Translational Equilibrium

Objectives

- Describe with examples Newton’s three laws of motion.
- Describe with examples the first condition for equilibrium.
- Draw free-body diagrams for objects in translational equilibrium.
- Apply the first condition for equilibrium to the solution of problems.

Newton's First Law

<u>Newton's First Law:</u> An object at rest or an object in motion at constant speed will remain at rest or at constant speed in the absence of a resultant force.

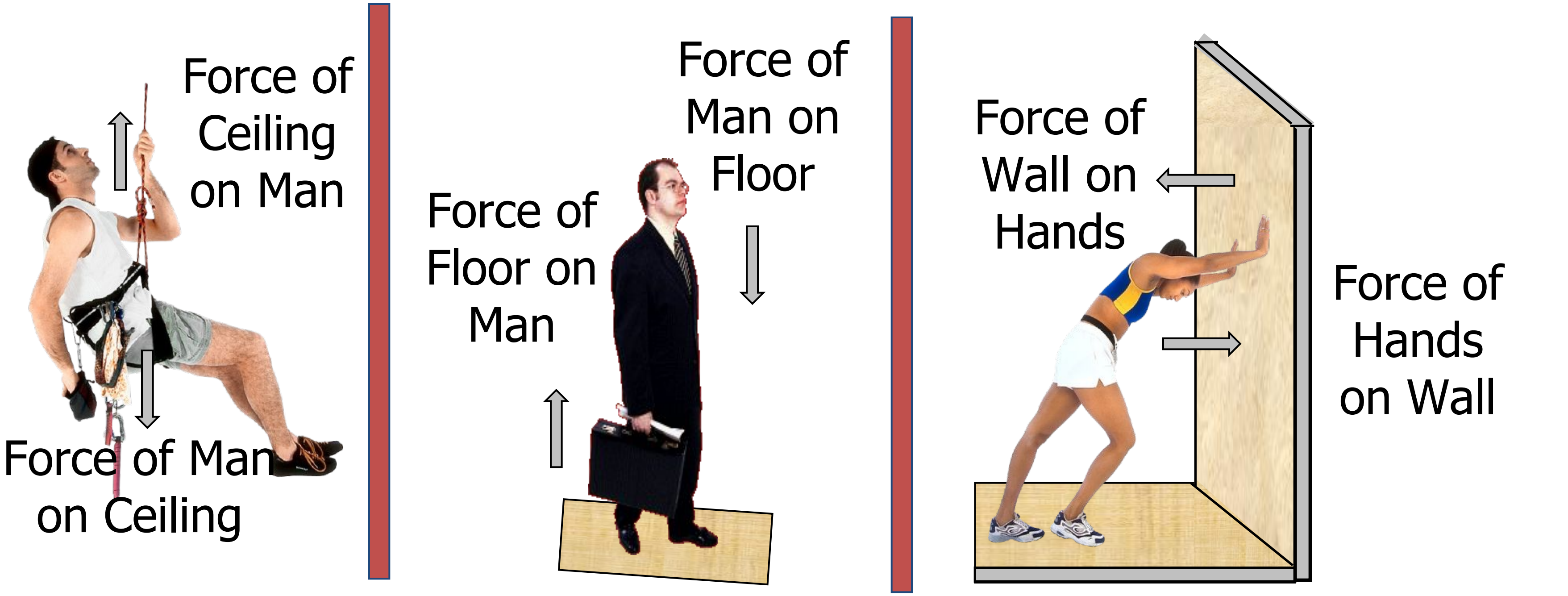
Newton's Second Law:

- Second Law: Whenever a resultant force acts on an object, it produces an acceleration - an acceleration that is directly proportional to the force and inversely proportional to the mass.

