

CLASS NOTE A

A1. Capital Market History and Risk & Return

- Returns
- The Historical Record
- Average Returns: The First Lesson
- The Variability of Returns: The Second Lesson
- Capital Market Efficiency

A2. Capital Market History and Risk & Return (continued)

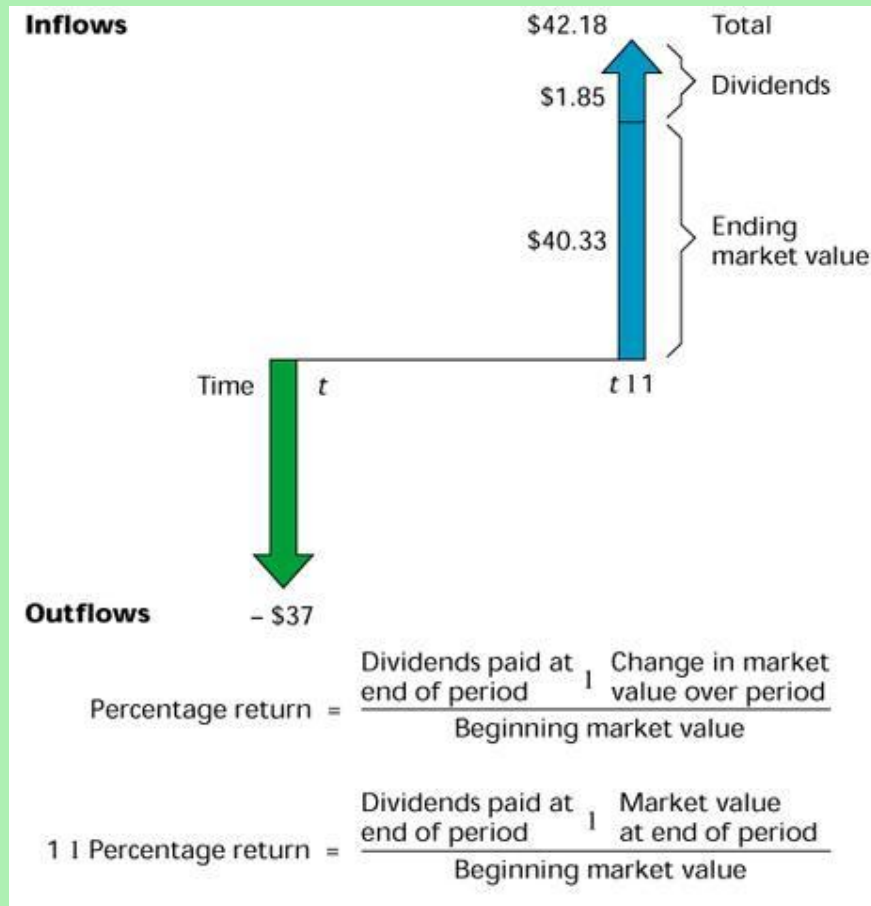
- Expected Returns and Variances
- Portfolios
- Announcements, Surprises, and Expected Returns
- Risk: Systematic and Unsystematic
- Diversification and Portfolio Risk
- Systematic Risk and Beta
- The Security Market Line
- The SML and the Cost of Capital: A Preview

A3. Risk, Return, and Financial Markets

“. . . Wall Street shapes Main Street. Financial markets transform factories, department stores, banking assets, film companies, machinery, soft-drink bottlers, and power lines from parts of the production process . . . into something easily convertible into money. Financial markets . . . not only make a hard asset liquid, they price that asset so as to promote its most productive use.”

Peter Bernstein, in his book, Capital Ideas

A4. Percentage Returns

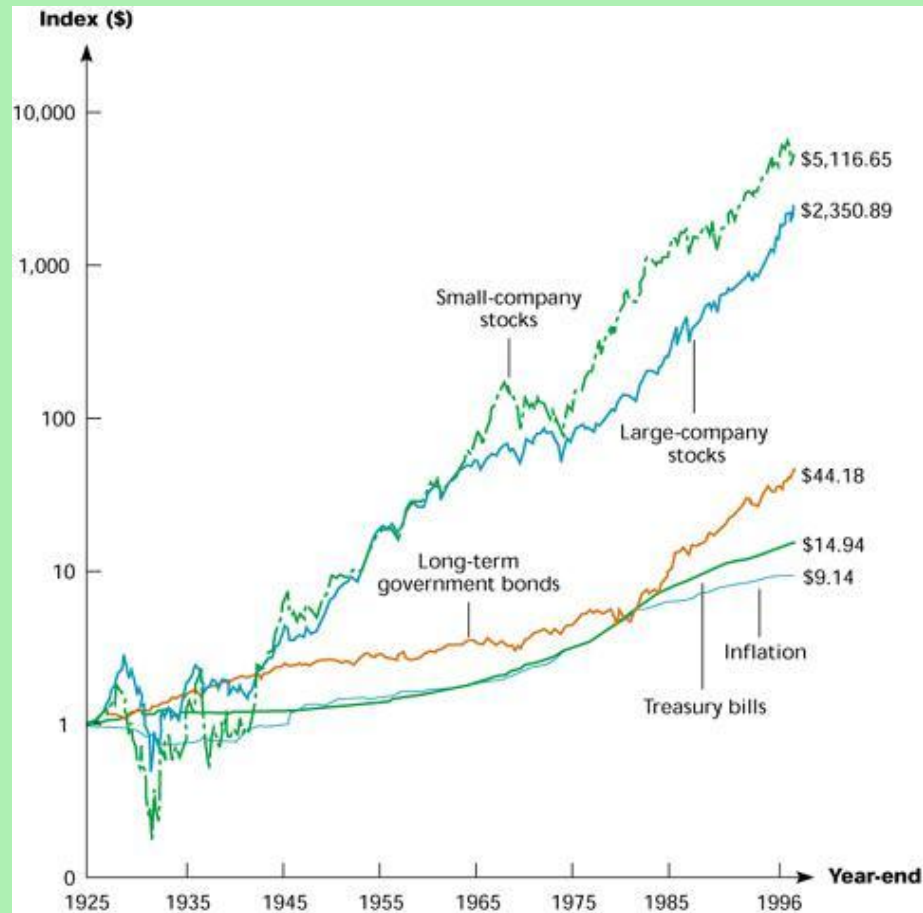


A5. Percentage Returns (concluded)

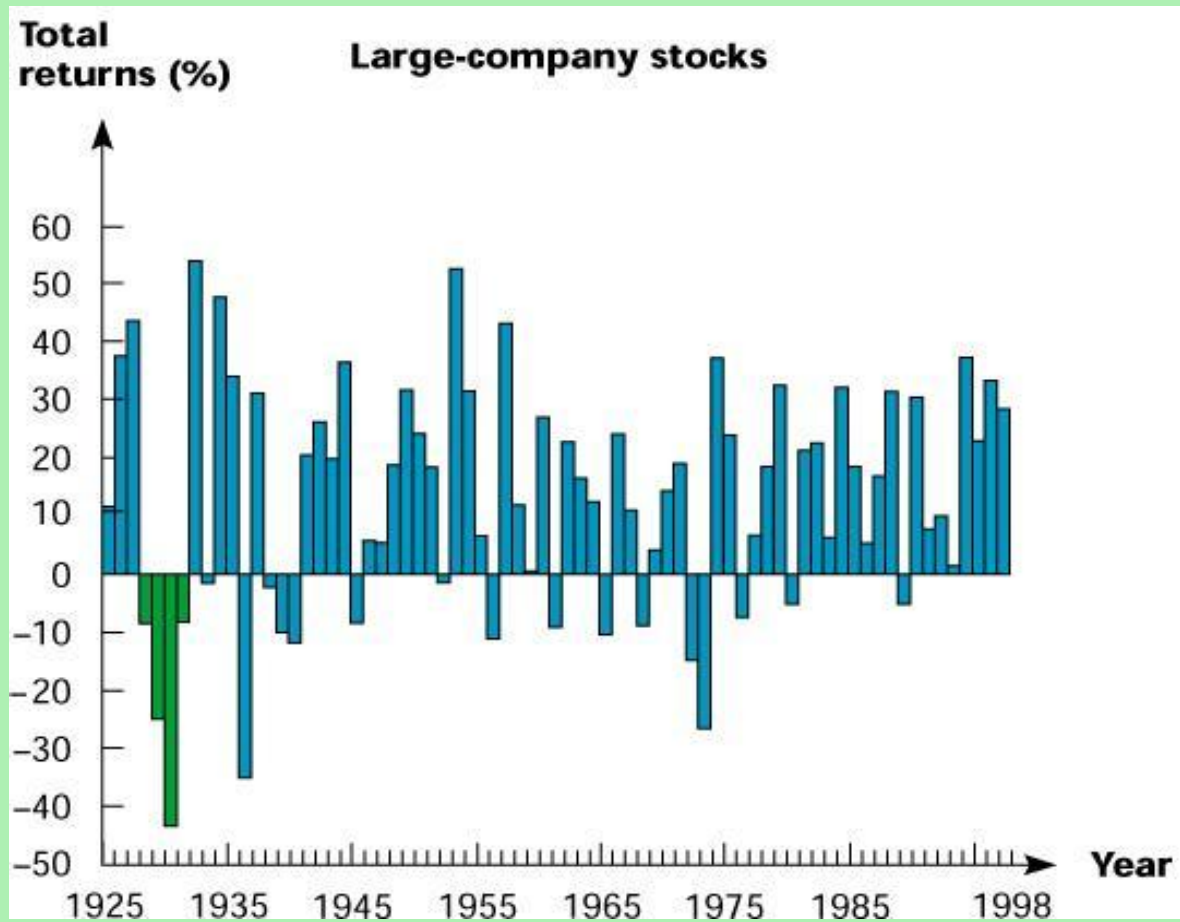
$$\text{Percentage return} = \frac{\begin{array}{l} \text{Dividends paid at} \\ \text{end of period} \end{array} + \begin{array}{l} \text{Change in market} \\ \text{value over period} \end{array}}{\text{Beginning market value}}$$

$$1 + \text{Percentage return} = \frac{\begin{array}{l} \text{Dividends paid at} \\ \text{end of period} \end{array} + \begin{array}{l} \text{Market value} \\ \text{at end of period} \end{array}}{\text{Beginning market value}}$$

