Chapter 4

The Time Value of Money

Chapter Outline

Chapter Outline (cont'd)

~~**4.6** Solving Problems with a Spreadsheet or Calculator~~

~~**4.7** Non-Annual Cash Flows~~

~~**4.8** Solving for the Cash Payments~~

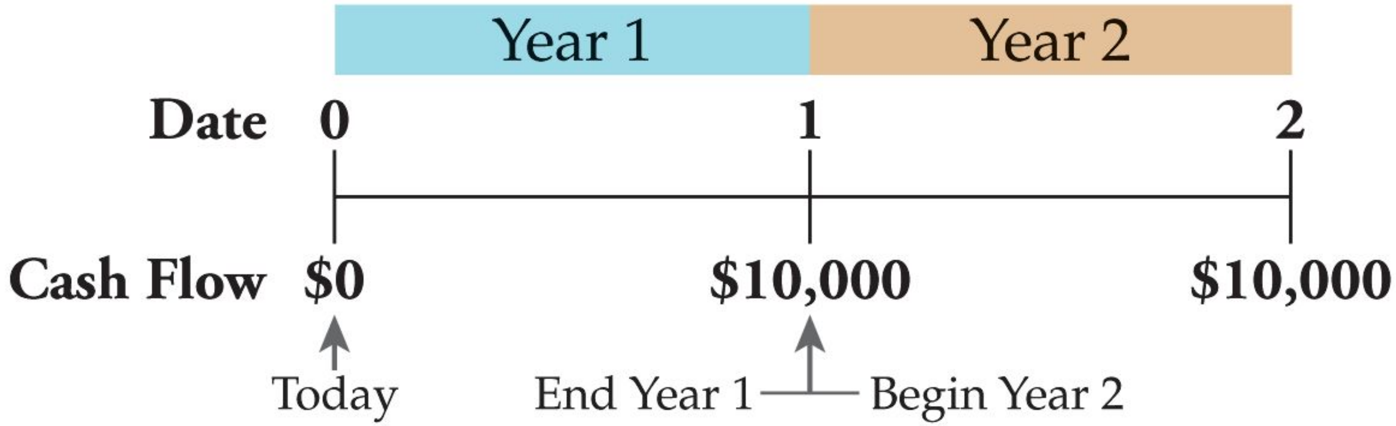
~~**4.9** _The Internal Rate of Return~~

4.1 The Timeline

- A timeline is a linear representation of the timing of potential cash flows.
- Drawing a timeline of the cash flows will help you visualize the financial problem.

4.1 The Timeline (cont'd)

- Assume that you made a loan to a friend. You will be repaid in two payments, one at the end of each year over the next two years.

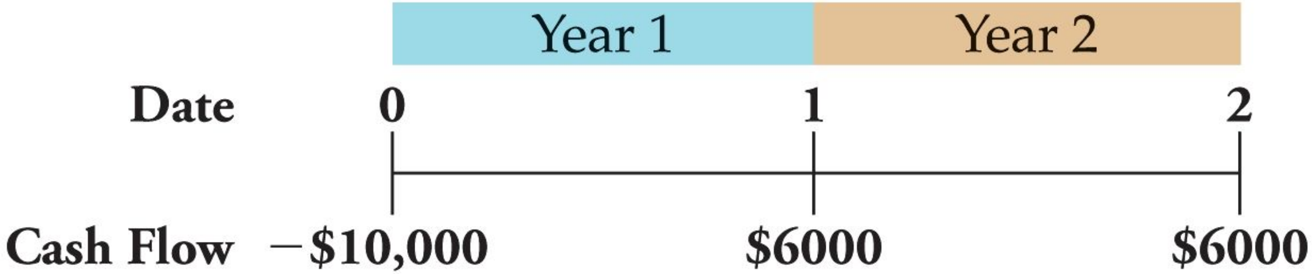


4.1 The Timeline (cont'd)

- Differentiate between two types of cash flows
  - Inflows are positive cash flows.
  - Outflows are negative cash flows, which are indicated with a – (minus) sign.

