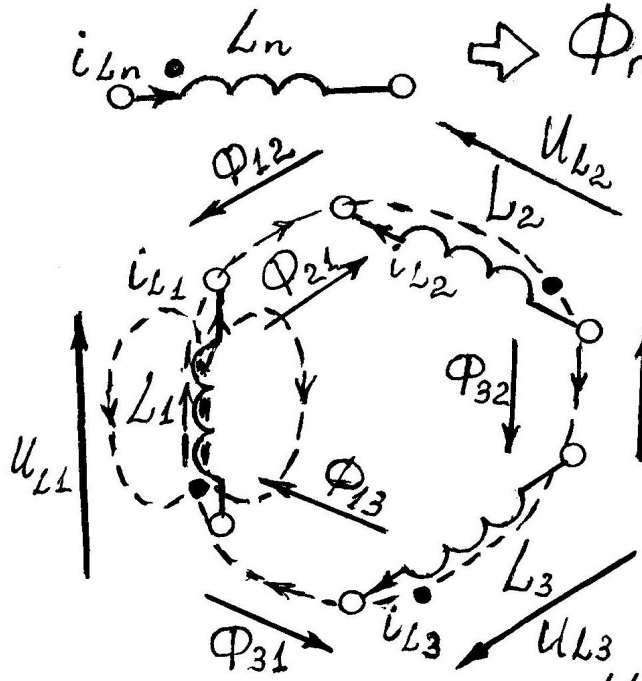
ЛІНІЙНІ КОЛА

Аналіз кіл з магнітним зв'язком при дії гармонічних сигналів

Трифазні кола

Зв'язані індуктивні елементи



$\Rightarrow \Phi_{nn}, \Phi_{in}$ → додатні ⊕

$\Phi_{iK} = M_{iK} \cdot i_{LK}, \quad M_{iK} = M_{Ki}$

M_{iK} - взаємна індуктивність

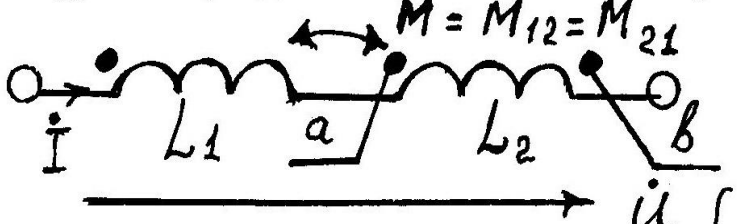
$$\Phi_{\Sigma,i} = \Phi_{ii} + \sum_{i \neq K}^{n} \Phi_{iK}$$

$$U_{Li} = L_i \frac{d(i_{Li})}{dt} + \sum_{i \neq K}^{n} M_{iK} \frac{d(i_{LK})}{dt}$$

$$\Phi_{\Sigma,1} = \Phi_{11} + \Phi_{12} - \Phi_{13}$$

$$U_{L1} = L_1 \frac{d(i_{L1})}{dt} + M_{12} \frac{d(i_{L2})}{dt} - M_{13} \frac{d(i_{L3})}{dt}$$

$i_L = Re(\dot{I}_L \cdot \exp(j\omega t)); \quad \dot{U}_{Li} = j\omega L_i \dot{I}_i + \sum_{i \neq K}^{K} j\omega M_{iK} \cdot \dot{I}_{LK}; \quad \dot{U}_{L1} = j\omega L_1 \cdot \dot{I}_1 \oplus$



a) $\dot{U} = j\omega (L_1 + L_2 + 2M)\dot{I}$

в) $\dot{U} = j\omega (L_1 + L_2 - 2M)\dot{I}$

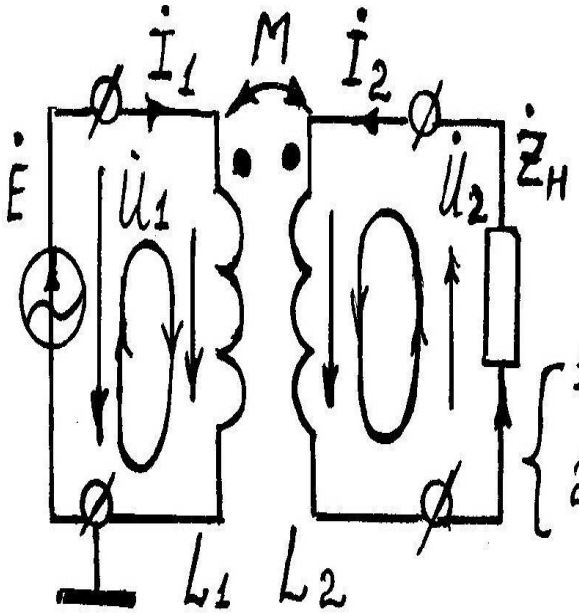
$\oplus j\omega M_{12} \cdot \dot{I}_2 - j\omega M_{13} \dot{I}_3$

