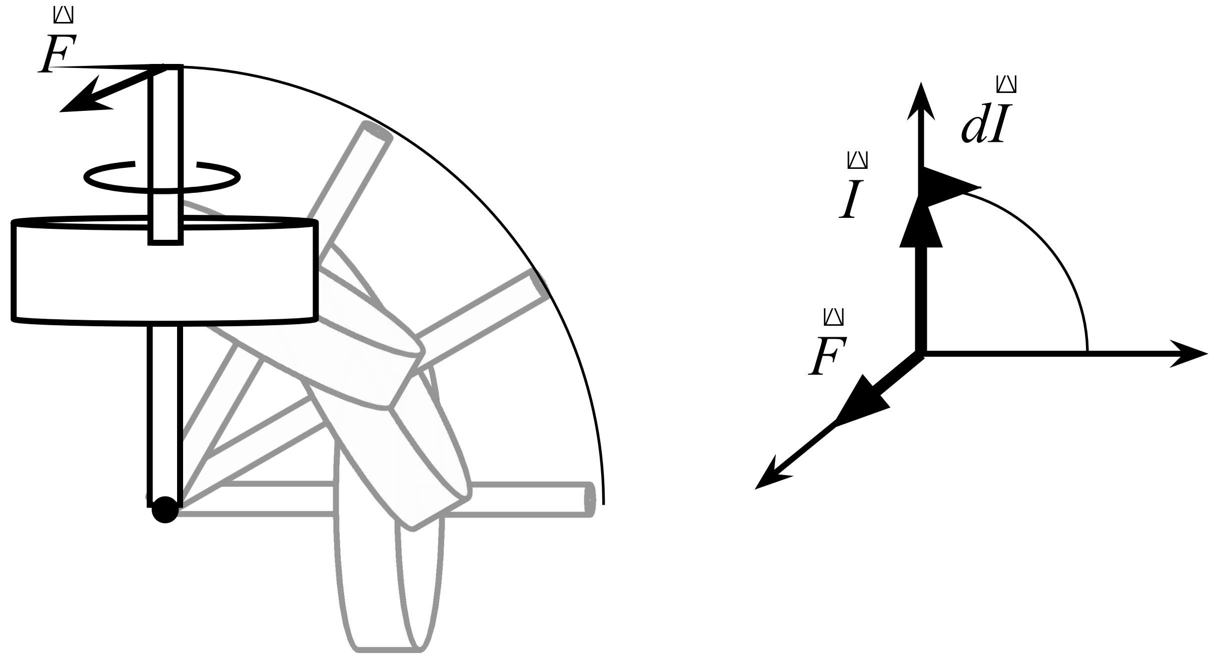
Reminder: classical angular momentum and magnetic dipole moment