$$\boxed{a \propto \frac{F}{m}}$$

Newton's Third Law

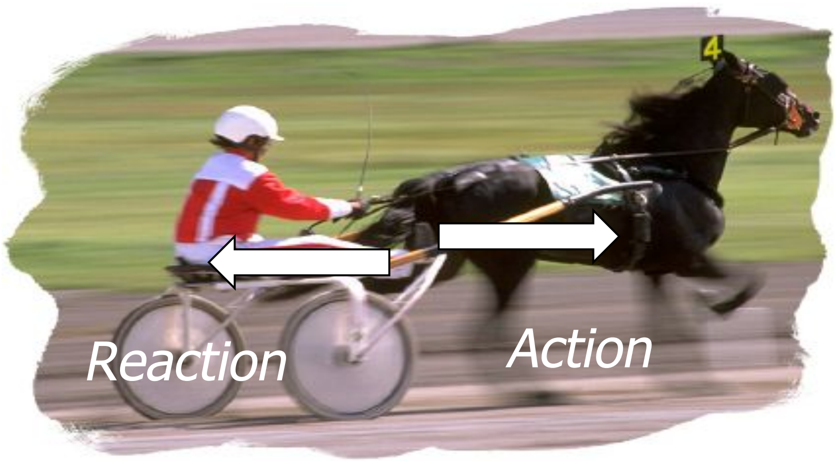
- To every action force there must be an equal and opposite reaction force.



Action and reaction forces act on different objects.

Newton's Third Law

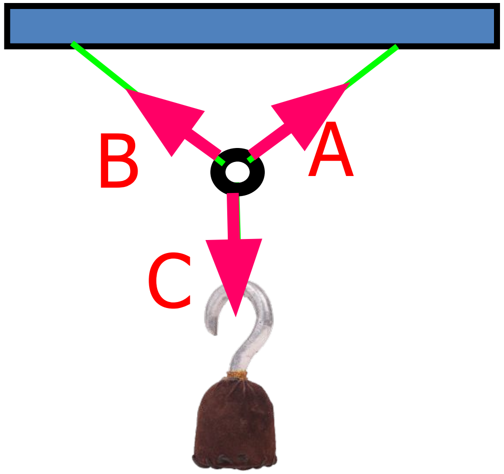
Examples:



Action and Reaction Forces Act on Different Objects. They Do Not Cancel Each Other!

Translational Equilibrium

- An object is said to be in Translational Equilibrium if and only if there is no resultant force.
- This means that the sum of all acting forces is zero.



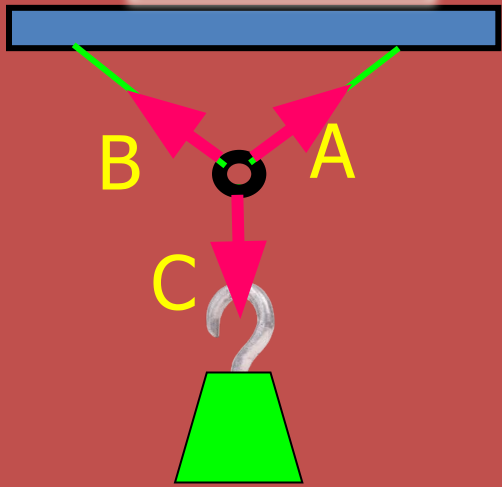
In the example, the resultant of the three forces A, B, and C acting on the ring must be zero.

Visualization of Forces

Force diagrams are necessary for studying objects in equilibrium.

Equilibrium:

$$\boxed{\Sigma F = 0}$$



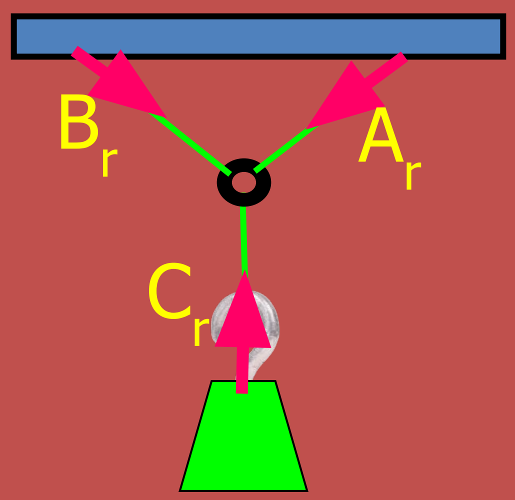
The action forces are each ON the ring.

- Force A: By ceiling on ring.
- Force B: By ceiling on ring.
- Force C: By weight on ring.

Visualization of Forces

Now let's look at the Reaction Forces for the same arrangement. They will be equal, but opposite, and they act on different objects.