4.1 The Timeline (cont'd)

- Assume that you are lending $10,000 today and that the loan will be repaid in two annual $6,000 payments.



- The first cash flow at date 0 (today) is represented as a negative sum because it is an outflow.
- Timelines can represent cash flows that take place at the end of any time period – a month, a week, a day, etc.

4.2 Three Rules of Time Travel

- Financial decisions often require combining cash flows or comparing values. Three rules govern these processes.

Table 4.1 The Three Rules of Time Travel

Rule 1	Only values at the same point in time can be compared or combined.	
Rule 2	To move a cash flow forward in time, you must compound it.	Future Value of a Cash Flow $FV_n = C \times (1 + r)^n$
Rule 3	To move a cash flow backward in time, you must discount it.	Present Value of a Cash Flow $\boxed{PV = C \div (1 + r)^n = \dfrac{C}{(1 + r)^n}}$

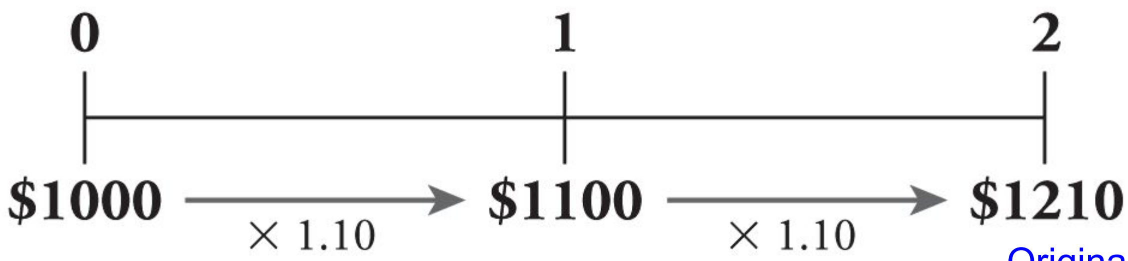
The 1st Rule of Time Travel

- A dollar today and a dollar in one year are not equivalent.
- It is only possible to compare or combine values at the same point in time.
  - Which would you prefer: A gift of $1,000 today or $1,210 at a later date?
  - To answer this, you will have to compare the alternatives to decide which is worth more. One factor to consider: How long is "later?"

The 2nd Rule of Time Travel

- To move a cash flow forward in time, you must compound it.
  - Suppose you have a choice between receiving $1,000 today or $1,210 in two years. You believe you can earn 10% on the $1,000 today, but want to know what the $1,000 will be worth in two years. The time line looks like this:

The 2nd Rule of Time Travel (cont'd)