Classical angular momentum



right hand grip rule



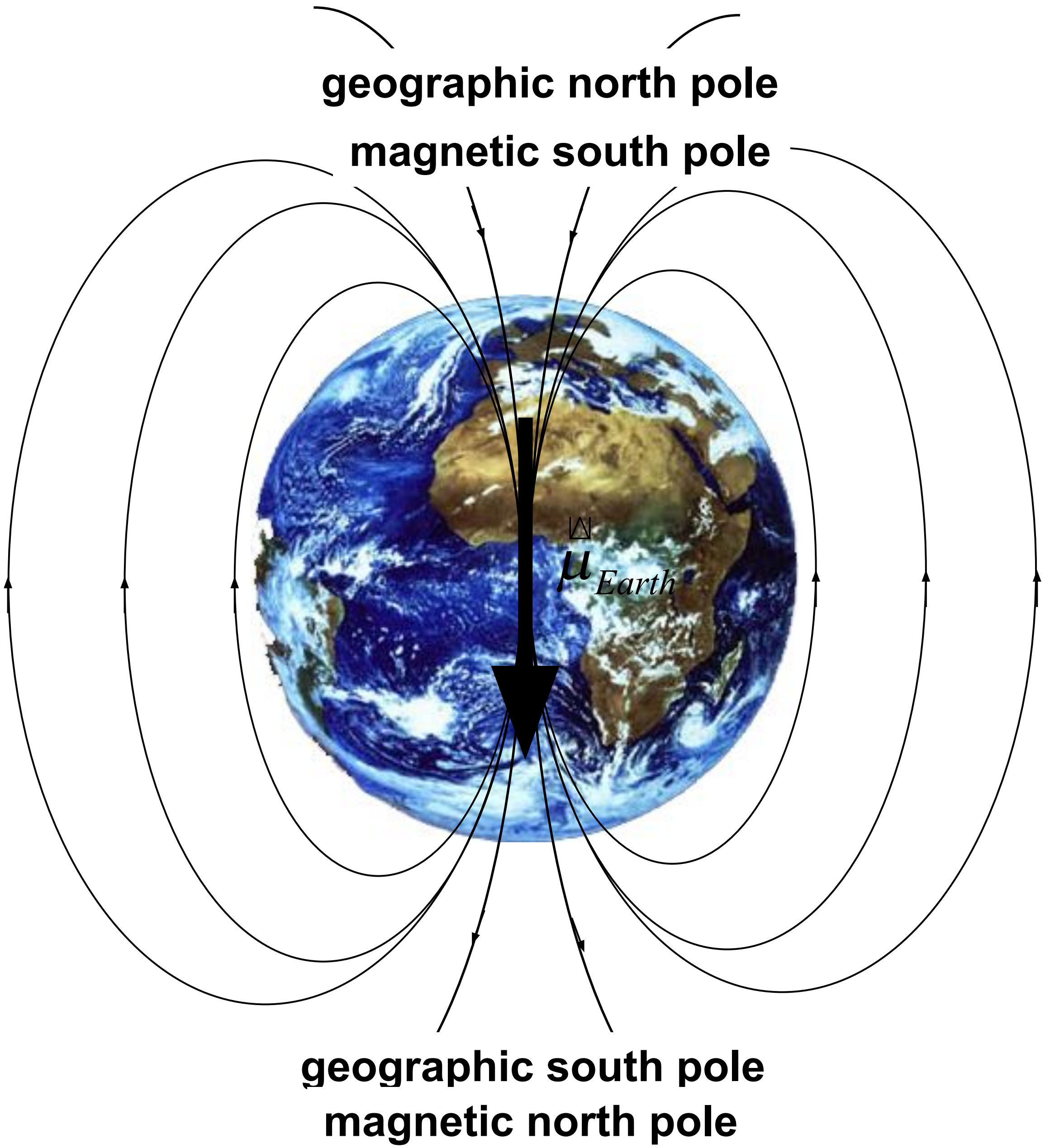
torque changes the anglular momentum

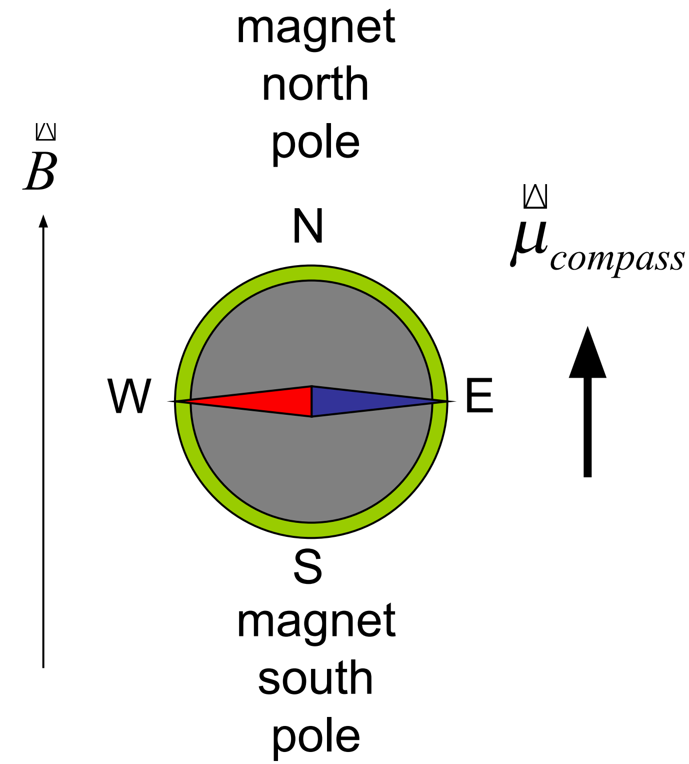
$\vec{I} = \vec{r} \times m\vec{V}$ – angular momentum