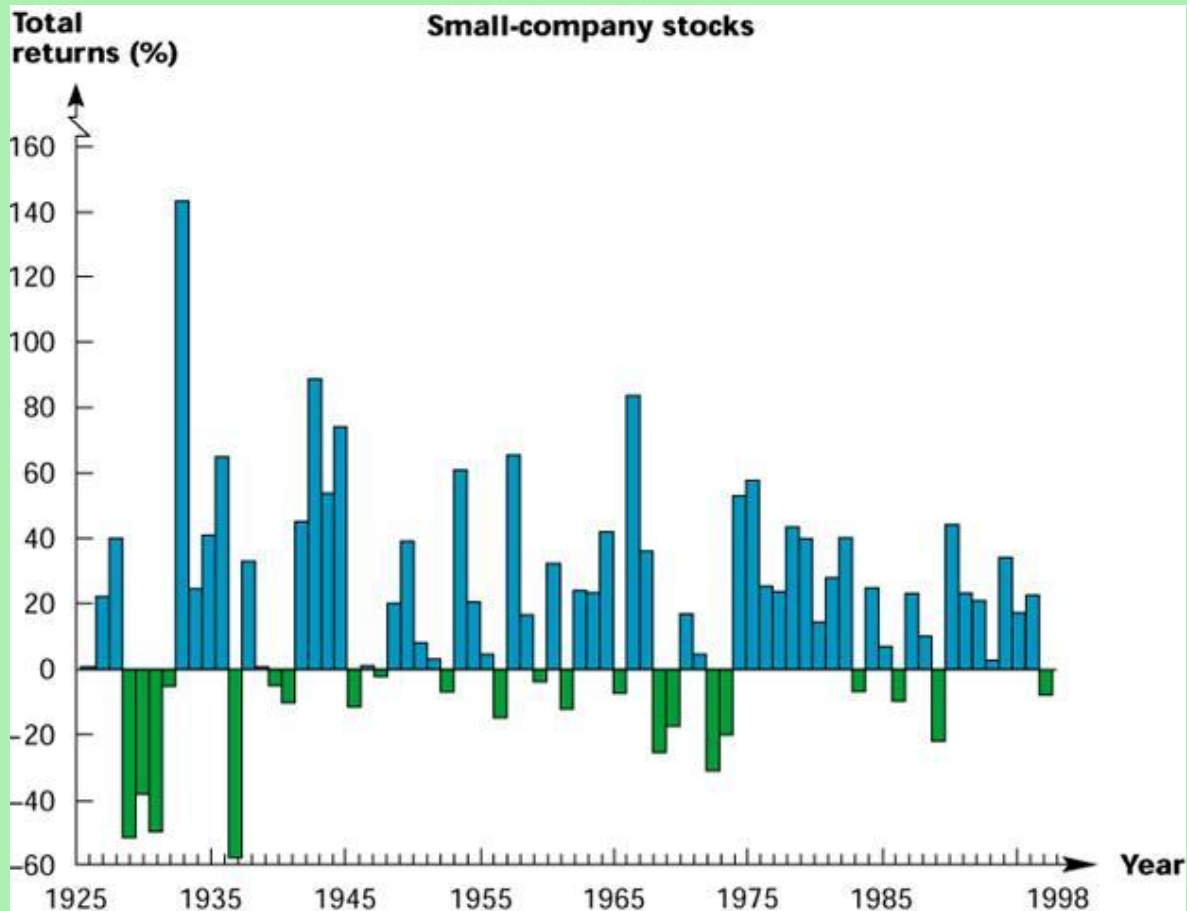
A6. A \$1 Investment in Different Types of Portfolios: 1926-1998



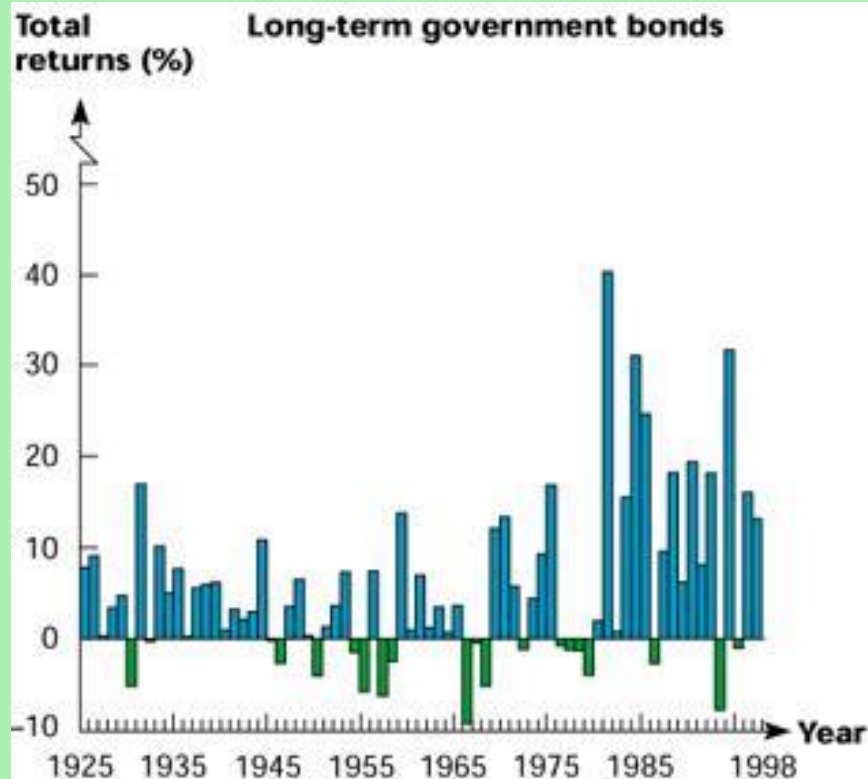
A7. Year-to-Year Total Returns on Large Company Common Stocks: 1926-1998