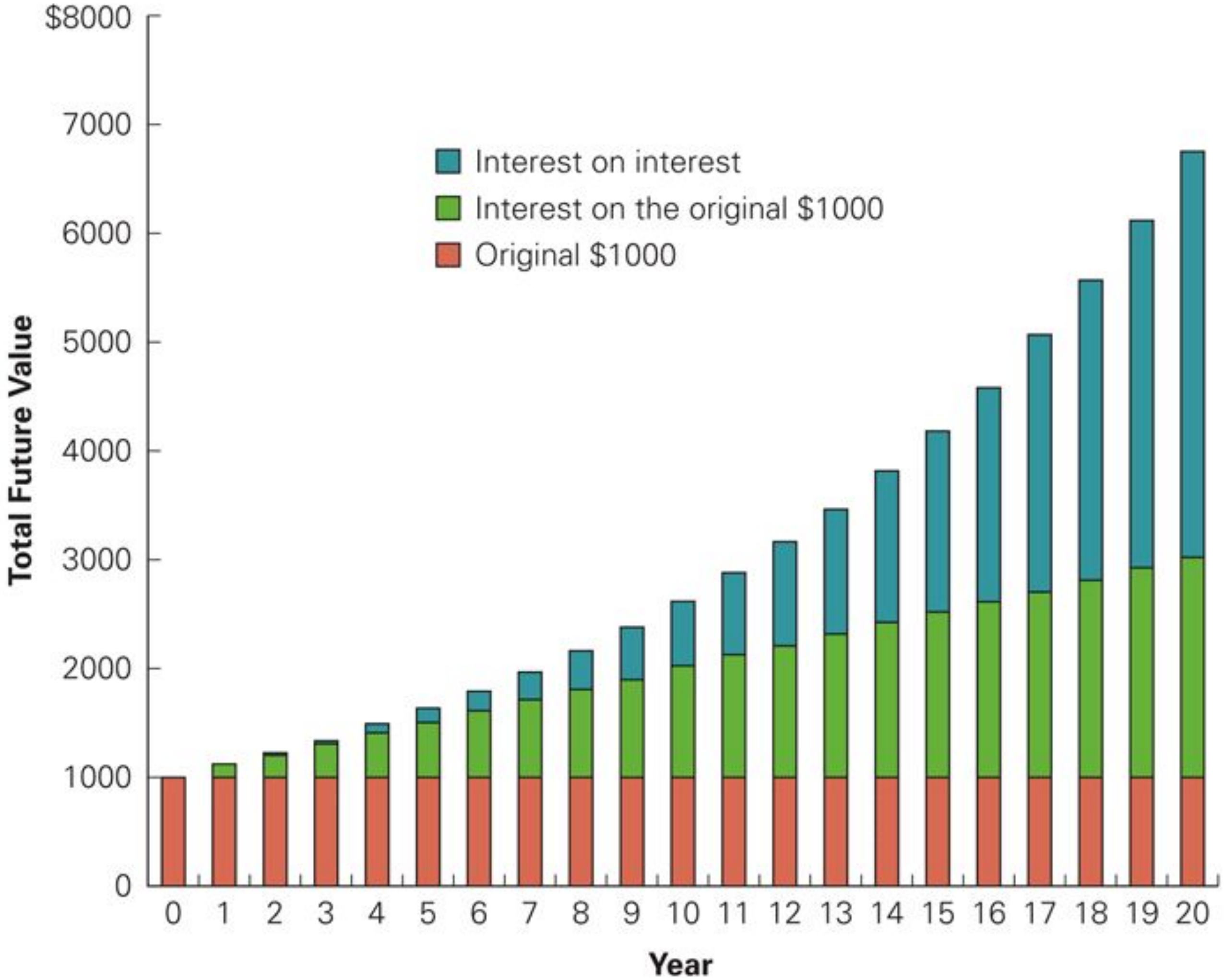
- Original capital: **$1,000**
- Interest on original capital: (1,000 x 10%) x 2 = **$200**
- Interest on interest: 100 x 10% = **$10**
- Total: 1,000+200+10 = **1,210**

- Future Value of a Cash Flow

Future Value of a Cash Flow

$$FV_n = C \times \underbrace{(1+r) \times (1+r) \times \cdots \times (1+r)}_{n \text{ times}} = C \times (1+r)^n \qquad (4.1)$$

Figure 4.1 The Composition of Interest Over Time

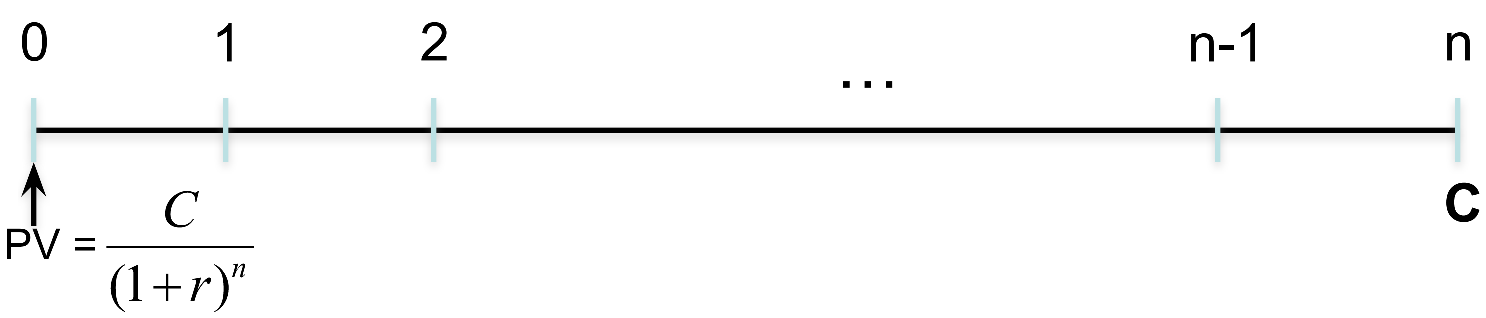


The 3rd Rule of Time Travel

- To move a cash flow backward in time, we must discount it.