$\vec{D} = \vec{r} \times \vec{F}$ – torque

$$\vec{D} = \frac{d\vec{I}}{dt}$$

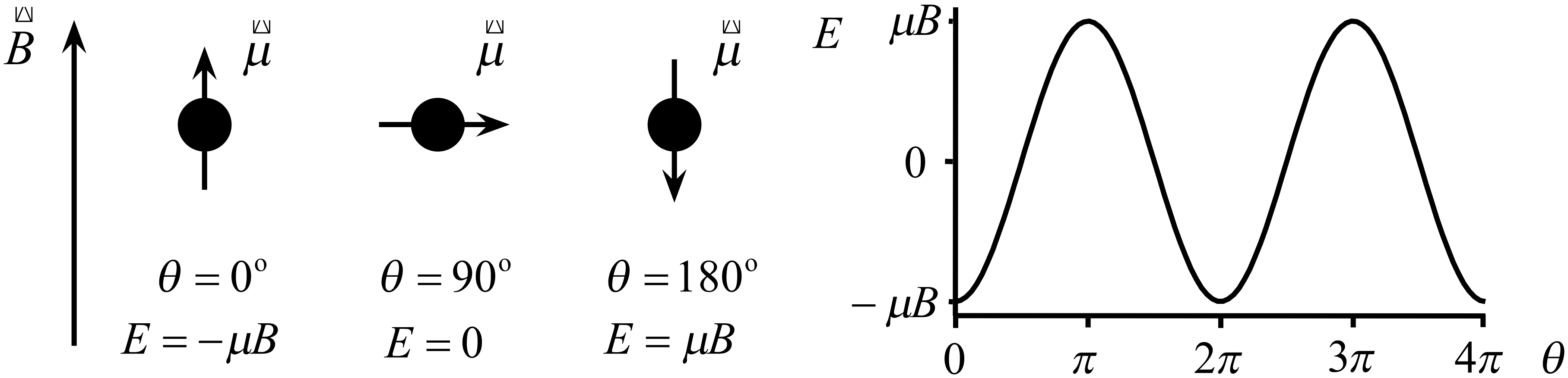
Magnetism


geographic north pole
magnetic south pole
$\vec{\mu}_{Earth}$
geographic south pole
magnetic north pole


$\vec{B}$
magnet
north
pole
N
W
E
S
magnet
south
pole
$\vec{\mu}_{compass}$

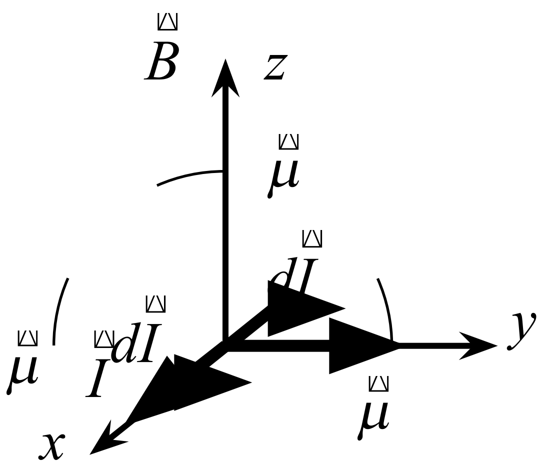
Classical magnetic dipole moment

$$E = -\vec{\mu}\cdot\vec{B} = -|\vec{\mu}||\vec{B}|\cos\theta = -\mu B\cos\theta$$

