Winston p. 313, # 2

2 Consider the following LP:

$$\begin{aligned} \max z &= -2x_1 - x_2 + x_3 \\ \text{s.t.} \quad x_1 + x_2 + x_3 &\le 3 \\ x_2 + x_3 &\ge 2 \\ x_1 + x_3 &= 1 \\ x_1, x_2, x_3 &\ge 0 \end{aligned}$$

- Write the dual of this LP. Use y_1, y_2, and y_3 for the dual variables of the constraints.

Winston p. 313, # 2

Dual:

min $3y_1 + 2y_2 + y_3$

subject to

$y_1 + y_3 \geq -2$

$y_1+y_2 \geq -1$

$y_1 + y_2 + y_3 \geq 1$

$y_1 \geq 0$, $y_2 \leq 0$, y_3 unrestricted

Winston p. 815, #11

TABLE 19

−5	−10	−1	−10	2	−1
−1	2	−10	7	−5	20
2	7	−5	−10	−10	7
7	20	−1	−1	−1	2
20	7	−10	7	−1	−10

- Find the floor and ceiling of the value of this game. Does it have a saddle point? If so, what's the value of the game?

Winston p. 815, #11

TABLE 19

							min
	−5	−10	−1	−10	2	−1	-1
	−1	2	−10	7	−5	20	-10
	2	7	−5	−10	−10	7	-10
	7	20	−1	−1	−1	2	-1
	20	7	−10	7	−1	−10	-10
max	20	20	-1	7	2	20	

This game has a saddle point; floor = value = ceiling = -1

Schrage #4

- The next 5 slides show the crop recourse homework problem I assigned, plus the MPL code
- Modify the code to compute the *worst-case* probabilities of a wet or dry season
- **Hint: you only have to add *one* variable, modify the objective function, and add *one* set of constraints; the other constraints in the model stay the same**
- What are the worst-case probabilities? How does the overall expected cost change?

Schrage #4 (Formulate Only)

Problem 4

A farmer has 1000 acres available for planting to either corn, sorghum or soybeans. The yields of the three crops, in bushels/acre, as a function of the two possible kinds of season are:

	Corn	Sorghum	Beans
Wet	100	43	45
Dry	45	35	33

The probability of a wet season is 0.6. The probability of a dry season is 0.4. Corn sells for $2/bushel, whereas sorghum and beans each sell for $4/bushel. The total production cost for any crop is $100/acre, regardless of type of season. The farmer can also raise livestock. One unit of livestock uses one hundred bushels of corn. The profit contribution of one unit of livestock, exclusive of its corn consumption, is $215. Corn can be purchased at any time on the market for $2.20/bushel. The decisions of how much to raise of livestock and of each crop must be made before the type of season is known.

(a) What should the farmer do?

(b) Formulate and solve the problem by the scenario-based stochastic programming approach.

Schrage Handout, #4

- Indices
  - s = season {wet,dry}
  - c = crops {corn, sorg, bean}
- Data
  - $YIELD_{cs}$ = yield/acre (bushels) of crop c in season s
  - $PROB_{s}$ = probablility of season s
  - $SPRICE_{s}$ = sale price ($) per bushel of crop c
  - PCOST = production cost/acre ($) for crops
  - $LCROP_{c}$ = bushels of crop c required per “unit” of livestock
  - LPROFIT = profit/unit ($) of livestock
  - $MCOST_{c}$ = cost/bushel ($) of crop c on open market
  - ACRES = total acreage available for planting
  - $MAXCROP_{c}$ = maximum bushels of crop c that can be bought on the open market
- Variables
  - live = units of livestock to raise
  - $pcrop_{c}$ = acres of crop c to plant
  - $bcrop_{cs}$ = bushels of crop c bought under scenario s
  - $csold_{cs}$ = bushels of crop c sold in scenario s