- Present Value of a Cash Flow

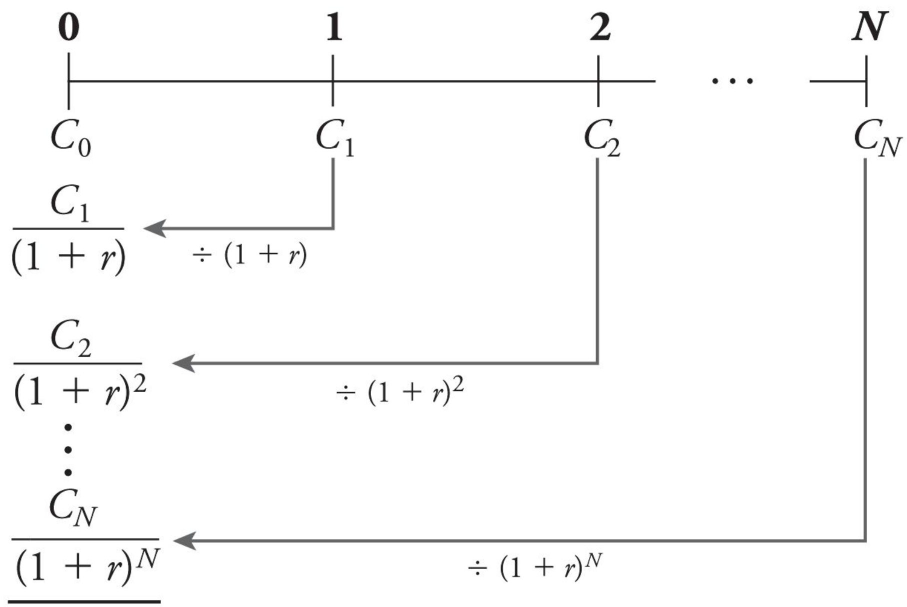
$$\boxed{PV = C \div (1+r)^n = \frac{C}{(1+r)^n}}$$



4.3 Valuing a Stream of Cash Flows

- Based on the first rule of time travel we can derive a general formula for valuing a stream of cash flows: if we want to find the present value of a stream of cash flows, we simply add up the present values of each.

4.3 Valuing a Stream of Cash Flows (cont'd)



$$C_0 + \frac{C_1}{(1+r)} + \frac{C_2}{(1+r)^2} + \ldots + \frac{C_N}{(1+r)^N}$$

- Present Value of a Cash Flow Stream

$$\boxed{PV = \sum_{n=0}^{N} PV(C_n) = \sum_{n=0}^{N} \frac{C_n}{(1+r)^n}}$$

4.4 Calculating the Net Present Value

- Calculating the NPV of future cash flows allows us to evaluate an investment decision.
- Net Present Value compares the present value of cash inflows (benefits) to the present value of cash outflows (costs).

Textbook Example 4.6

Net Present Value of an Investment Opportunity

Problem

You have been offered the following investment opportunity: If you invest $1000 today, you will receive $500 at the end of each of the next three years. If you could otherwise earn 10% per year on your money, should you undertake the investment opportunity?

Textbook Example 4.6 (cont'd)

Solution

As always, we start with a timeline. We denote the upfront investment as a negative cash flow (because it is money we need to spend) and the money we receive as a positive cash flow.

0	1	2	3
−$1000	$500	$500	$500

To decide whether we should accept this opportunity, we compute the NPV by computing the present value of the stream:

$$NPV = -1000 + \frac{500}{1.10} + \frac{500}{1.10^2} + \frac{500}{1.10^3} = \$243.43$$ > 0 Accept!