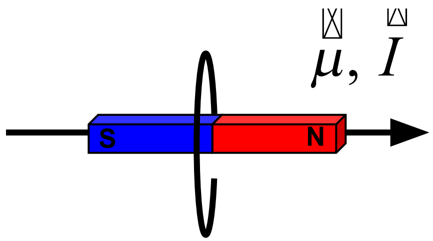


oscillations of magnetic moment in magnetic field

$$\vec{D} = \vec{\mu}\times\vec{B} = \frac{d\vec{I}}{dt}$$

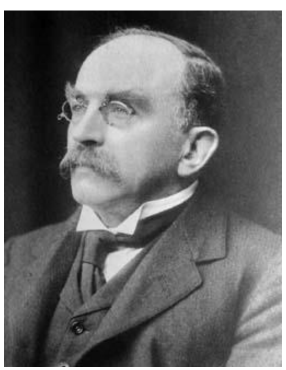


Classical magnetic dipole with angular momentum

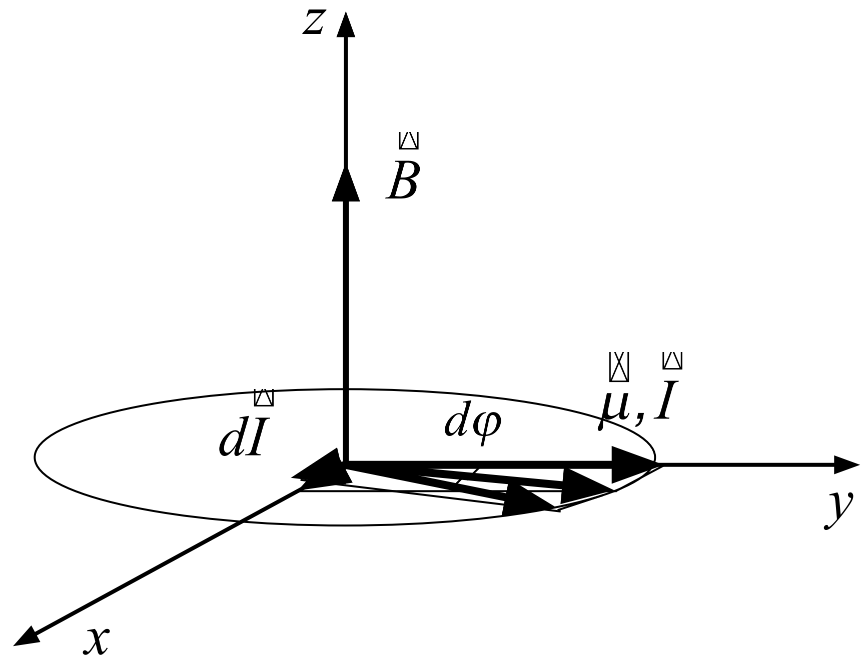


$\mu = \gamma \vec{I}$

γ – gyromagnetic ratio



Joseph Larmor
1857-1942
Cambridge University



precession with $\omega = -\gamma B$

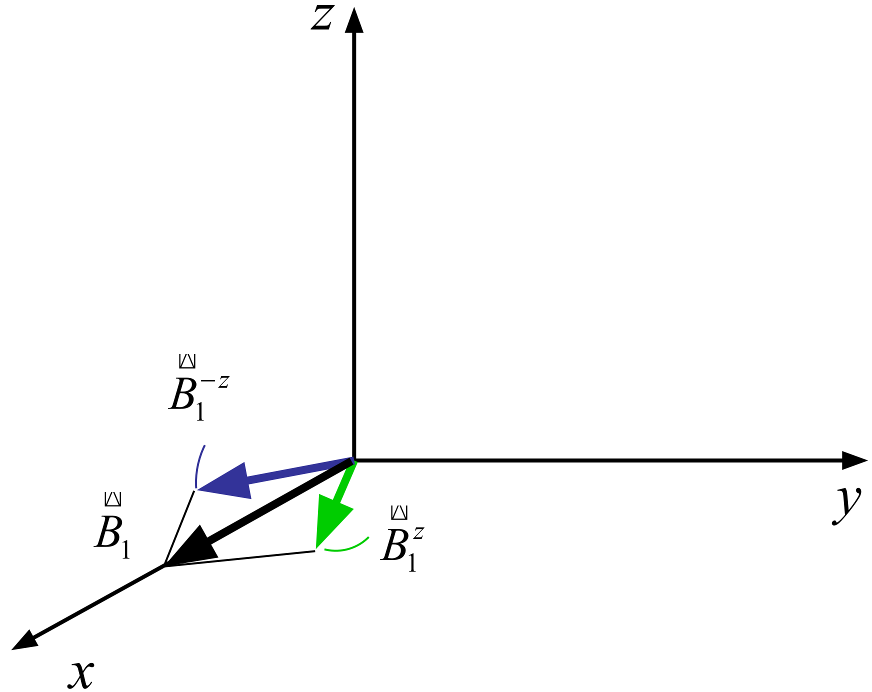
ω – Larmor frequency

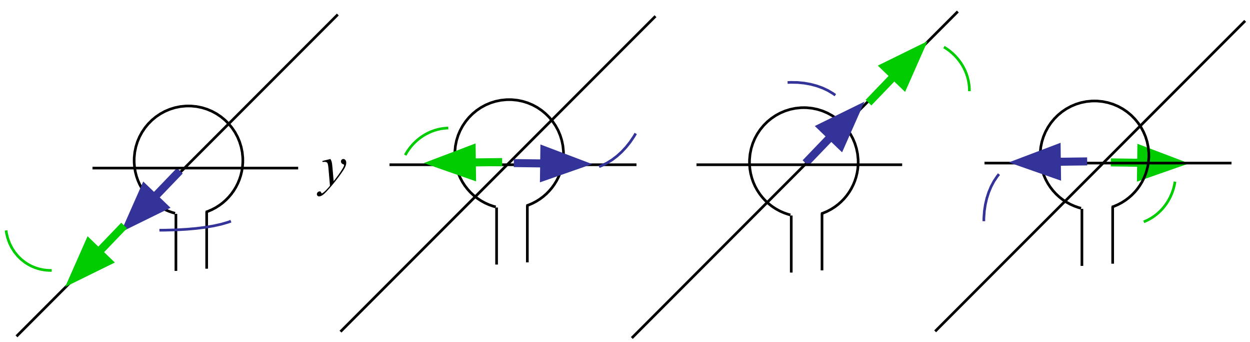
Magnetic field of the pulse

$$\vec{B}_1 = \begin{pmatrix} 2B_1\cos\omega_{ref}t \\ 0 \\ 0 \end{pmatrix} = \vec{B}_1^{z} + \vec{B}_1^{-z}$$