Reaction forces:

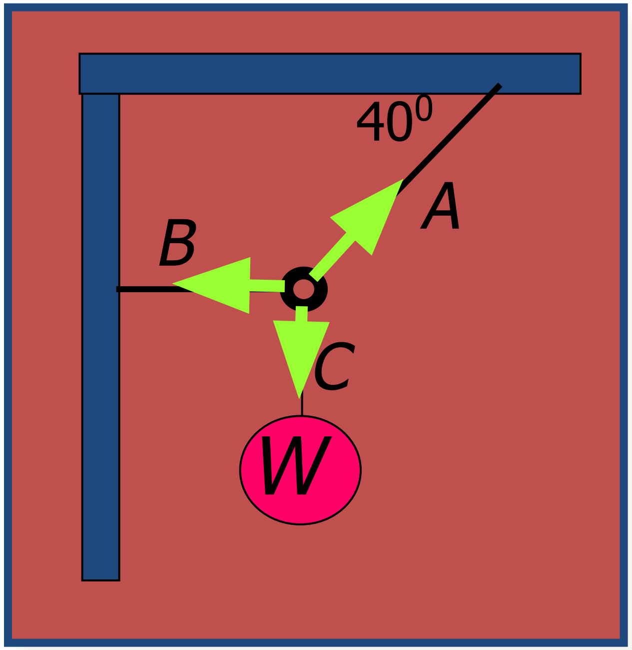


Reaction forces are each exerted: BY the ring.

- Force A_r: By ring on ceiling.
- Force B_r: By ring on ceiling.
- Force C_r: By ring on weight.

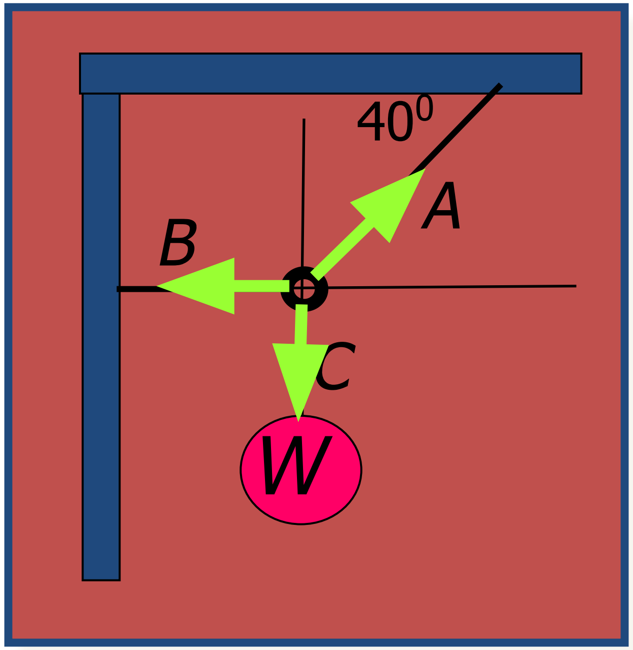
Vector Sum of Forces

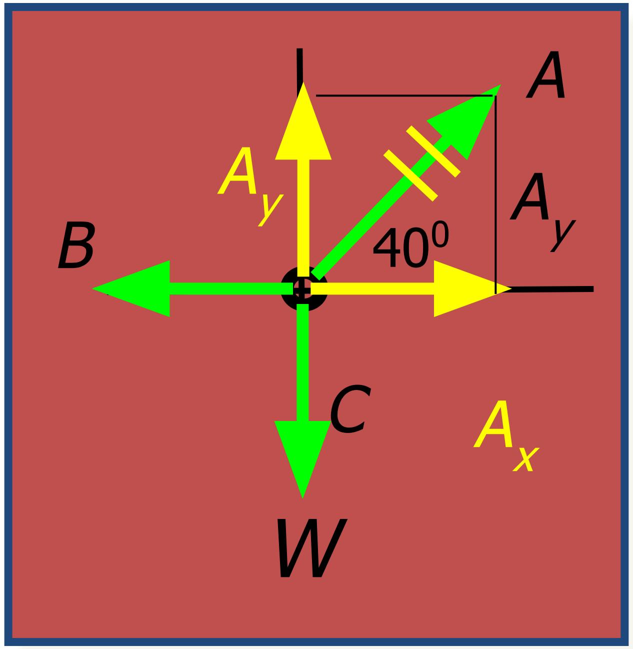
- An object is said to be in Translational Equilibrium if and only if there is no resultant force.
- The vector sum of all forces acting on the ring is zero in this case.



