Boolean algebra. Logic operations. Formula and their conversion.

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Have you ever wondered…

- How can we communicate with our computers or laptops?
- How is it possible that my SMS from my mobile phone be sent hundreds of miles from my location?
- How does televisions be able to project images on a screen?
- Why does robots be able to do specific (and even complicated) tasks?

An introduction

- A statement is true if it agrees with reality, false if it doesn’t.
- Two-state logic assumes that each statement is either true or false.
- The Greeks, especially Aristotle, worked out the theory of two-state logic in great detail.
- In 1854, George Boole came up with symbolic logic, better known as the Boolean Algebra. Boolean algebra uses letters and symbols to represent statements and their logical connections.
- Each variable in Boolean algebra has either of two values: true or false. (this is why it is called a two-state or binary algebra)
- Boolean algebra was a far-out subject until 1938, when Claude Shannon used it to analyze and design telephone switching circuits.
- “He let the variables represents closed and open relays.
- Boolean algebra has become one of the major design tools of digital and computer electronics

When to use Boolean Algebra?

- At least one (1) or more inputs of either logic 1 (true) or logic 0 (false) and a single desired output (either a 1 or a 0, depending on the inputs)
- Examples:
  - F = a+b
  - F = a*b
  - F = (a+b)*c’
  - F = abc’+(bd)’+ab+a’cd
- Note that inputs a, b, c, and d should have a value either a logic 1 or logic 0 and the output F should acquire a value either 1 and 0.

Axioms in Boolean Algebra

- a+b = b+a
- a*b = b*a

COMMUTATIVE LAW

- a+0 = a
- a*1 = a

IDENTITY LAW

DISTRIBUTIVE LAW

- a+(b*c) = (a+b)*(a+c)
- a*(b+c) = (a*b)+(a*c)

COMPLEMENT LAW

- a+a′ = 1
- a*a′ = 0

What is the output if we have *?

Let *a* be the values on the column side and *b* be the values on the row side.

*	0	1
0	0	0
1	0	1

What is the result of the unary operation?

Simply change the input 0 to an output of 1 and the input 1 to an output 0!!

`	0	1
	1	0

Other special theorems

Let a be any element of a Boolean algebra B.

Theorem 6: Uniqueness of compliment:

If $a+x = 1$ and $a*x = 0$, then $x = a'$

Theorem 7: Involution Law:

$(a')' = a$

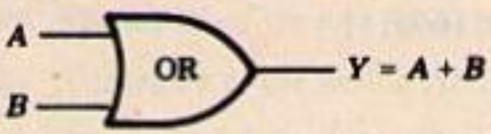
Theorem 8: *Inversion Law:

$0' = 1$ $\qquad$ $1' = 0$

Theorem 9: DeMorgan's Laws:

$(a+b)' = a' * b'$ $\qquad$ $(a*b)' = a' + b'$

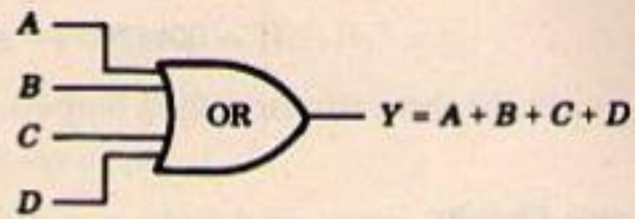
OR Gate



(a) OR gate

A	B	A + B
1	1	1
1	0	1
0	1	1
0	0	0

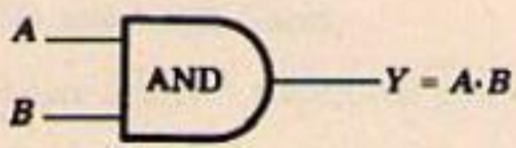
(b)



(c)

If at least one of the inputs has a value of logic high (1) , the output is logic high (1). Else, the output is logic low (0)