Schrage Handout, #4 (cont'd)

$$\max \quad totprofit = (LPROFIT * live) + \qquad \{\text{livestock profit}\}$$
$$\left(\sum_{cs} PROB_s * SPRICE_c * csold_{cs} \right) - \qquad \{\text{expected crop profit}\}$$
$$\left(PCOST * \sum_{c} pcrop_c \right) - \qquad \{\text{planting cost}\}$$
$$\left(\sum_{cs} PROB_s * MCOST_c * bcrop_{cs} \right) \qquad \{\text{expected crops bought on the market}\}$$

$$\sum_{c} pcrop_c \le ACRES \qquad \{\text{acreage constraint}\}$$

$$YIELD_{cs} * pcrop_c + bcrop_{cs} - LCROP_c * live = csold_{cs} \quad \text{for all } c, s \quad \{\text{crop balance constraints}\}$$

$$pcrop_c \ge 0 \quad \text{for all } c$$
$$csold_{cs} \ge 0 \quad \text{for all } c, s$$
$$0 \le bcrop_{cs} \le MAXBUY_c \quad \text{for all } c, s$$
$$live \ge 0$$

Schrage #4 (Sample MPL Code)

```
TITLE
    RecourseCrops;

INDEX

    s := (wet,dry);             { seasons }

    c := (corn,sorghum,beans);  { crops }


DATA

    PROB[s]  := (.6,.4); { probability of season }

    TACRES := 1000;  { total acreage }

    YIELD[c,s] := (100,45,
                   43,35,
                   45,33);    { yield per acre of c in season s }

    SPRICE[c] := (2,4,4);  { sale price/bushel of c in dollars }

    PCOST := 100; { production cost per acre planted }

    LCROP[c] := (100,0,0); { bushels of c required/unit of livestock raised }

    LPROFIT := 215; { profit per unit of livestock raised }

    MCOST[c] := (2.2,0,0); { cost to buy crop on open market }

    MAXBUY[c] := (1000000,0,0);  { maximum bushels of crop c that can be bought }
```

Schrage #4 (Sample MPL code)

```
DECISION VARIABLES

    live;        { units of livestock to raise }
    pcrop[c];    { number of acres of c to plant }
    bcrop[c,s];  { bushels of c to buy on market under scenario s }
    csold[c,s];  { bushels of c grown and sold (excess of livestock requirements ) }

MODEL

    MAX  totexpcost = LPROFIT*live +                              { livestock profit }
                      SUM(c,s: PROB[s]*SPRICE[c]*csold[c,s]) -    { expected crop sales profit }
                      PCOST*SUM(c: pcrop[c]) -                    { planting cost }
                      SUM(c,s: PROB[s]*MCOST[c]*bcrop[c,s]);      { expected corn bought }

SUBJECT TO

    acres:  { acreage constraints}

      SUM(c: pcrop[c])  <  TACRES;

    balance[c,s]:

      YIELD[c,s]*pcrop[c] + bcrop[c] - LCROP[c]*live = csold[c,s];

BOUNDS

  bcrop[c] < MAXBUY[c]; WE HAVE TO DO THIS TO DISALLOW BEAN AND SORGHUM BUYS

END
```

Schrage #4 (solution)

```
DECISION VARIABLES

    live;        { units of livestock to raise }
    pcrop[c];    { number of acres of c to plant }
    bcrop[c,s]; { bushels of c to buy on market under scenario s }
    csold[c,s]; { bushels of c grown and sold (excess of livestock requirements ) }

FREE VARIABLES

    u          ; { worst case expected corn profit - cost; can be positive, 0, or negative }

MODEL

    MAX  totexpcost = LPROFIT*live -                          { livestock profit }
                      PCOST*SUM(c: pcrop[c]) +                { planting cost }
                      u;                                      { worst case expected corn bought cost }

SUBJECT TO

    acres:  { acreage constraints}

      SUM(c: pcrop[c])  <  TACRES;

    balance[c,s]:

      YIELD[c,s]*pcrop[c] + bcrop[c] - LCROP[c]*live = csold[c,s];

    worstcase[s]:

      u < SUM(c: SPRICE[c]*csold[c,s]) - sum(c: MCOST[c]*bcrop[c,s]);

BOUNDS

  bcrop[c] < MAXBUY[c];

END
```

Old solution: 66,000

New solution: 40,000
worst case is p(dry season) = 1.0, farmer does nothing but plant sorghum