A8. Year-to-Year Total Returns on Small Company Common Stocks: 1926-1998

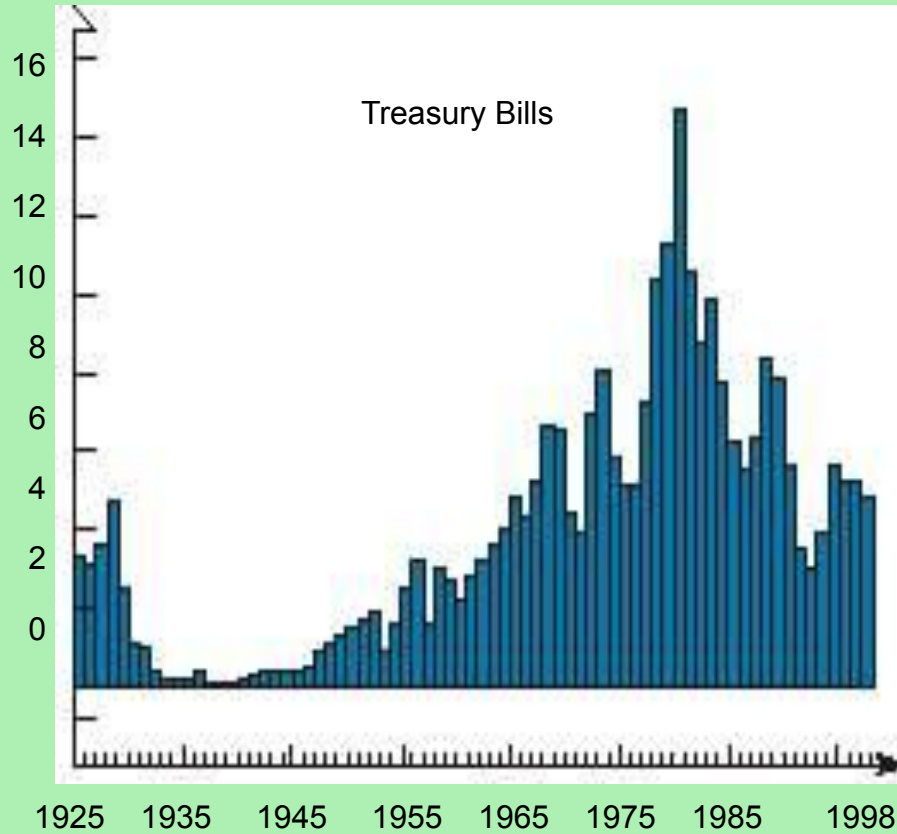


A9. Year-to-Year Total Returns on Bonds and Bills: 1926-1998

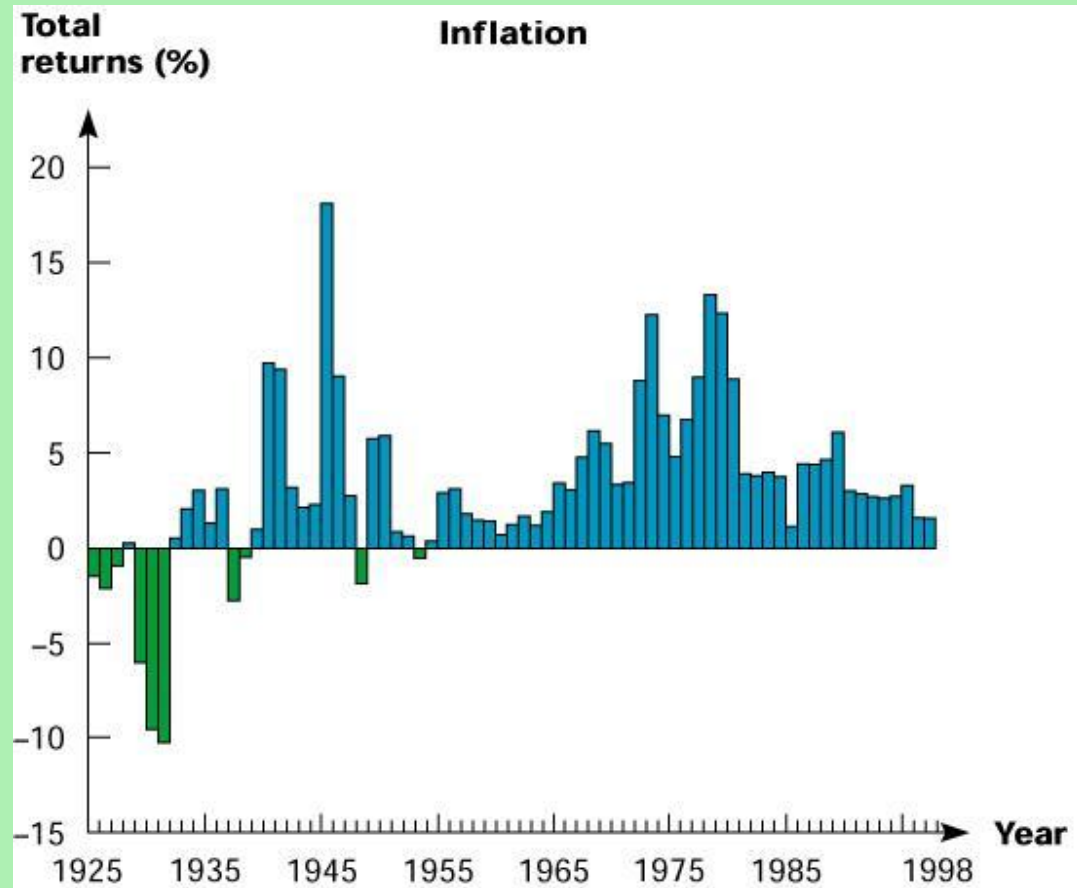


A10. Year-to-Year Total Returns on Bonds and Bills: 1926-1998 (concluded)

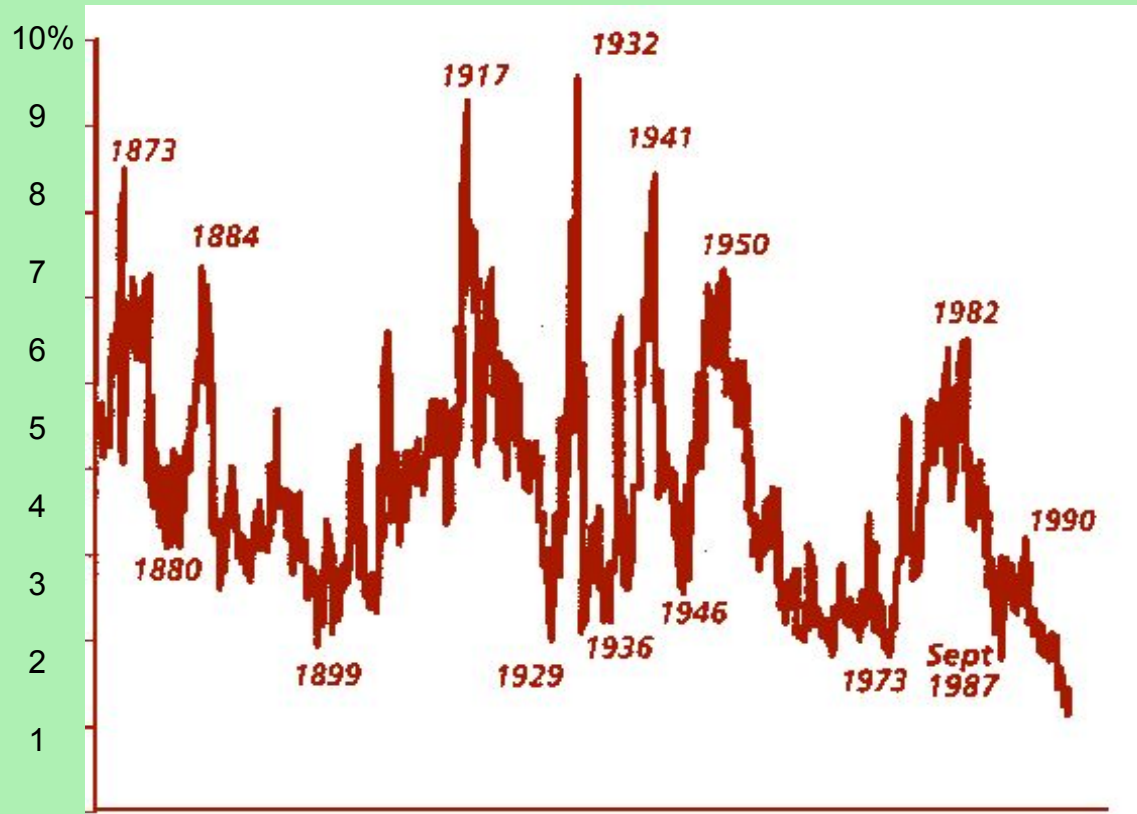
Total Returns (%)