$$\vec{B}_1^{z} = \begin{pmatrix} B_1\cos\omega_{ref}t \\ B_1\sin\omega_{ref}t \\ 0 \end{pmatrix}$$

$$\vec{B}_1^{-z} = \begin{pmatrix} B_1\cos\omega_{ref}t \\ -B_1\sin\omega_{ref}t \\ 0 \end{pmatrix}$$

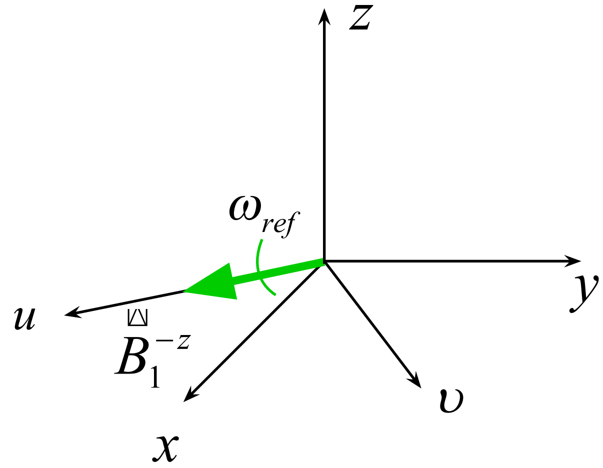


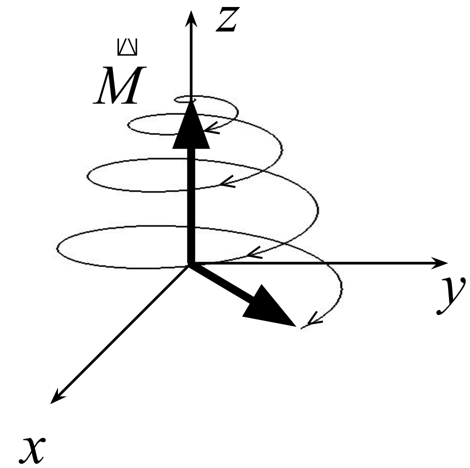


Pulse

laboratory frame

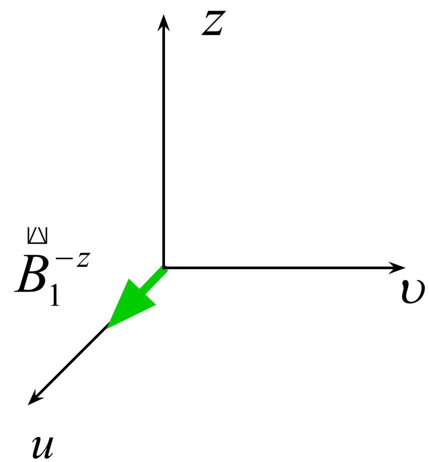
$$\begin{pmatrix} B_1 \cos \omega_{ref} t \\ -B_1 \sin \omega_{ref} t \\ 0 \end{pmatrix}$$