Because the NPV is positive, the benefits exceed the costs and we should make the investment. Indeed, the NPV tells us that taking this opportunity is like getting an extra $243.43 that you can spend today. To illustrate, suppose you borrow $1000 to invest in the opportunity and an extra $243.43 to spend today. How much would you owe on the $1243.43 loan in three years? At 10% interest, the amount you would owe would be

$$FV = (\$1000 + \$243.43) \times (1.10)^3 = \$1655 \text{ in three years}$$

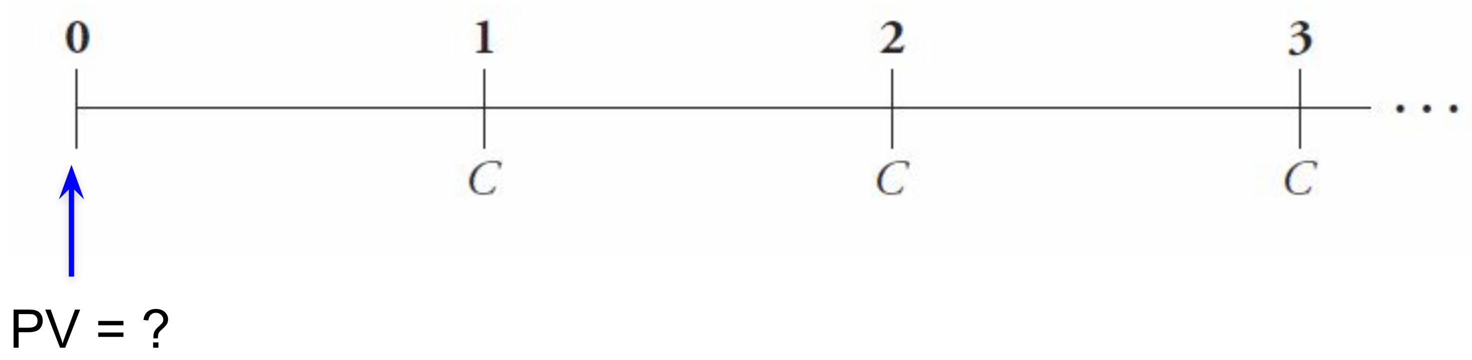
At the same time, the investment opportunity generates cash flows. If you put these cash flows into a bank account, how much will you have saved three years from now? The future value of the savings is

$$FV = (\$500 \times 1.10^2) + (\$500 \times 1.10) + \$500 = \$1655 \text{ in three years}$$

As you see, you can use your bank savings to repay the loan. Taking the opportunity therefore allows you to spend $243.43 today at no extra cost.

4.5 Perpetuities and Annuities

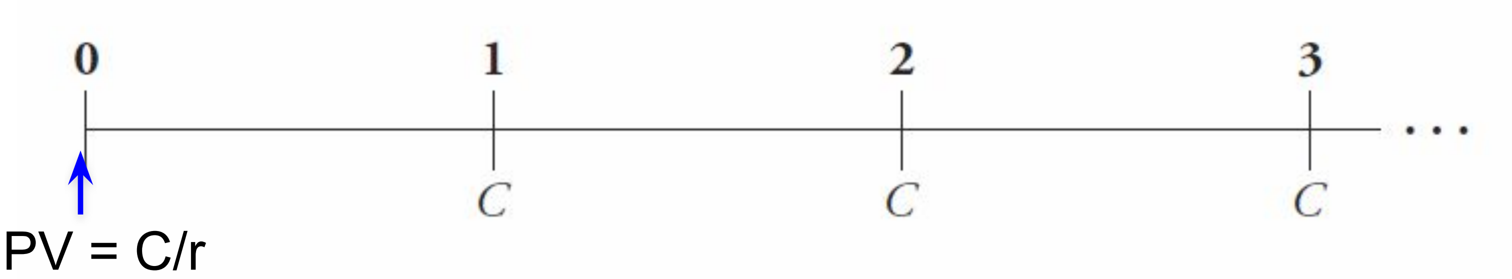
- Perpetuities
  - When a constant cash flow will occur at regular intervals forever it is called a perpetuity.