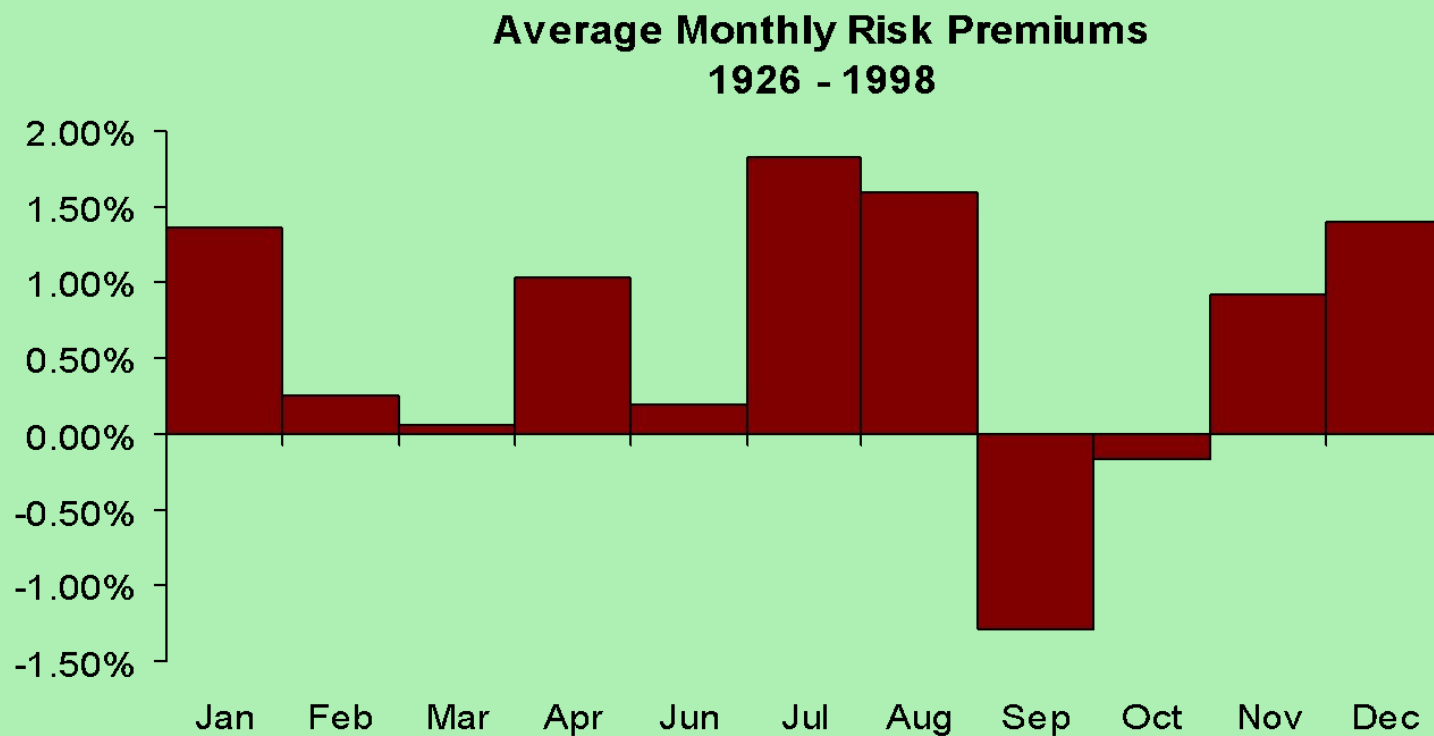
A11. Year-to-Year Inflation: 1926-1998



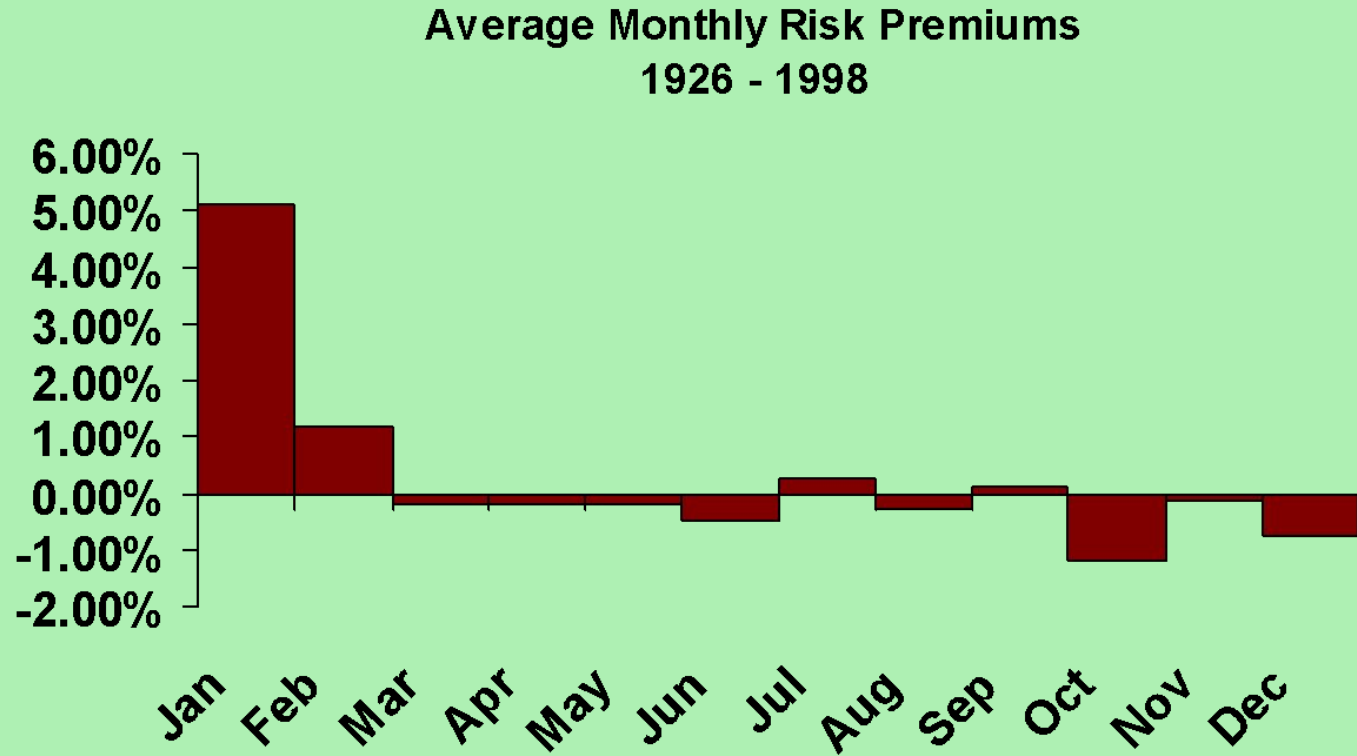
A12. Historical Dividend Yield on Common Stocks



A13. S&P 500 Risk Premiums: 1926-1998



A14. Small Stock Risk Premiums: 1926-1998



A15. Using Capital Market History

- Now let's use our knowledge of capital market history to make some financial decisions. Consider these questions:
 - ◆ Suppose the current T-bill rate is 5%. An investment has “average” risk relative to a typical share of stock. It offers a 10% return. Is this a good investment?
 - ◆ Suppose an investment is similar in risk to buying small company equities. If the T-bill rate is 5%, what return would you demand?

A16. Using Capital Market History (continued)

- Risk premiums: First, we calculate risk premiums. The risk premium is the difference between a risky investment's return and that of a riskless asset. Based on historical data:

Investment	Average return	Standard deviation	Risk premium
Common stocks	13.2%	20.3%	____%
Small stocks	17.4%	33.8%	____%
LT Corporates	6.1%	8.6%	____%
Long-term Treasury bonds	5.7%	9.2%	____%
Treasury bills	3.8%	3.2%	____%

A17. Using Capital Market History (continued)

- Risk premiums: First, we calculate risk premiums. The risk premium is the difference between a risky investment's return and that of a riskless asset. Based on historical data:

Investment	Average return	Standard deviation	Risk premium
Common stocks	13.2%	20.3%	9.4%
Small stocks	17.4%	33.8%	13.6%
LT Corporates	6.1%	8.6%	2.3%
Long-term Treasury bonds	5.7%	9.2%	1.9%
Treasury bills	3.8%	3.2%	0%

A18. Using Capital Market History (concluded)

- Let's return to our earlier questions.
 - ◆ Suppose the current T-bill rate is 5%. An investment has “average” risk relative to a typical share of stock. It offers a 10% return. Is this a good investment?

No - the average risk premium is 9.4%; the risk premium of the stock above is only $(10\% - 5\%) = 5\%$.

- ◆ Suppose an investment is similar in risk to buying small company equities. If the T-bill rate is 5%, what return would you demand?

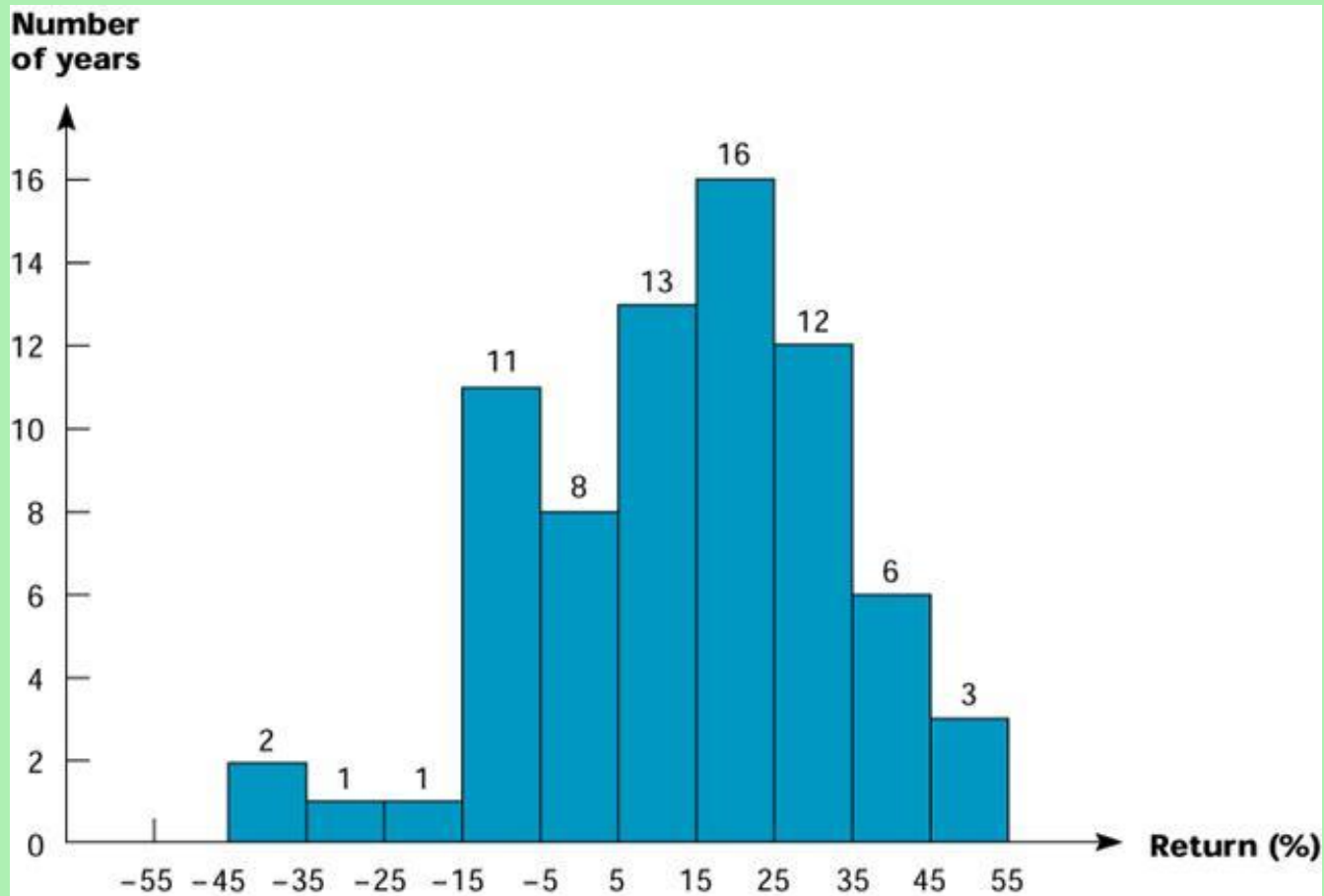
Since the risk premium has been 13.6%, we would demand 18.6%.