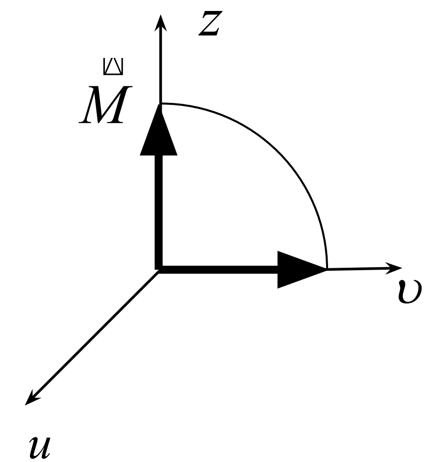




rotating frame

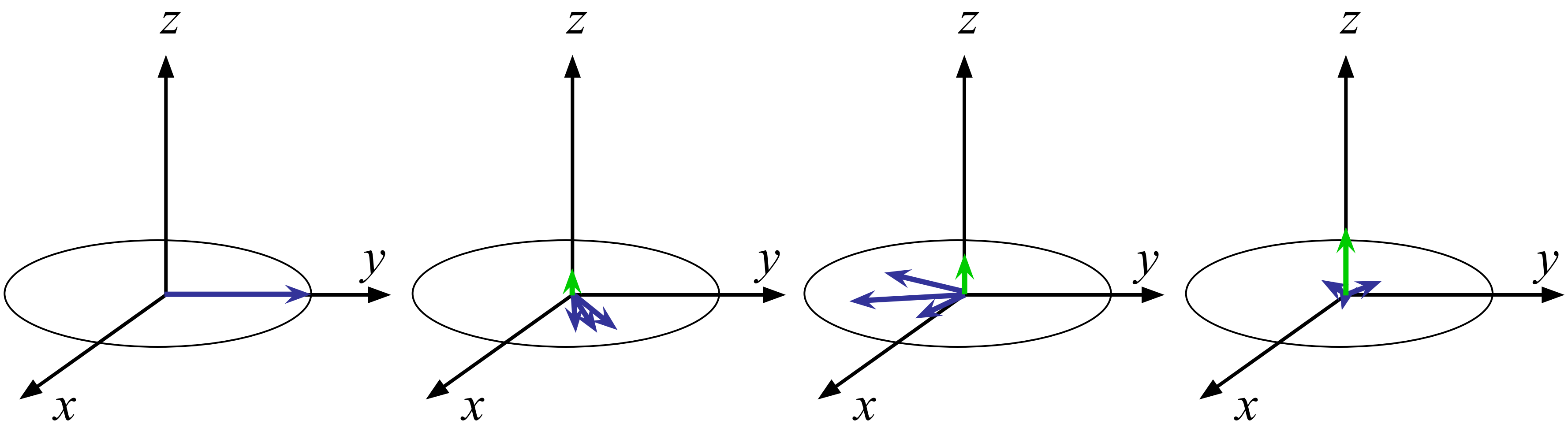
$$\begin{pmatrix} B_1 \\ 0 \\ 0 \end{pmatrix}$$