Vector sum: $\Sigma F = A + B + C = 0$

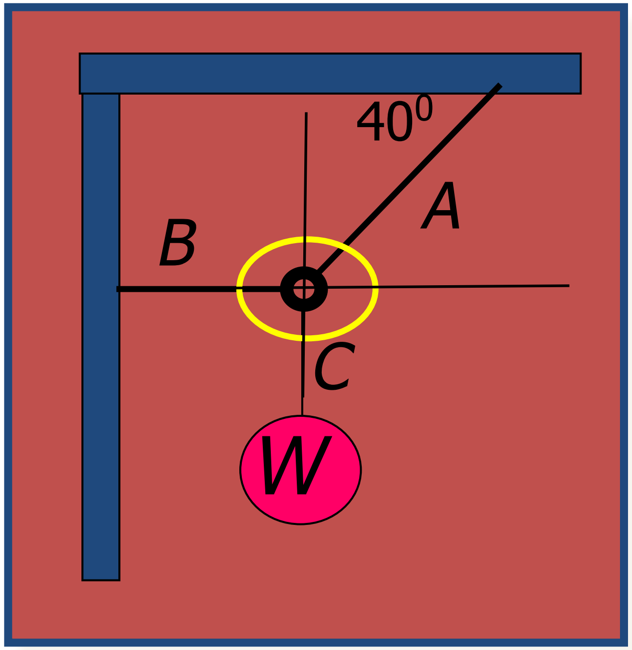
Vector Force Diagram

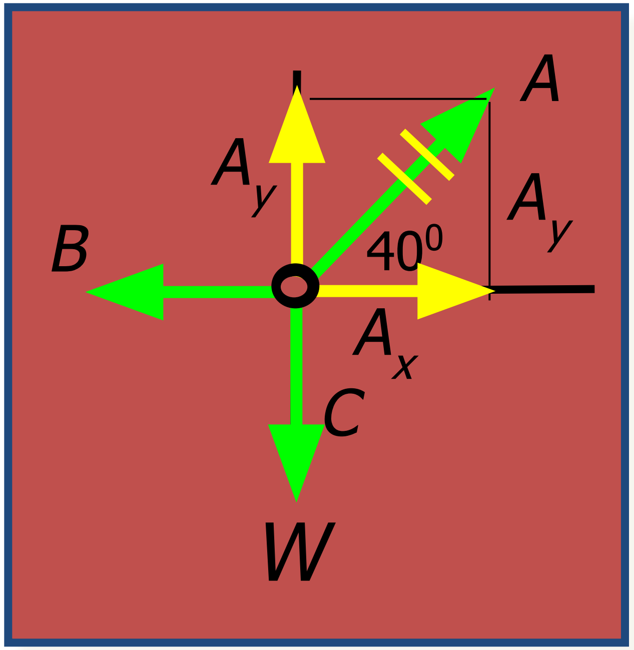




A free-body diagram is a force diagram showing all the elements in this diagram: axes, vectors, components, and angles.