A19. Average Annual Returns and Risk Premiums: 1926-1998

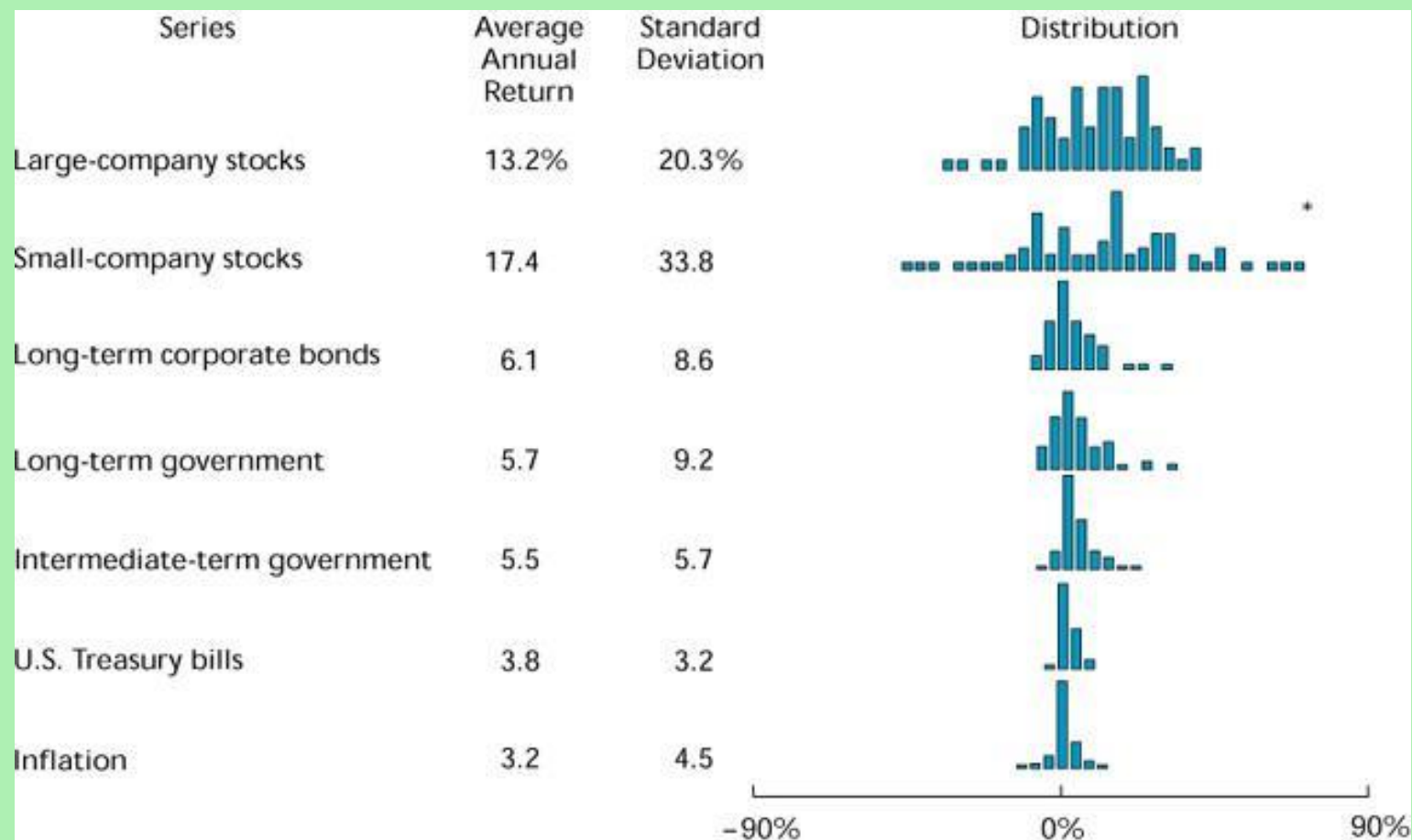
Investment	Average Return		Risk Premium
Large-company stocks	13.2%		9.4%
Small-company stocks	17.4	13.6	
Long-term corporate bonds	6.1	2.3	
Long-term government bonds	5.7	1.9	
U.S. Treasury bills	3.8	0.0	

Source: © *Stocks, Bonds, Bills and Inflation 1998 Yearbook*™, Ibbotson Associates, Inc. Chicago (annually updates work by Roger G. Ibbotson and Rex A. Sinquefeld). All rights reserved

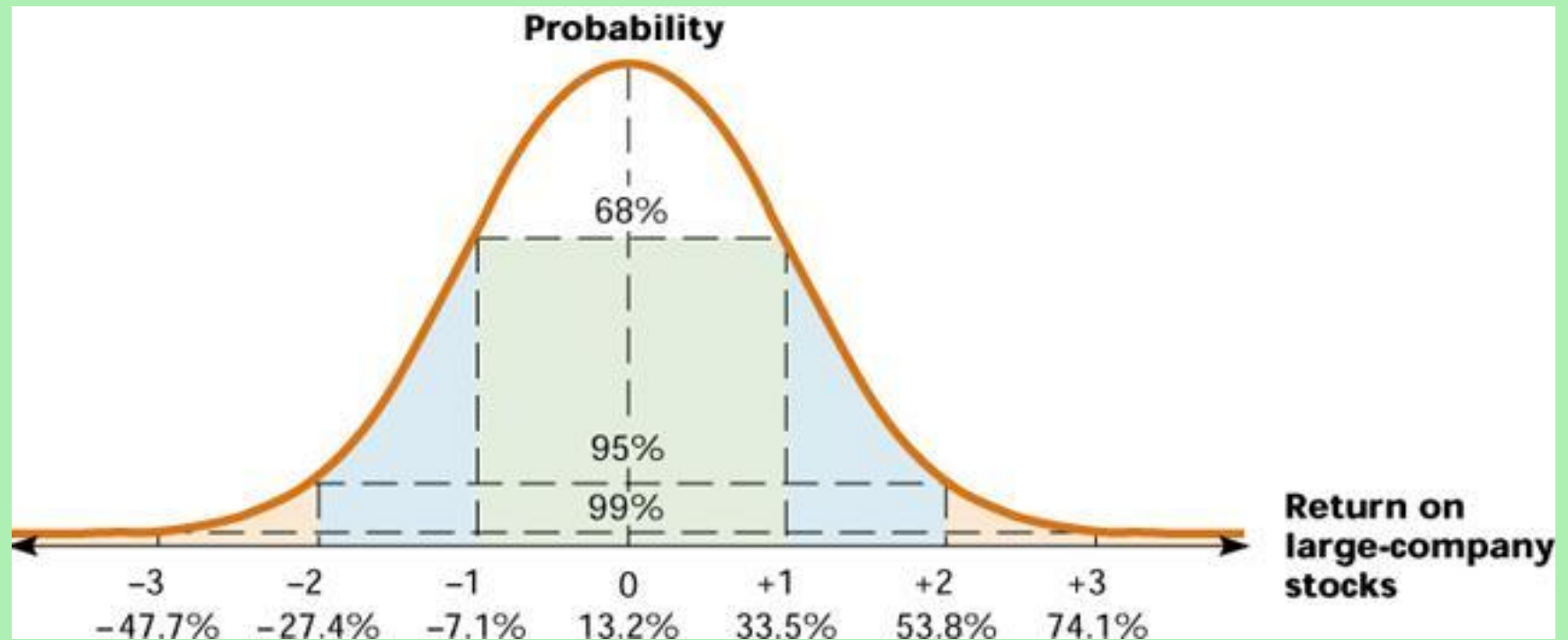
A20. Frequency Distribution of Returns on Common Stocks, 1926-1998



A21. Historical Returns, Standard Deviations, and Frequency Distributions: 1926-1998



A22. The Normal Distribution



A23. Two Views on Market Efficiency

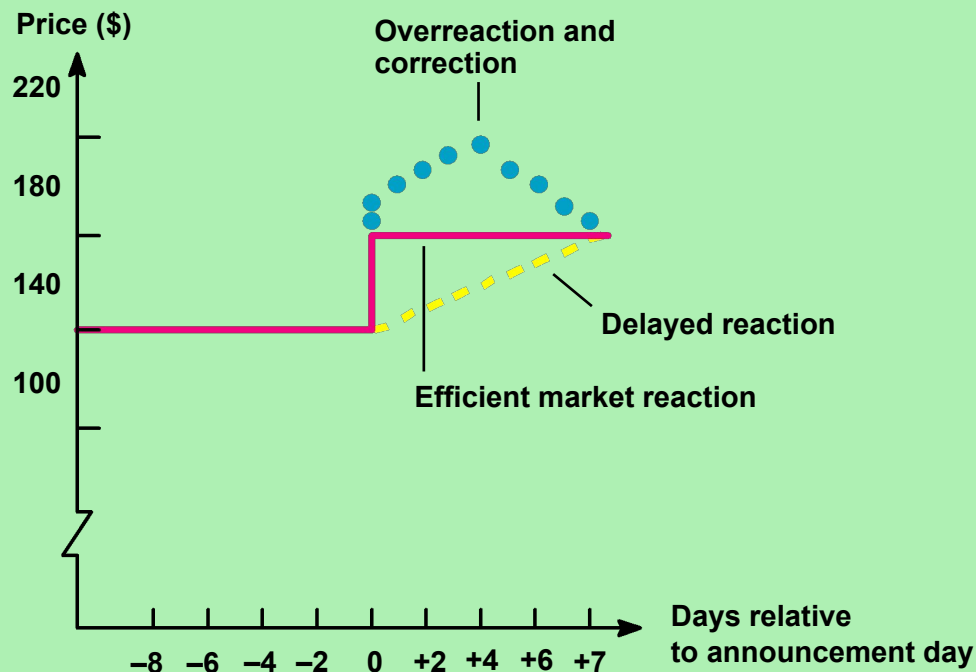
“ . . . in price movements . . . the sum of every scrap of knowledge available to Wall Street is reflected as far as the clearest vision in Wall Street can see.”

Charles Dow, founder of Dow-Jones, Inc. and first editor of The Wall Street Journal (1903)

“In an efficient market, prices ‘fully reflect’ available information.”

Professor Eugene Fama, financial economist (1976)

A24. Stock Price Reaction to New Information in Efficient and Inefficient Markets



Efficient market reaction: The price instantaneously adjusts to and fully reflects new information; there is no tendency for subsequent increases and decreases.

Delayed reaction: The price partially adjusts to the new information; 8 days elapse before the price completely reflects the new information

Overreaction: The price overadjusts to the new information; it “overshoots” the new price and subsequently corrects.