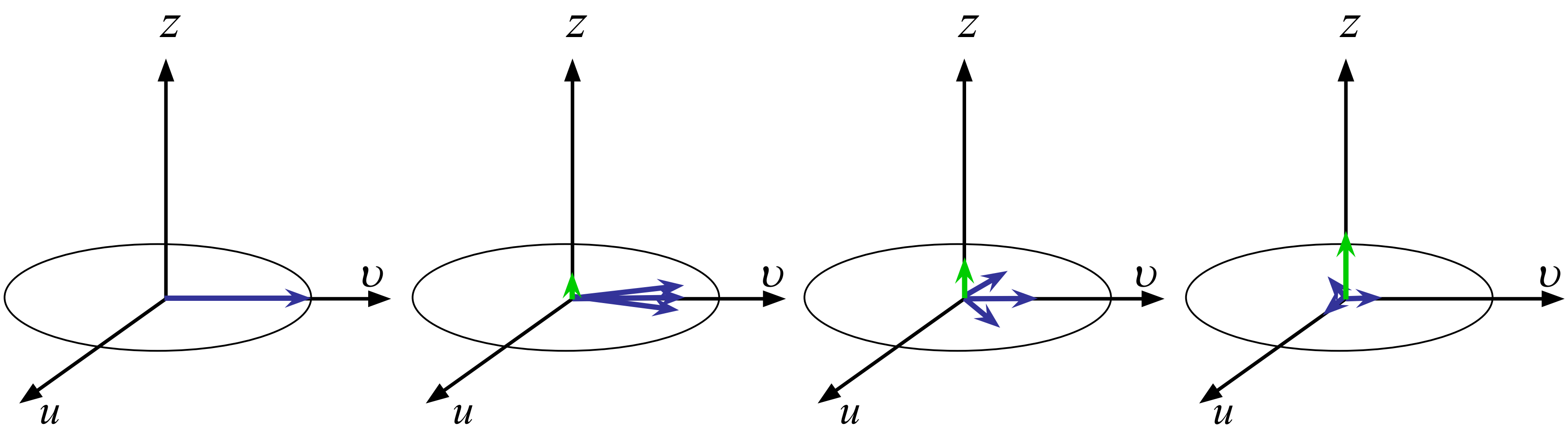


longitudinal and transverse relaxation

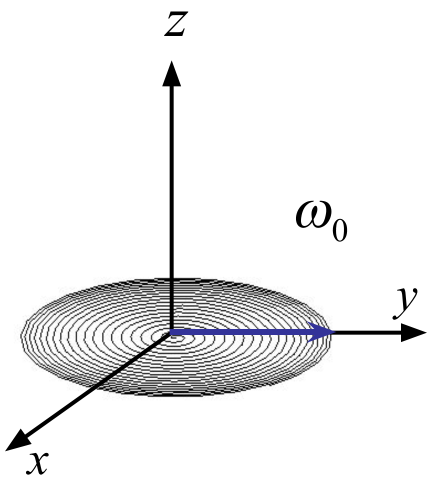
laboratory frame



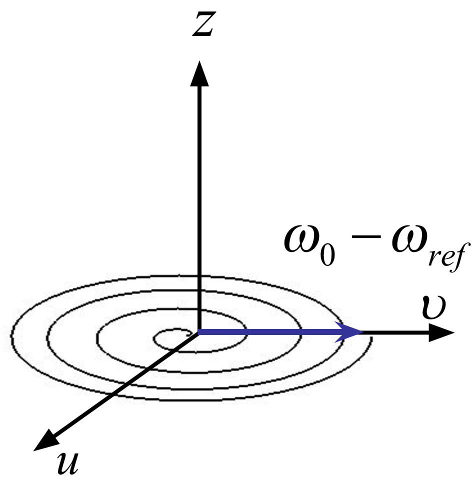
rotating frame



Free Induction Decay



laboratory frame



rotating frame

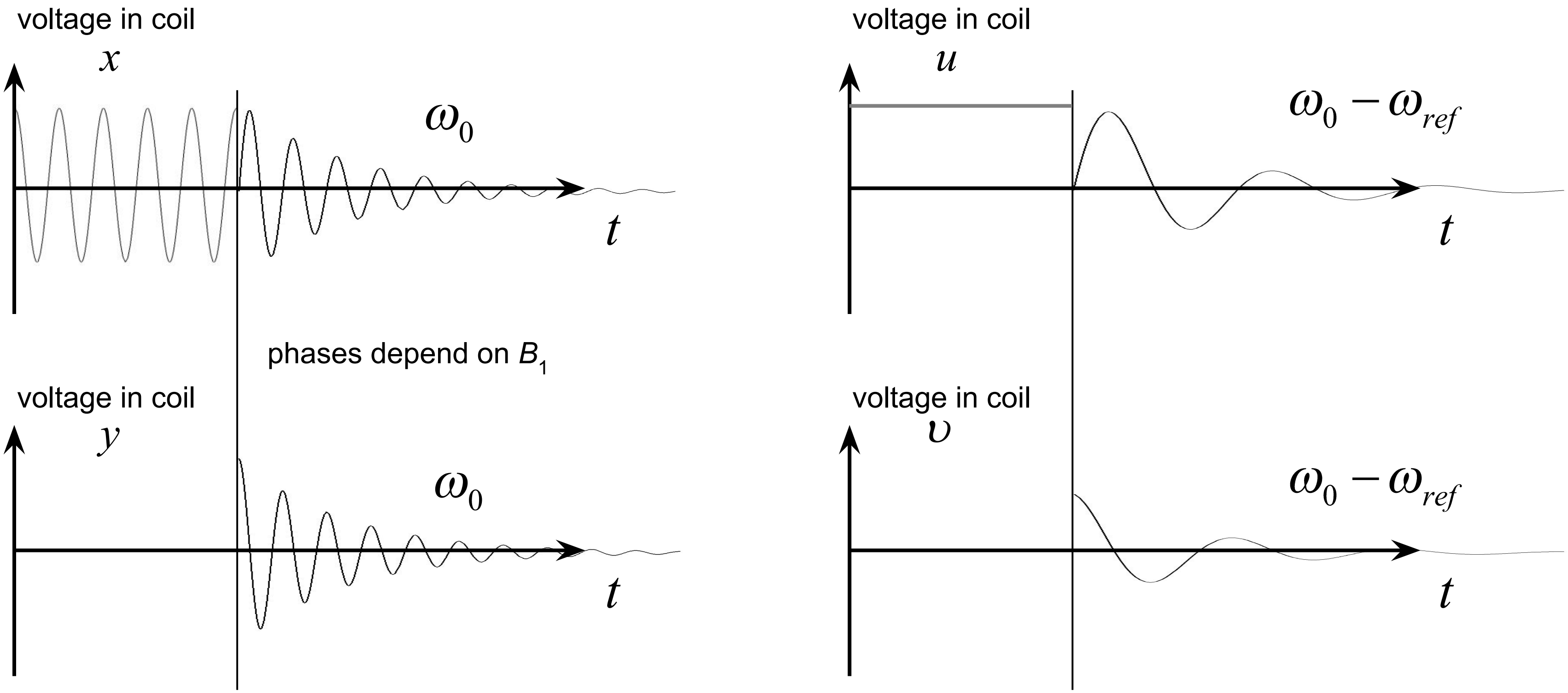
$$M_x = M_\infty e^{-t/T_2} \sin \omega_0 t$$