Look Again at Previous Arrangement

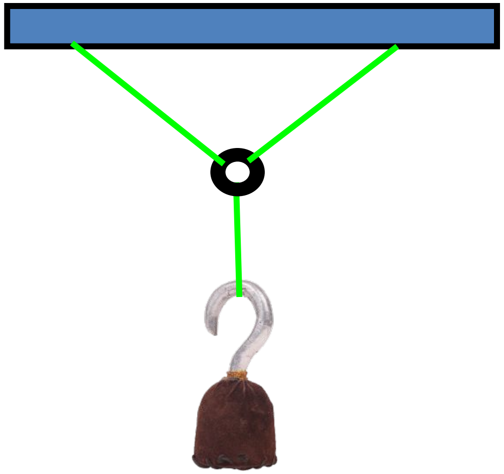




1. Isolate point.
2. Draw x,y axes.
3. Draw vectors.
4. Label components.
5. Show all given information.

Translational Equilibrium

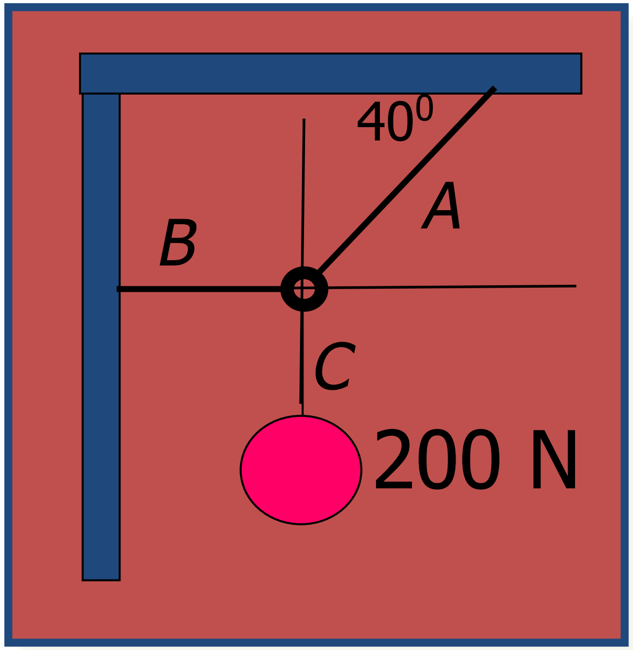
- The First Condition for Equilibrium is that there be no resultant force.
- This means that the sum of all acting forces is zero.

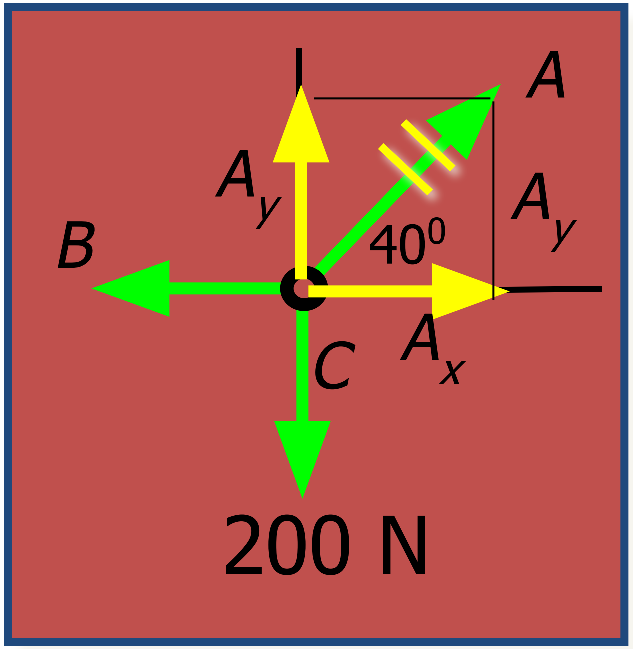


$$\boxed{\Sigma F_x = 0}$$

$$\boxed{\Sigma F_y = 0}$$

Example 2. Find the tensions in ropes A and B for the arrangement shown.





The Resultant Force on the ring is zero:

$$R = \Sigma F = 0$$

$$R_x = A_x + B_x + C_x = 0$$

$$R_y = A_y + B_y + C_y = 0$$

Example 2. Continued . . .