$$\begin{cases} L_1 + L_2 - 2M \geq 0 \\ (L_1 + L_2)/2 \geq \sqrt{L_1 L_2} \end{cases} \Rightarrow M \leq \sqrt{L_1 \cdot L_2} \Rightarrow K_{3в}^{m} = 0 \ldots 1$$

$$M = K_{3в}^{m} \cdot \sqrt{L_1 L_2}$$

$$K_{3в}^{m} = \frac{M}{\sqrt{L_1 L_2}};$$

Передача потужності між зв'язаними індуктивностями



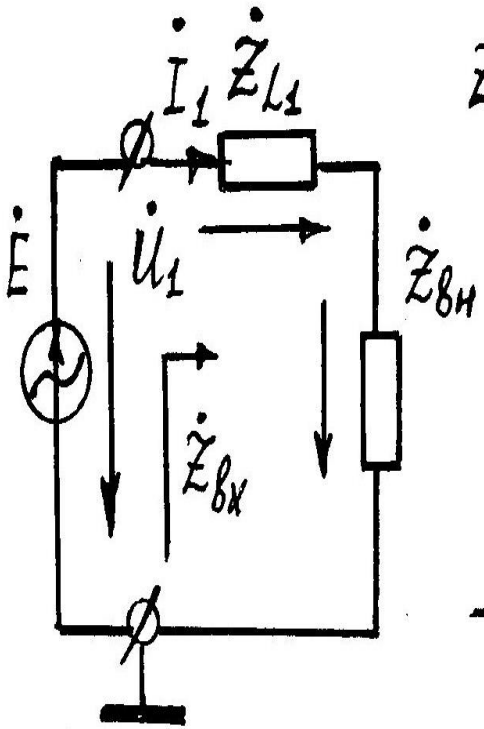
$$P_{12}=Re\left(\frac{1}{2}\dot{U}_{12}\cdot\dot{I}_2^*\right)=Re\left(\frac{1}{2}j\omega M\cdot\dot{I}_1\cdot\dot{I}_2^*\right)\Rightarrow -P-jQ=\dot{S}$$

$$P_{21}=Re\left(\frac{1}{2}\dot{U}_{21}\cdot\dot{I}_1^*\right)=Re\left(\frac{1}{2}j\omega M\cdot\dot{I}_2\cdot\dot{I}_1^*\right)\Rightarrow P+jQ=-\dot{S}$$

$$\begin{cases}1)\ \dot{U}_1=j\omega L_1\cdot\dot{I}_1+j\omega M\cdot\dot{I}_2\\ 2)\ 0=j\omega L_2\cdot\dot{I}_2+j\omega M\cdot\dot{I}_1+\dot{Z}_H\cdot\dot{I}_2\end{cases}\Rightarrow\quad \dot{I}_2=\frac{-j\omega M}{j\omega L_2+\dot{Z}_H}\cdot\dot{I}_1$$

$$\dot{U}_1=\left(j\omega L_1+\frac{\omega^2M^2}{j\omega L_2+\dot{Z}_H}\right)\cdot\dot{I}_1$$

Внесений опір та трансформація опору



$$\dot{Z}_{BX}=\frac{\dot{U}_1}{\dot{I}_1}=j\omega L_1+\frac{\omega^2M^2}{j\omega L_2+\dot{Z}_H}\Rightarrow \dot{Z}_{L1}=j\omega L_1,\ \dot{Z}_{BH}=\frac{\omega^2M^2}{j\omega L_2+\dot{Z}_H}$$

$$\sqrt{-1}=j\Rightarrow -(j)^2=1$$

При $|\dot{Z}_H|\to\infty$ $\quad \dot{Z}_{BX}^{XX}=j\omega L_1$

При $|\dot{Z}_H|\to 0$ $\quad \dot{Z}_{BX}^{K3}=j\omega L_1+\frac{-(j)^2\omega^2M^2}{j\omega L_2}\cdot\frac{L_1}{L_1}=j\omega L_1\left(1-\left(k_{3B}^M\right)^2\right)$