$$M_y = M_\infty e^{-t/T_2} \cos \omega_0 t$$

$$M_u = M_\infty e^{-t/T_2} \sin(\omega_0 - \omega_{ref})t$$

$$M_\upsilon = M_\infty e^{-t/T_2} \cos(\omega_0 - \omega_{ref})t$$

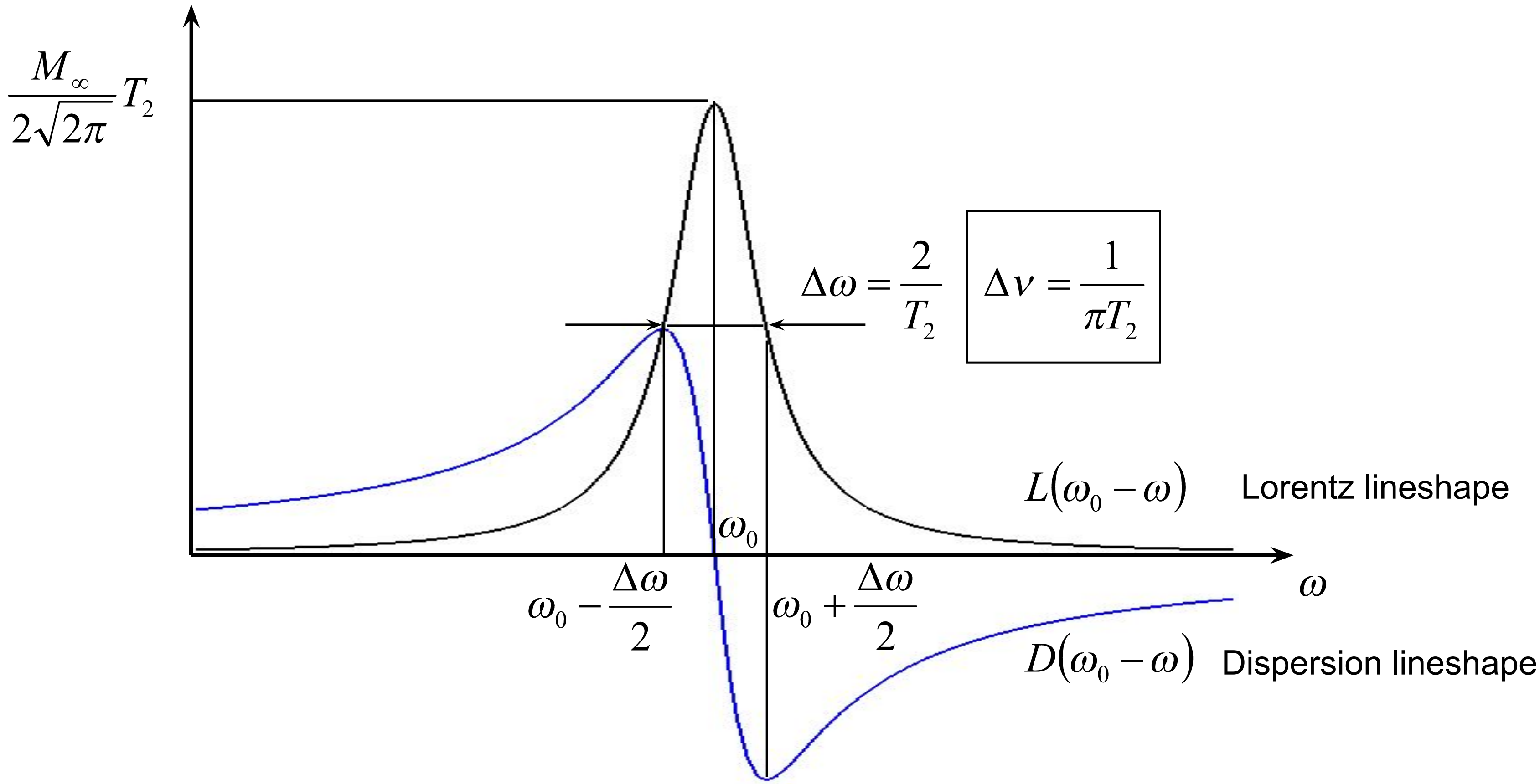
Free Induction Decay



laboratory frame

rotating frame

Spectral lines



$$\int_{-\infty}^{+\infty} L(\omega_0 - \omega)d\omega = \frac{1}{2}\sqrt{\frac{\pi}{2}}M_0$$