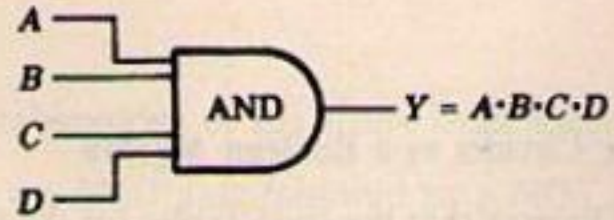
AND Gate



(a) AND gate

A	B	A · B
1	1	1
1	0	0
0	1	0
0	0	0

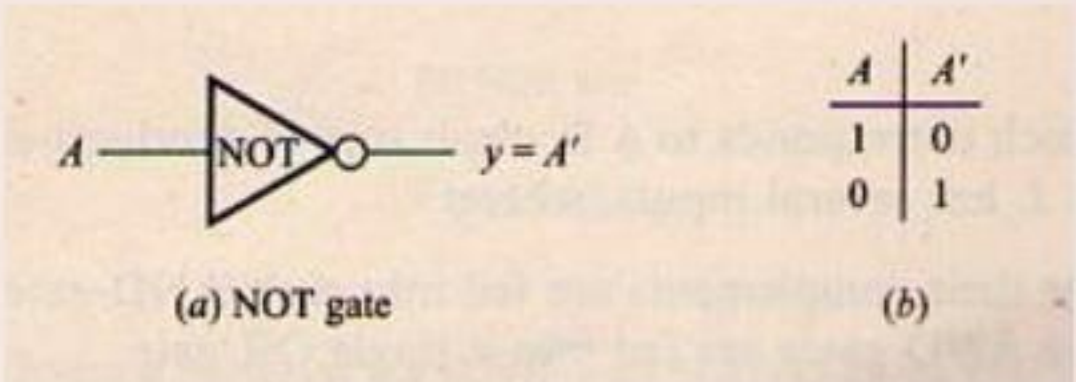
(b)



(c)

If all of the inputs has a value of logic high (1) , the output is logic high (1). Else, the output is logic low (0)

NOT Gate

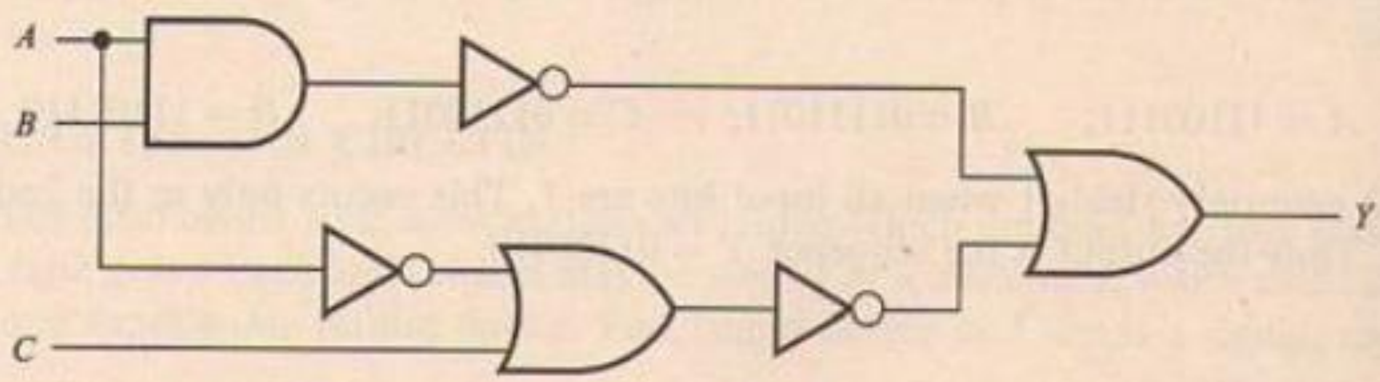


(a) NOT gate (b)

If the input is logic high (1), the output is logic low (0). And, vice versa.

Logic Circuit

$$Y = (A \cdot B)' + (A' + C)'$$



Integration of AND, OR, and NOT gates.

Let’s try this one…

Problem: Design a three input minimal AND-OR circuit L with the following table:

T = [A, B, C; L] = [00001111, 00110011, 01010101; 11001101]