$k_{3B}^M=0\ldots 1$

$\dot{Z}_{BX}^{K3}=j\omega L_1\ldots 0 \Leftarrow$

$\left(k_{3B}^M\right)^2=M^2/L_1L_2$

а) Трансф. опору:

$$\dot{Z}_{BX}=j\omega L_1-\frac{(j)^2\omega^2M^2}{(j\omega L_2+\dot{Z}_H)}\cdot\frac{L_1/j\omega L_2\cdot L_1}{j/j\cdot j\omega L_2}=j\omega L_1-\frac{j\omega L_1\left(k_{3B}^M\right)^2}{1-j\dot{Z}_H/\omega L_2}\left(\frac{1}{1-X}\approx 1+X \text{ при } X\ll 1\right)$$

$$\dot{Z}_{BX}=j\omega L_1-\left(1+j\dot{Z}_H/\omega L_2\right)\cdot\left(k_{3B}^M\right)^2\cdot j\omega L_1=j\omega L_1\left(1-\left(k_{3B}^M\right)^2\right)+\left(k_{3B}^M\right)^2\cdot\dot{Z}_H\frac{L_1}{L_2}$$

При $k_{3B}^M=1$, $n^2=L_2/L_1$ $\quad \dot{Z}_{BX}=\dot{Z}_{BH}=\dot{Z}_H/n^2$, $n=\sqrt{L_2/L_1}$ – коефіцієнт трансформації

Трансформація струму та напруги

в) Трансформація струму: $\frac{\dot{I}_2}{\dot{I}_1} = \frac{-j\omega M}{j\omega L_2 + \dot{Z}_н}$;

При: $\omega L_2 \gg |\dot{Z}_н|$ $\dot{I}_2/\dot{I}_1 = -M/L_2$;

$M = k_{зв}^m \cdot \sqrt{L_1 \cdot L_2} \Rightarrow \dot{I}_2/\dot{I}_1 = -k_{зв}^m \cdot \sqrt{L_1/L_2}$

$n = \sqrt{L_2/L_1} \Rightarrow \boxed{\dot{I}_2/\dot{I}_1 = -k_{зв}^m \cdot \frac{1}{n}}$

При: $k_{зв}^m = 1 \Rightarrow \boxed{\dot{I}_2/\dot{I}_1 = -1/n}$

с) Трансформація напруги:

При $|\dot{Z}_н| \to \infty$ $\dot{I}_1 = \frac{\dot{u}_1}{j\omega L_1}$, $\dot{u}_2 = -j\omega M \cdot \dot{I}_1$

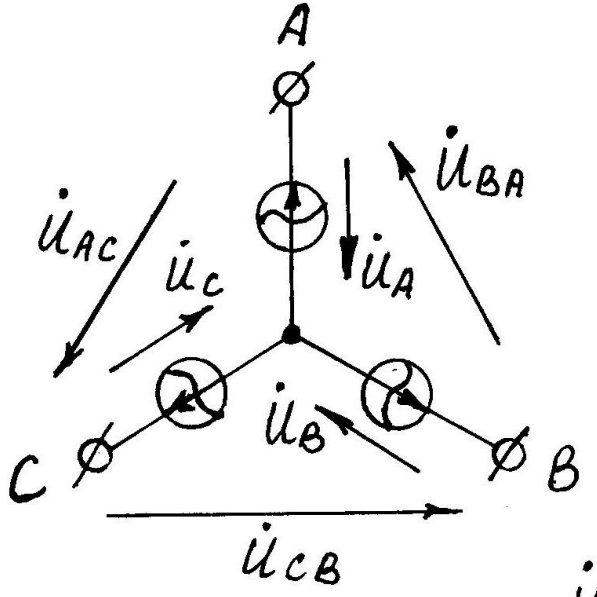
$|\dot{I}_2| \to 0$. $\dot{u}_1 = \dot{I}_1 j\omega L_1$

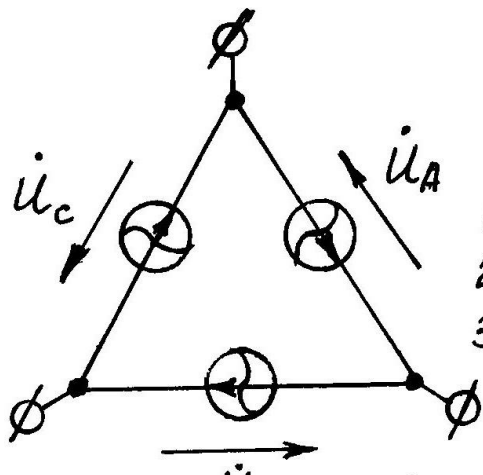
$M = k_{зв}^m \cdot \sqrt{L_1 L_2}$ $u_1/u_2 = -M/L_1$