A25. A Quick Quiz

Here are three questions that should be easy to answer (if you've been paying attention, that is).

1. How are average annual returns measured?

2. How is volatility measured?

3. Assume your portfolio has had returns of 11%, -8%, 20%, and -10% over the last four years. What is the average annual return?

A26. Chapter 12 Quick Quiz (continued)

1. How are average annual returns measured?

*Annual returns are often measured as **arithmetic** averages.*

An arithmetic average is found by summing the annual returns and dividing by the number of returns. It is most appropriate when you want to know the mean of the distribution of outcomes.

A27. Chapter 12 Quick Quiz (continued)

2. How is volatility measured?

Given a normal distribution, volatility is measured by the “spread” of the distribution, as indicated by its variance or standard deviation.

When using historical data, variance is equal to:

$$\frac{1}{T - 1} [(R_1 - R)^2 + \dots + (R_T - R)^2]$$

And, of course, the standard deviation is the square root of the variance.

A28. Chapter 12 Quick Quiz (concluded)

3. Assume your portfolio has had returns of 11%, -8%, 20%, and -10% over the last four years. What is the average annual return?

Your average annual return is simply:

$$[.11 + (-.08) + .20 + (-.10)]/4 = .0325 = 3.25\% \text{ per year.}$$

A29. A Few Examples

- Suppose a stock had an initial price of \$58 per share, paid a dividend of \$1.25 per share during the year, and had an ending price of \$45. Compute the percentage total return.
- The percentage total return (R) =
$$[\$1.25 + (\$45 - 58)]/\$58 = -20.26\%$$
- The dividend yield = $\$1.25/\$58 = 2.16\%$
- The capital gains yield = $(\$45 - 58)/\$58 = -22.41\%$

A30. A Few Examples (continued)

- Suppose a stock had an initial price of \$58 per share, paid a dividend of \$1.25 per share during the year, and had an ending price of \$75. Compute the percentage total return.
- The percentage total return (R) =
$$[\$1.25 + (\$75 - 58)]/\$58 = 31.47\%$$
- The dividend yield = $\$1.25/\$58 = 2.16\%$
- The capital gains yield = $(\$75 - 58)/\$58 = 29.31\%$

A31. A Few Examples (continued)

- Using the following returns, calculate the average returns, the variances, and the standard deviations for stocks X and Y.

Returns

Year	X	Y
1	18%	28%
2	11	-7
3	-9	-20
4	13	33
5	7	16

A32. A Few Examples (continued)

$$\text{Mean return on X} = (.18 + .11 - .09 + .13 + .07)/5 = \underline{\hspace{2cm}}.$$

$$\text{Mean return on Y} = (.28 - .07 - .20 + .33 + .16)/5 = \underline{\hspace{2cm}}.$$

$$\begin{aligned} \text{Variance of X} &= [(.18-.08)^2 + (.11-.08)^2 + (-.09-.08)^2 \\ &\quad + (.13-.08)^2 + (.07-.08)^2]/(5 - 1) = \underline{\hspace{2cm}}. \end{aligned}$$

$$\begin{aligned} \text{Variance of Y} &= [(.28-.10)^2 + (-.07-.10)^2 + (-.20-.10)^2 \\ &\quad + (.33-.10)^2 + (.16-.10)^2]/(5 - 1) = \underline{\hspace{2cm}}. \end{aligned}$$

$$\text{Standard deviation of X} = (\underline{\hspace{2cm}})^{1/2} = \underline{\hspace{2cm}}\%.$$

$$\text{Standard deviation of Y} = (\underline{\hspace{2cm}})^{1/2} = \underline{\hspace{2cm}}\%.$$

A33. A Few Examples (concluded)

Mean return on X = $(.18 + .11 - .09 + .13 + .07)/5 = .08$.

Mean return on Y = $(.28 - .07 - .20 + .33 + .16)/5 = .10$.

Variance of X = $[(.18-.08)^2 + (.11-.08)^2 + (-.09-.08)^2 + (.13-.08)^2 + (.07-.08)^2]/(5 - 1) = .0106$.

Variance of Y = $[(.28-.10)^2 + (-.07-.10)^2 + (-.20-.10)^2 + (.33-.10)^2 + (.16-.10)^2]/(5 - 1) = .05195$.

Standard deviation of X = $(.0106)^{1/2} = 10.30\%$.

Standard deviation of Y = $(.05195)^{1/2} = 22.79\%$.

A34. Expected Return and Variance: Basic Ideas

- The quantification of risk and return is a crucial aspect of modern finance. It is not possible to make “good” (i.e., value-maximizing) financial decisions unless one understands the relationship between risk and return.
- **Rational** investors like returns and dislike risk.
- Consider the following proxies for return and risk:

Expected return - weighted average of the distribution of possible returns in the future.

Variance of returns - a measure of the *dispersion* of the distribution of possible returns in the future.

How do we calculate these measures? Stay tuned.

A35. Example: Calculating the Expected Return

State of Economy	Probability p_i	Return in of state i state i R_i
+1% change in GNP	.25	-5%
+2% change in GNP	.50	15%
+3% change in GNP	.25	35%

A36. Example: Calculating the Expected Return (concluded)

$$\frac{\sum_i (p_i \times R_i)}{}$$

$$i = 1 - 1.25\%$$

$$i = 27.50\%$$

$$i = 38.75\%$$

$$\text{Expected return} = (-1.25 + 7.50 + 8.75)$$

$$= 15\%$$

A37. Calculation of Expected Return

		<u>Stock L</u>			<u>Stock U</u>		
	(3)	(5)					
(1) Probability	(2) Rate of	(3) Rate of	(4) Return	(6) Return	(2) × (3)	(2) × (5)	
State of Economy	of State of Economy	if State Occurs	(2) × (3)	Product if State Occurs	(2) × (5)	(2) × (5)	Product
Recession	.80	-.20	-.16	.30	.24		
Boom	.20	.70	.14	.10	.02		
26%		$E(R_L) = -2\%$			$E(R_U) =$		

A38. Example: Calculating the Variance

State of Economy	Probability p_i	Return in of state i r_i
+1% change in GNP	.25	-5%
+2% change in GNP	.50	15%
+3% change in GNP	.25	35%

$$E(R) = \bar{R} = 15\% = .15$$

A39. Calculating the Variance (concluded)

	i	$(R_i - \bar{R})^2$	$p_i^- \times (R_i - \bar{R})^2$	$\bar{}$
i=1	.04	.01		
i=2	0	0		
i=3	.04	.01		

$$\text{Var}(R) = .02$$

What is the standard deviation?

$$\text{The standard deviation} = (.02)^{1/2} = .1414 .$$

A40. Example: Expected Returns and Variances

State of the economy	Probability of state	Return on asset A	Return on asset B
Boom	0.40	30%	-5%
Bust	0.60	-10%	25%
	1.00		

■ A. Expected returns

$$E(R_A) = 0.40 \times (.30) + 0.60 \times (-.10) = .06 = 6\%$$

$$E(R_B) = 0.40 \times (-.05) + 0.60 \times (.25) = .13 = 13\%$$

A41. Example: Expected Returns and Variances (concluded)

■ B. Variances

$$\text{Var}(R_A) = 0.40 \times (.30 - .06)^2 + 0.60 \times (-.10 - .06)^2 = .0384$$

$$\text{Var}(R_B) = 0.40 \times (-.05 - .13)^2 + 0.60 \times (.25 - .13)^2 = .0216$$

■ C. Standard deviations

$$\text{SD}(R_A) = (.0384)^{1/2} = .196 = 19.6\%$$

$$\text{SD}(R_B) = (.0216)^{1/2} = .147 = 14.7\%$$

A42. Example: Portfolio Expected Returns and Variances

- Portfolio *weights*: put 50% in Asset A and 50% in Asset B:

State of the economy	Probability of state	Return on A	Return on B	Return on portfolio
Boom	0.40	30%	-5%	12.5%
Bust	<u>0.60</u>	-10%	25%	7.5%
	1.00			

A43. Example: Portfolio Expected Returns and Variances (continued)

- A. $E(R_p) = 0.40 \times (.125) + 0.60 \times (.075) = .095 = 9.5\%$
- B. $\text{Var}(R_p) = 0.40 \times (.125 - .095)^2 + 0.60 \times (.075 - .095)^2 = .0006$
- C. $\text{SD}(R_p) = (.0006)^{1/2} = .0245 = 2.45\%$
- Note: $E(R_p) = .50 \times E(R_A) + .50 \times E(R_B) = 9.5\%$
- BUT: $\text{Var}(R_p) \neq .50 \times \text{Var}(R_A) + .50 \times \text{Var}(R_B)$

A44. Example: Portfolio Expected Returns and Variances (concluded)

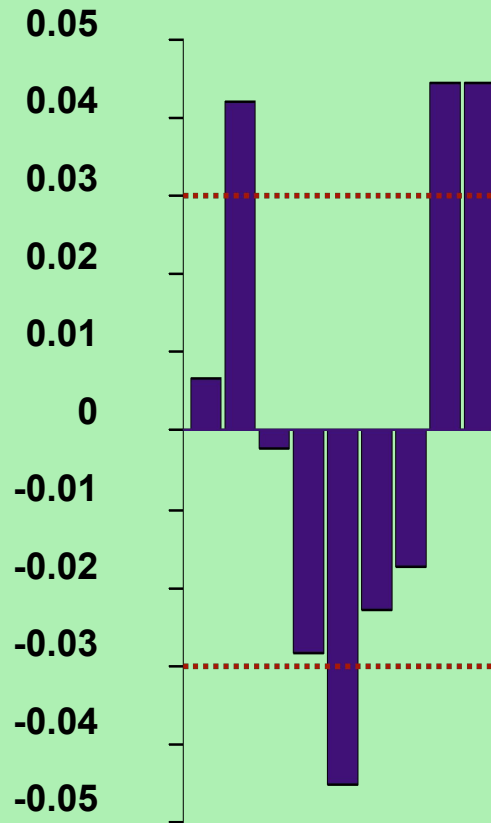
- New portfolio weights: put 3/7 in A and 4/7 in B:

