

простейших тригонометрических выражений.

Лекция 1. Основные формулы тригонометрии

Формулы приведения

- $\sin(t + 2\pi k) = \sin t; \cos(t + 2\pi k) = \cos t,$
 $k \in \mathbb{Z};$
- $\sin(t + \pi) = -\sin t; \cos(t + \pi) = -\cos t;$
- $\sin(-t) = -\sin t; \cos(-t) = \cos t;$
- $\sin(\pi - t) = \sin t; \cos(\pi - t) = -\cos t;$
- $\sin\left(\frac{\pi}{2} - t\right) = \cos t; \cos\left(\frac{\pi}{2} - t\right) = \sin t;$

- $\operatorname{tg}(t + k\pi) = \operatorname{tg} t; \operatorname{ctg}(t + k\pi) = \operatorname{ctg} t; k \in \mathbf{Z};$
- $\operatorname{tg}(-t) = -\operatorname{tg} t; \operatorname{ctg}(-t) = -\operatorname{ctg} t;$
- $\operatorname{tg}(\pi - t) = \operatorname{tg}(-t) = -\operatorname{tg} t; \operatorname{ctg}(\pi - t) = -\operatorname{ctg} t;$
- $\operatorname{tg}\left(\frac{\pi}{2} - t\right) = \operatorname{ctg} t; \operatorname{ctg}\left(\frac{\pi}{2} - t\right) = \operatorname{tg} t.$

Формулы сложения

- $\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta;$
- $\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta;$
- $\sin(\alpha + \beta) = \sin \alpha \cos \beta + \sin \beta \cos \alpha;$
- $\sin(\alpha - \beta) = \sin \alpha \cos \beta - \sin \beta \cos \alpha;$
- $\operatorname{tg}(\alpha + \beta) = \frac{\operatorname{tg} \alpha + \operatorname{tg} \beta}{1 - \operatorname{tg} \alpha \operatorname{tg} \beta};$
- $\operatorname{tg}(\alpha - \beta) = \frac{\operatorname{tg} \alpha - \operatorname{tg} \beta}{1 + \operatorname{tg} \alpha \operatorname{tg} \beta}.$

Формулы удвоения

- $\sin 2\alpha = 2\sin\alpha\cos\alpha$; $\cos 2\alpha = 1 - 2\sin^2\alpha$;
- $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$; $\cos 2\alpha = 2\cos^2\alpha - 1$;
- $\operatorname{tg} 2\alpha = \frac{2\operatorname{tg}\alpha}{1 - \operatorname{tg}^2\alpha}$.

Формулы половинного угла

$$\cos^2 \alpha = \frac{1}{2}(1 + \cos 2\alpha); \quad \sin^2 \alpha = \frac{1}{2}(1 - \cos 2\alpha).$$

Примеры

Формулы приведения

- $\sin 370^\circ = \sin(10^\circ + 360^\circ) = \sin 10^\circ;$
- $\cos 190^\circ = \cos(10^\circ + 180^\circ) = -\cos 10^\circ;$
- $\operatorname{tg} \frac{9\pi}{8} = \operatorname{tg}\left(\frac{\pi}{8} + \pi\right) = \operatorname{tg} \frac{\pi}{8};$
- $\sin 80^\circ = \sin(90^\circ - 10^\circ) = \cos 10^\circ.$

Формулы сложения

$$\begin{aligned} \bullet \cos 105^\circ &= \cos(60^\circ + 45^\circ) = \\ &= \cos 60^\circ \cos 45^\circ - \sin 60^\circ \sin 45^\circ = \\ &= \frac{\sqrt{2}}{2 \cdot 2} - \frac{\sqrt{3} \cdot \sqrt{2}}{2 \cdot 2} = \frac{\sqrt{2}(1 - \sqrt{3})}{4}; \end{aligned}$$

$$\begin{aligned} \bullet \sin 20^\circ \cos 40^\circ + \cos 20^\circ \sin 40^\circ &= \\ &= \sin 60^\circ = \frac{\sqrt{3}}{2}; \end{aligned}$$

$$\begin{aligned} \bullet \operatorname{tg} 75^\circ &= \operatorname{tg}(45^\circ + 30^\circ) = \\ &= \frac{\operatorname{tg} 45^\circ + \operatorname{tg} 30^\circ}{1 - \operatorname{tg} 45^\circ \operatorname{tg} 30^\circ} = \frac{1 + \frac{\sqrt{3}}{3}}{1 - \frac{\sqrt{3}}{3}} = \\ &= \frac{\sqrt{3} + 1}{\sqrt{3} - 1} = \frac{(\sqrt{3} + 1)^2}{2} = 2 + \sqrt{3}. \end{aligned}$$

1. Вычислите:

1) $\sin 2^\circ \cos 28^\circ + \sin 28^\circ \cos 2^\circ;$

2) $\cos 73^\circ \cos 13^\circ + \sin 73^\circ \sin 13^\circ;$

3) $\sin 50^\circ \sin 5^\circ + \cos 50^\circ \cos 5^\circ;$

4) $\cos \frac{3\pi}{8} \sin \frac{5\pi}{24} - \cos \frac{5\pi}{24} \sin \frac{3\pi}{8};$

5) $\cos 100^\circ \sin 10^\circ - \sin 100^\circ \cos 10^\circ;$

6) $\cos 170^\circ \sin 35^\circ - \cos 35^\circ \sin 170^\circ;$

7) $\cos^2 \frac{\pi}{8} + \sin \frac{\pi}{8} \cos \frac{3\pi}{8};$

8) $\sin 105^\circ \sin 75^\circ + \sin 15^\circ \cos 105^\circ;$

9) $\cos 20^\circ \cos 25^\circ - \cos 70^\circ \sin 25^\circ;$

10) $\cos 43^\circ \cos 17^\circ - \cos 47^\circ \cos (-73^\circ).$

2. Докажете тождества:

$$1) \ 2\sin\left(\frac{\pi}{2} - \alpha\right)\sin\alpha = \sin 2\alpha;$$

$$2) \ \sin^4\alpha - \cos^4\alpha = -\cos 2\alpha;$$

$$3) \ \sin^4\alpha + \cos^4\alpha = \frac{1 + \cos^2 2\alpha}{2};$$

$$4) \ \left(\sin\frac{\alpha}{2} - \cos\frac{\alpha}{2}\right)^2 = 1 - \sin\alpha;$$

$$5) \ 2\cos^4\alpha + \sin^2 2\alpha + 2\sin^4\alpha = 2;$$

$$6) \ 1 + \sin\alpha = 2\cos^2\left(\frac{\pi}{4} - \frac{\alpha}{2}\right);$$

$$7) \ 1 - \sin\alpha = 2\cos^2\left(\frac{\pi}{4} + \frac{\alpha}{2}\right);$$

$$8) \ \sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha;$$

$$9) \ \sin\alpha + \cos\alpha = \sqrt{2}\left(\alpha + \frac{\pi}{4}\right);$$

$$10) \ \operatorname{ctg}\alpha - \operatorname{tg}\alpha = 2\operatorname{ctg} 2\alpha.$$