$n = \sqrt{L_1/L_2} \Rightarrow \boxed{\dot{u}_1/\dot{u}_2 = -k_{зв}^m \cdot n}$

При $k_{зв}^m = 1 \Rightarrow \boxed{\dot{u}_1/\dot{u}_2 = -n}$

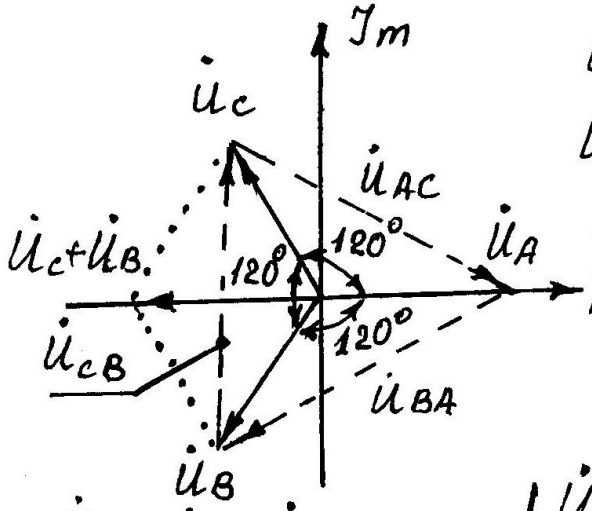
Трифазні кола





Трифазні системи змінного струму

220/127 (В); $f_c = 50$ Гц
220/127 (В); $f_c = 60$ Гц
380/220 (В); $f_c = 50$ Гц
660/380 (В); $f_c = 50$ Гц



$\dot{U}_A = U_g \exp(j0°)$
$\dot{U}_B = U_g \exp(-j120°)$
$\dot{U}_C = U_g \exp(+j120°)$

$\dot{U}_{AC} = \dot{U}_A - \dot{U}_C = U_g\sqrt{3}\exp(-j30°)$
$\dot{U}_{BC} = \dot{U}_B - \dot{U}_C = U_g\sqrt{3}\exp(j90°)$
$\dot{U}_{BA} = \dot{U}_B - \dot{U}_A = U_g\sqrt{3}\exp(-j150°)$

$\dot{U}_{AC} = \dot{U}_A - \dot{U}_C = U_g \exp(j0°) - U_g \exp(j120°)$

$\mathrm{Re}\ \dot{U}_{AC} = \dot{U}_g(\cos 0° + j\sin 0° - \cos(120°) - j\sin(120°))$

$\dot{U}_{AC} = U_g\left(1 + \frac{1}{2} - j\frac{\sqrt{3}}{2}\right) = U_g\left(\frac{3}{2} - j\frac{\sqrt{3}}{2}\right);$

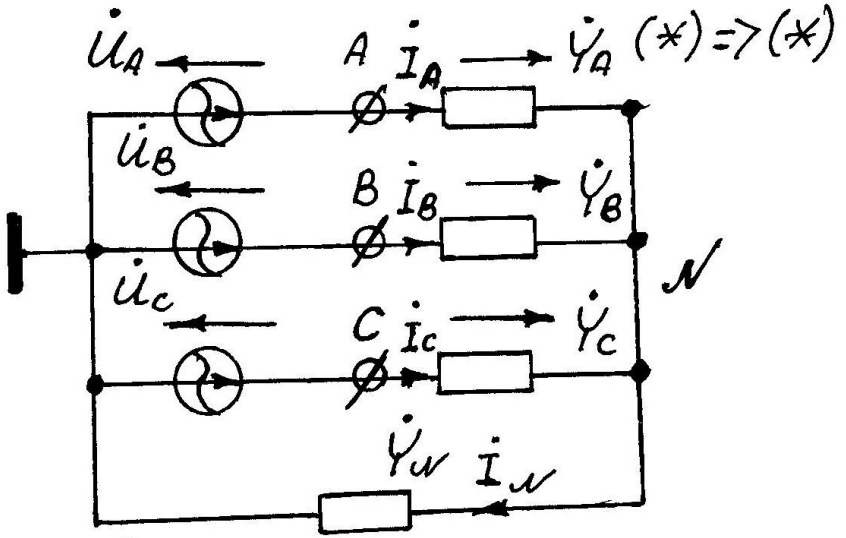
$|\dot{U}_{AC}| = U_g\sqrt{\left(\frac{3}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = U_g\cdot\sqrt{3};\ \varphi_{AC} = \mathrm{arctg}\left(\frac{-\sqrt{3}/2}{3/2}\right) = -30°$

$|\dot{U}_{AC}| = |\dot{U}_{BC}| = |\dot{U}_{BA}| = U_g\sqrt{3};\ \varphi_{CB} = 90°;\ \varphi_{BA} = -150°$