State of the economy	Probability of state	Return on A	Return on B	Return on portfolio
Boom	0.40	30%	-5%	10%
Bust	<u>0.60</u>	-10%	25%	10%
	1.00			

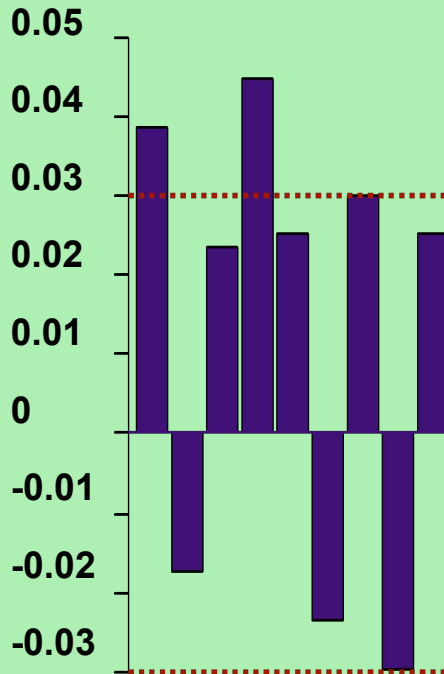
- A. $E(R_p) = 10\%$
- B. $SD(R_p) = 0\%$ (Why is this zero?)

A45. The Effect of Diversification on Portfolio Variance

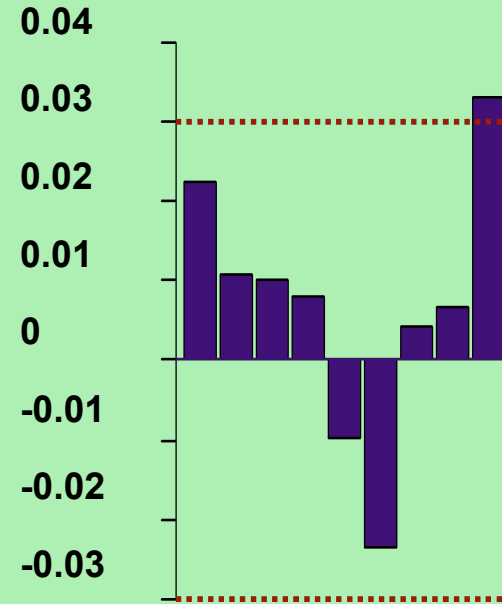
Stock A returns



Stock B returns



**Portfolio returns:
50% A and 50% B**



A46. Announcements, Surprises, and Expected Returns

- Key issues:
 - ◆ What are the components of the total return?
 - ◆ What are the different types of risk?

- Expected and Unexpected Returns

Total return = Expected return + Unexpected return

$$R = E(R) + U$$

- Announcements and News

Announcement = Expected part + Surprise

A47. Risk: Systematic and Unsystematic

- Systematic and Unsystematic Risk

- ◆ Types of surprises

1. Systematic or “market” risks
2. Unsystematic/unique/asset-specific risks

- ◆ Systematic and unsystematic components of return

Total return = Expected return + Unexpected return

$$R = E(R) + U$$

$$= E(R) + \text{systematic portion} + \text{unsystematic portion}$$

A48. Peter Bernstein on Risk and Diversification

“Big risks are scary when you cannot diversify them, especially when they are expensive to unload; even the wealthiest families hesitate before deciding which house to buy. Big risks are not scary to investors who can diversify them; big risks are interesting. No single loss will make anyone go broke . . . by making diversification easy and inexpensive, financial markets enhance the level of risk-taking in society.”

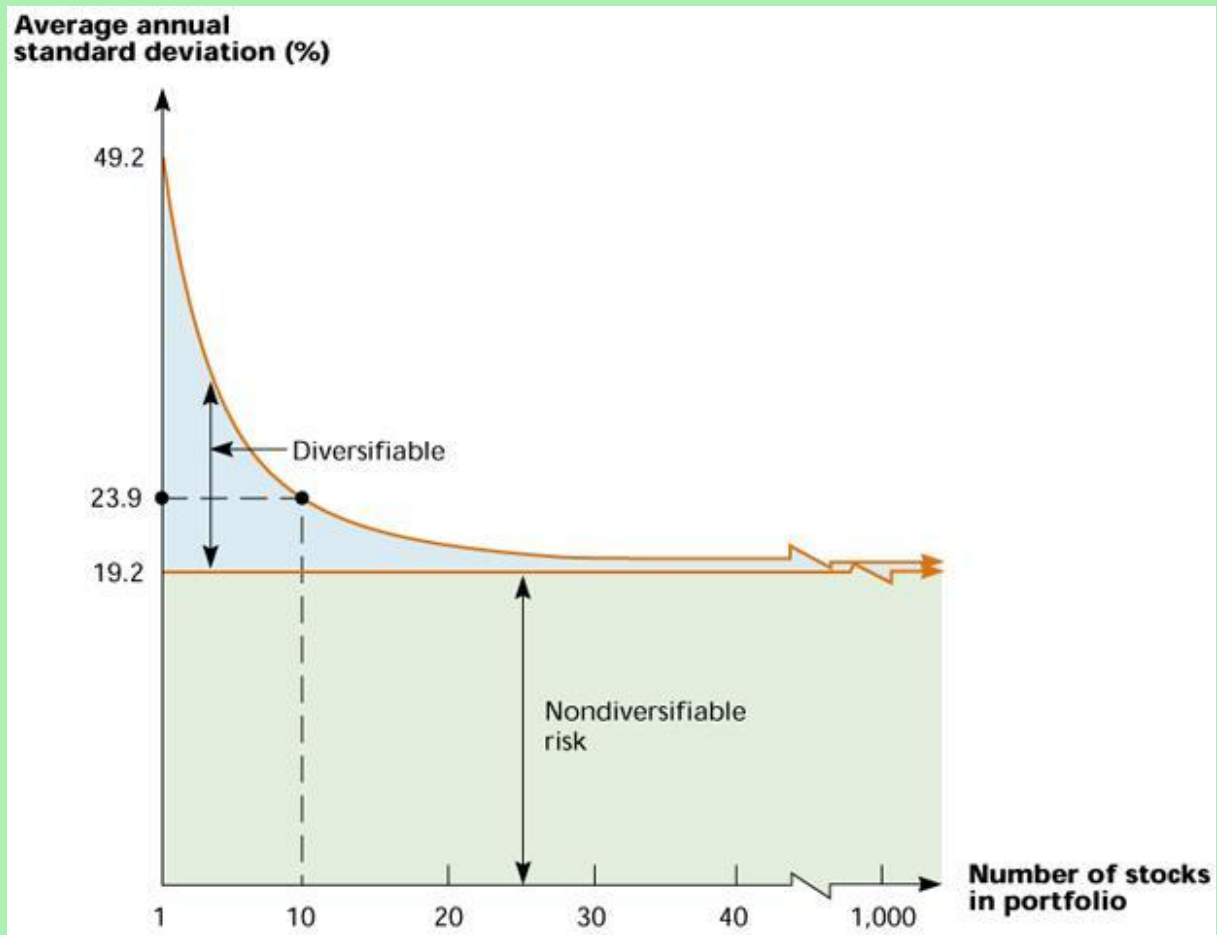
Peter Bernstein, in his book, Capital Ideas

A49. Standard Deviations of Annual Portfolio Returns

(3)		
(1)	(2)	(3)
Number of Stocks in Portfolio	Average Standard Deviation of Annual Portfolio Returns	Ratio of Portfolio Standard Deviation to Standard Deviation of a Single Stock
1	49.24%	1.00
10	23.93	0.49
50	20.20	0.41
100	19.69	0.40
300	19.34	0.39
500	19.27	0.39
1,000	19.21	0.39

These figures are from Table 1 in Meir Statman, "How Many Stocks Make a Diversified Portfolio?" *Journal of Financial and Quantitative Analysis* 22 (September 1987), pp. 353–64. They were derived from E. J. Elton and M. J. Gruber, "Risk Reduction and Portfolio Size: An Analytic Solution," *Journal of Business* 50 (October 1977), pp. 415–37.

A50. Portfolio Diversification



A51. Beta Coefficients for Selected Companies

Company	Beta Coefficient
American Electric Power	.65
Exxon	.80
IBM	.95
Wal-Mart	1.15
General Motors	1.05
Harley-Davidson	1.20
Papa Johns	1.45
America Online	1.65

Source: From Value Line *Investment Survey*, May 8, 1998.

A52. Example: Portfolio Beta Calculations

Amount		Portfolio		
Stock Invested		Weights	Beta	
(1)	(2)	(3)	(4)	(3) × (4)
Haskell Mfg.	\$ 6,000	50%	0.900	0.450
Cleaver, Inc.	4,000	33%	1.100	0.367
Rutherford Co.	2,000	17%	1.300	0.217
Portfolio \$12,000		100%		1.034

A53. Example: Portfolio Expected Returns and Betas

- Assume you wish to hold a portfolio consisting of asset A and a riskless asset. Given the following information, calculate portfolio expected returns and portfolio betas, letting the proportion of funds invested in asset A range from 0 to 125%.

Asset A has a beta of 1.2 and an expected return of 18%.

The risk-free rate is 7%.

Asset A weights: 0%, 25%, 50%, 75%, 100%, and 125%.