4.5 Perpetuities and Annuities (cont'd)

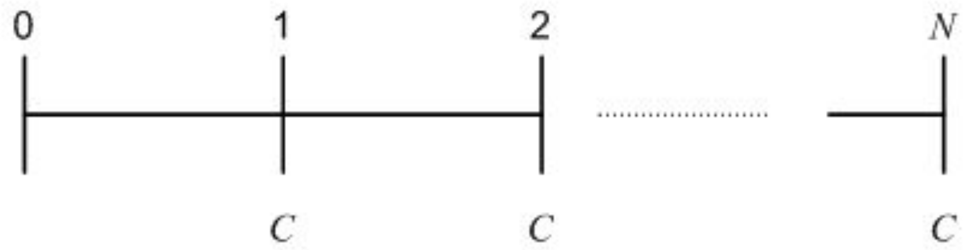
- The value of a perpetuity is simply the cash flow divided by the interest rate.
- Present Value of a Perpetuity

$$\boxed{PV(C \text{ in perpetuity}) = \frac{C}{r}}$$



4.5 Perpetuities and Annuities (cont'd)

- Annuities
  - When a constant cash flow will occur at regular intervals for a finite number of *N* periods, it is called an annuity.



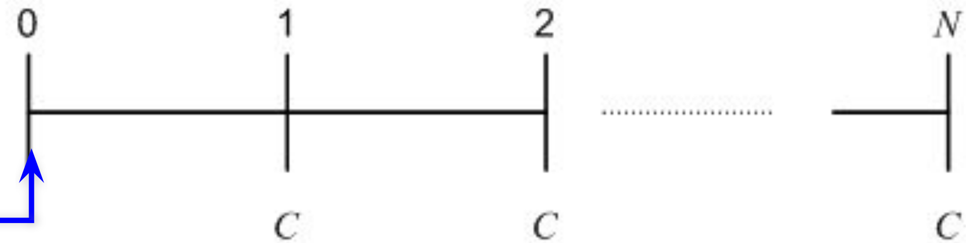
  - Present Value of an Annuity

$$PV = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \ldots + \frac{C}{(1+r)^N} = \sum_{n=1}^{N} \frac{C}{(1+r)^n}$$

Present Value of an Annuity

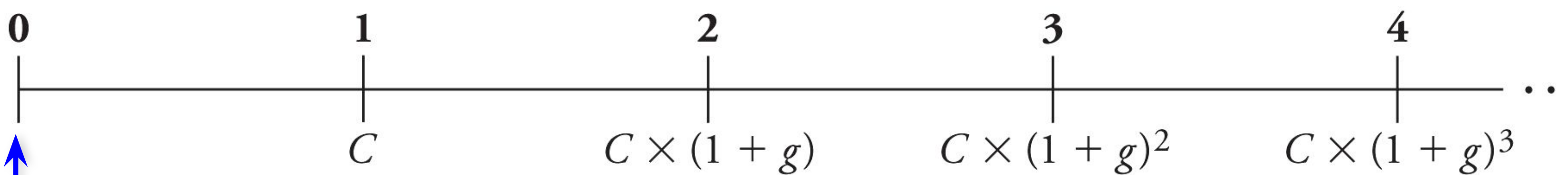
- For the general formula, substitute P for the principal value and:

$$
\begin{aligned}
&\text{PV(annuity of C for N periods)}\\
&= \text{P} - \text{PV(P in period N)}\\
&= \text{P} - \frac{\text{P}}{(1+\text{r})^{\text{N}}} = \text{P}\left(1 - \frac{1}{(1+\text{r})^{\text{N}}}\right) = \frac{C}{r}\left[1 - \frac{1}{(1+r)^{N}}\right]
\end{aligned}
$$



Growing Cash Flows

- Growing Perpetuity
  - Assume you expect the amount of your perpetual payment to increase at a constant rate, *g*.



- Present Value of a Growing Perpetuity

$$PV \text{ (growing perpetuity)} = \frac{C}{r - g}$$