$\dot{U}_A + \dot{U}_C + \dot{U}_B = 0$

$\dot{U}_{BA} + \dot{U}_{AC} + \dot{U}_{CB} = 0$

Розрахунок трифазних кіл



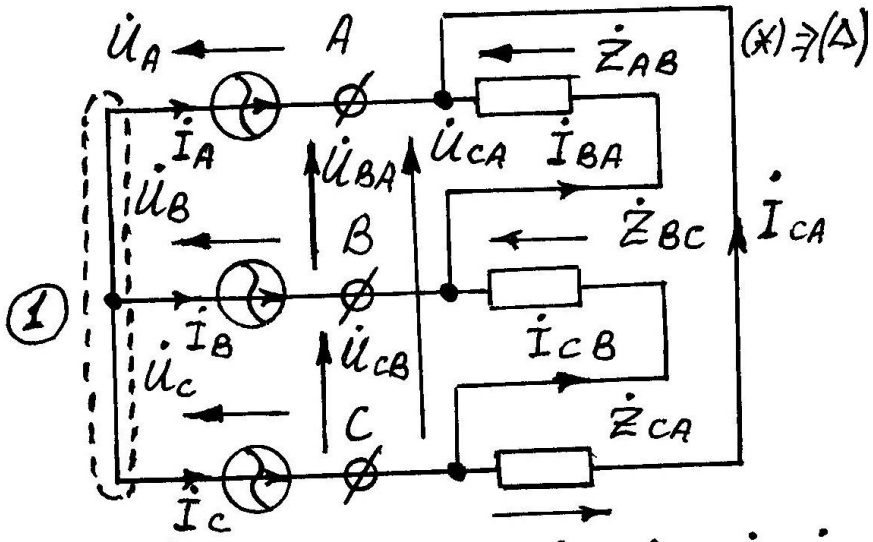
(N) $-\dot{Y}_A(\dot{U}_A-\dot{U}_N)-\dot{Y}_B(\dot{U}_B-\dot{U}_N)-\dot{Y}_C(\dot{U}_C-\dot{U}_N)$ (+)

(+) $\dot{Y}_N\cdot\dot{U}_N=0$

$$\dot{U}_N=\frac{\dot{Y}_A\dot{U}_A+\dot{Y}_B\dot{U}_B+\dot{Y}_C\cdot\dot{U}_C}{\dot{Y}_A+\dot{Y}_B+\dot{Y}_C+\dot{Y}_N};$$

При: $\dot{Y}_A=\dot{Y}_B=\dot{Y}_C$, $\dot{U}_A+\dot{U}_B+\dot{U}_C=0$ $\Rightarrow \dot{U}_N=0$

$$\dot{I}_A=\dot{Y}_A(\dot{U}_A-\dot{U}_N);\ \dot{I}_B=\dot{Y}_B(\dot{U}_B-\dot{U}_N);\ \dot{I}_C=\dot{Y}_C(\dot{U}_C-\dot{U}_N)$$



$\dot{U}_{BA}=\dot{U}_B-\dot{U}_A;\ \dot{U}_{CB}=\dot{U}_C-\dot{U}_B;\ \dot{U}_{CA}=\dot{U}_C-\dot{U}_A;$

(A) $\begin{cases} -\dot{I}_A-\dot{I}_{BA}-\dot{I}_{CA}=0 \\ \dot{I}_{BA}=\dfrac{\dot{U}_{BA}}{\dot{Z}_{AB}};\ \dot{I}_{CA}=\dfrac{\dot{U}_{CA}}{\dot{Z}_{CA}}; \end{cases}$

$$\dot{I}_A=-\dot{I}_{BA}-\dot{I}_{CA}=-\frac{\dot{U}_{BA}}{\dot{Z}_{AB}}-\frac{\dot{U}_{CA}}{\dot{Z}_{CA}};$$

(B) $-\dot{I}_B+\dot{I}_{BA}-\dot{I}_{CB}=0$

$$\dot{I}_B=-\dot{I}_{BA}+\dot{I}_{CB}=-\frac{\dot{U}_{BA}}{\dot{Z}_{AB}}+\frac{\dot{U}_{CB}}{\dot{Z}_{BC}};$$

(C) $-\dot{I}_C+\dot{I}_{CA}+\dot{I}_{CB}=0$

$$\dot{I}_C=\dot{I}_{CA}+\dot{I}_{CB}=\frac{\dot{U}_{CA}}{\dot{Z}_{CA}}+\frac{\dot{U}_{CB}}{\dot{Z}_{BC}};$$