A54. Example: Portfolio Expected Returns and Betas (concluded)

Proportion Invested in Asset A (%)	Proportion Invested in Risk-free Asset (%)	Portfolio Expected Return (%)	Portfolio Beta
0	100	7.00	0.00
25	75	9.75	0.30
50	50	12.50	0.60
75	25	15.25	0.90
100	0	18.00	1.20
125	-25	20.75	1.50

A55. Return, Risk, and Equilibrium

- Key issues:
 - ◆ What is the relationship between risk and return?
 - ◆ What does security market equilibrium look like?

The fundamental conclusion is that the ratio of the risk premium to beta is the same for every asset. In other words, the reward-to-risk ratio is *constant* and equal to

$$\text{Reward/risk ratio} = \frac{E(R_i) - R_f}{\beta}$$

A56. Return, Risk, and Equilibrium (concluded)

- Example:

Asset A has an expected return of 12% and a beta of 1.40. Asset B has an expected return of 8% and a beta of 0.80. Are these assets valued correctly relative to each other if the risk-free rate is 5%?

a. For A, $(.12 - .05)/1.40 =$ _____

b. For B, $(.08 - .05)/0.80 =$ _____

- What would the risk-free rate have to be for these assets to be correctly valued?

$$(.12 - R_f)/1.40 = (.08 - R_f)/0.80$$

$$R_f = \underline{\hspace{2cm}}$$

A57. Return, Risk, and Equilibrium (concluded)

- Example:

Asset A has an expected return of 12% and a beta of 1.40. Asset B has an expected return of 8% and a beta of 0.80. Are these assets valued correctly relative to each other if the risk-free rate is 5%?

a. For A, $(.12 - .05)/1.40 = .05$

b. For B, $(.08 - .05)/0.80 = .0375$

- What would the risk-free rate have to be for these assets to be correctly valued?

$$(.12 - R_f)/1.40 = (.08 - R_f)/0.80$$

$$R_f = .02666$$

A58. The Capital Asset Pricing Model

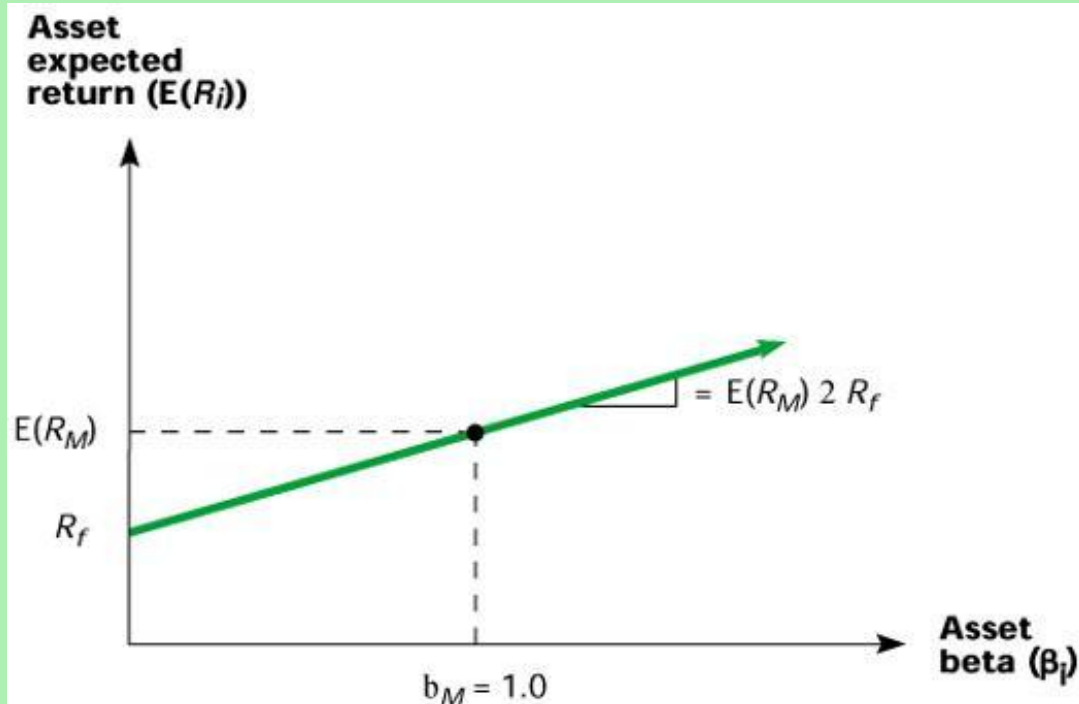
- The Capital Asset Pricing Model (CAPM) - an equilibrium model of the relationship between risk and return.

What determines an asset's expected return?

- ◆ The risk-free rate - the pure time value of money
- ◆ The market risk premium - the reward for bearing systematic risk
- ◆ The beta coefficient - a measure of the amount of systematic risk present in a particular asset

$$\text{The CAPM: } E(R_i) = R_f + [E(R_M) - R_f] \times \beta_i$$

A59. The Security Market Line (SML)



The slope of the security market line is equal to the market risk premium; i.e., the reward for bearing an average amount of systematic risk. The equation describing the SML can be written:

$$E(R_i) = R_f + b_i [E(R_M) - R_f]$$

which is the capital asset pricing model (CAPM).

A60. Summary of Risk and Return

I. Total risk - the variance (or the standard deviation) of an asset's return.

II. Total return - the expected return + the unexpected return.

III. Systematic and unsystematic risks

Systematic risks are unanticipated events that affect almost all assets to some degree because the effects are economywide.

Unsystematic risks are unanticipated events that affect single assets or small groups of assets. Also called *unique* or *asset-specific* risks.

IV. The effect of diversification - the elimination of unsystematic risk via the combination of assets into a portfolio.

V. The systematic risk principle and beta - the reward for bearing risk depends *only* on its level of systematic risk.

VI. The reward-to-risk ratio - the ratio of an asset's risk premium to its beta.

VII. The capital asset pricing model - $E(R_i) = R_f + [E(R_M) - R_f] \times \beta_i$.

A61. Another Quick Quiz

1. Assume: the historic market risk premium has been about 8.5%. The risk-free rate is currently 5%. GTX Corp. has a beta of .85. What return should you expect from an investment in GTX?

$$E(R_{\text{GTX}}) = 5\% + \underline{\hspace{2cm}} \times .85\% = 12.225\%$$

2. What is the effect of diversification?

3. The is the *equation* for the SML; the *slope* of the SML = .

A62. Another Quick Quiz (continued)

1. Assume: the historic market risk premium has been about 8.5%. The risk-free rate is currently 5%. GTX Corp. has a beta of .85. What return should you expect from an investment in GTX?

$$E(R_{\text{GTX}}) = 5\% + 8.5 \times .85 = 12.225\%$$

2. What is the effect of diversification?

Diversification reduces *unsystematic* risk.

3. The CAPM is the *equation* for the SML; the *slope* of the SML = $E(R_M) - R_f$.

A63. An Example

- Consider the following information:

State of Economy	Prob. of State of Economy	Stock A Return	Stock B Return	Stock C Return
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Boom 0.35 0.14 0.15 0.33

Bust 0.65 0.12 0.03 -0.06

- What is the expected return on an equally weighted portfolio of these three stocks?
- What is the variance of a portfolio invested 15 percent each in A and B, and 70 percent in C?

A64. Solution to the Example

- Expected returns on an equal-weighted portfolio
- a. Boom $E[R_p] = (.14 + .15 + .33)/3 = .2067$

Bust: $E[R_p] = (.12 + .03 - .06)/3 = .0300$

so the overall portfolio expected return must be

$$E[R_p] = .35(.2067) + .65(.0300) = .0918$$

A65. Solution to the Example (continued)

■ b. Boom: $E[R_p] = \underline{\hspace{1cm}} (.14) + .15(.15) + .70(.33) =$

$\underline{\hspace{1cm}}$
Bust: $E[R_p] = .15(.12) + .15(.03) + .70(-.06) = \underline{\hspace{1cm}}$

$$E[R_p] = .35(\underline{\hspace{1cm}}) + .65(\underline{\hspace{1cm}}) = \underline{\hspace{1cm}}$$

so

$$\sigma_p^2 = .35(\underline{\hspace{1cm}} - \underline{\hspace{1cm}})^2 + .65(\underline{\hspace{1cm}} - \underline{\hspace{1cm}})^2$$

$$= \underline{\hspace{1cm}}$$

A66. Solution to the Example (concluded)

■ b. Boom: $E[R_p] = .15(.14) + .15(.15) + .70(.33) = .2745$

Bust: $E[R_p] = .15(.12) + .15(.03) + .70(-.06) = -.0195$

$$E[R_p] = .35(.2745) + .65(-.0195) = .0834$$

so

$$\sigma_p^2 = .35(.2745 - .0834)^2 + .65(-.0195 - .0834)^2$$

$$= .01278 + .00688 = .01966$$

A67. Another Example

- Using information from capital market history, determine the return on a portfolio that was equally invested in large-company stocks and long-term government bonds.
- What was the return on a portfolio that was equally invested in small company stocks and Treasury bills?

A68. Solution to the Example

Solution

- The average annual return on common stocks over the period 1926-1998 was 13.2 percent, and the average annual return on long-term government bonds was 5.7 percent. So, the return on a portfolio with half invested in common stocks and half in long-term government bonds would have been:

$$E[R_{p1}] = .50(13.2) + .50(5.7) = 9.45\%$$

If on the other hand, one would have invested in the common stocks of *small* firms and in Treasury bills in equal amounts over the same period, one's portfolio return would have been:

$$E[R_{p2}] = .50(17.4) + .50(3.8) = 10.6\%.$$