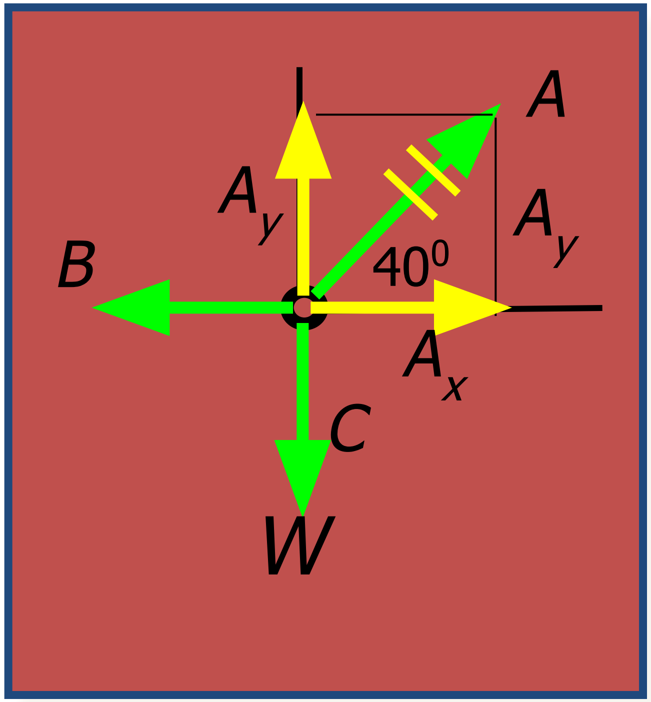
Components

$A_x = A \cos 40^0$

$A_y = A \sin 40^0$

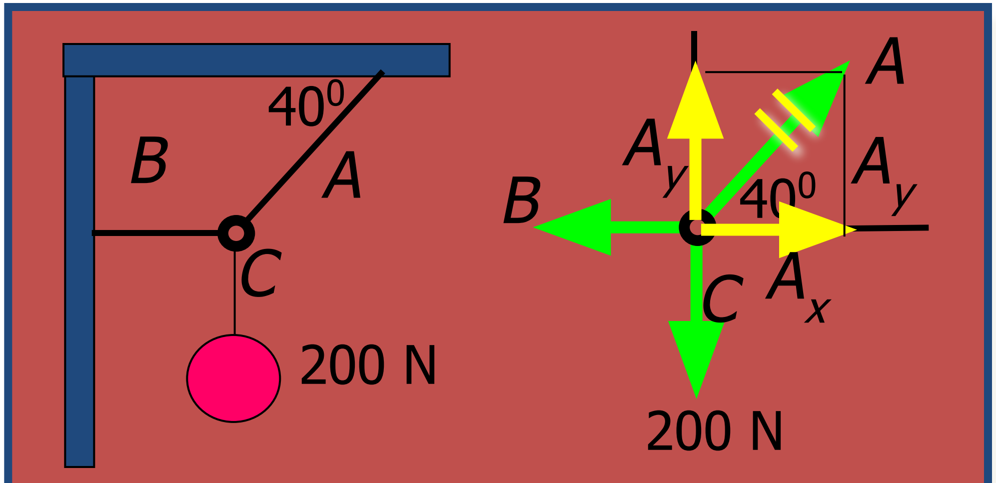
$B_x = B; \quad B_y = 0$

$C_x = 0; \quad C_y = W$



A *free-body diagram* must represent all forces as components along x and y-axes. It must also show all given information.

Example 2. Continued . . .



$\Sigma F_x = 0$ $\Sigma F_y = 0$

Components

$A_x = A \cos 40^0$

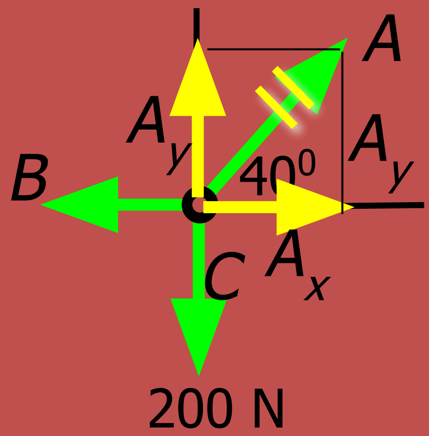
$A_y = A \sin 40^0$

$B_x = B$; $B_y = 0$

$C_x = 0$; $C_y = W$

$$\sum F_x = A\cos 40^0 - B = 0; \quad \text{or} \quad B = A\cos 40^0$$

Example 2. Continued . . .



Two equations; two unknowns

Solve first for A

$$A = \frac{200\text{ N}}{\sin 40^0} = 311\text{ N}$$

Solve Next for B

$$B = A\cos 40^0 = (311\text{ N})\cos 40^0; \quad B = 238\text{ N}$$

The tensions in A and B are

A = 311 N; B = 238 N

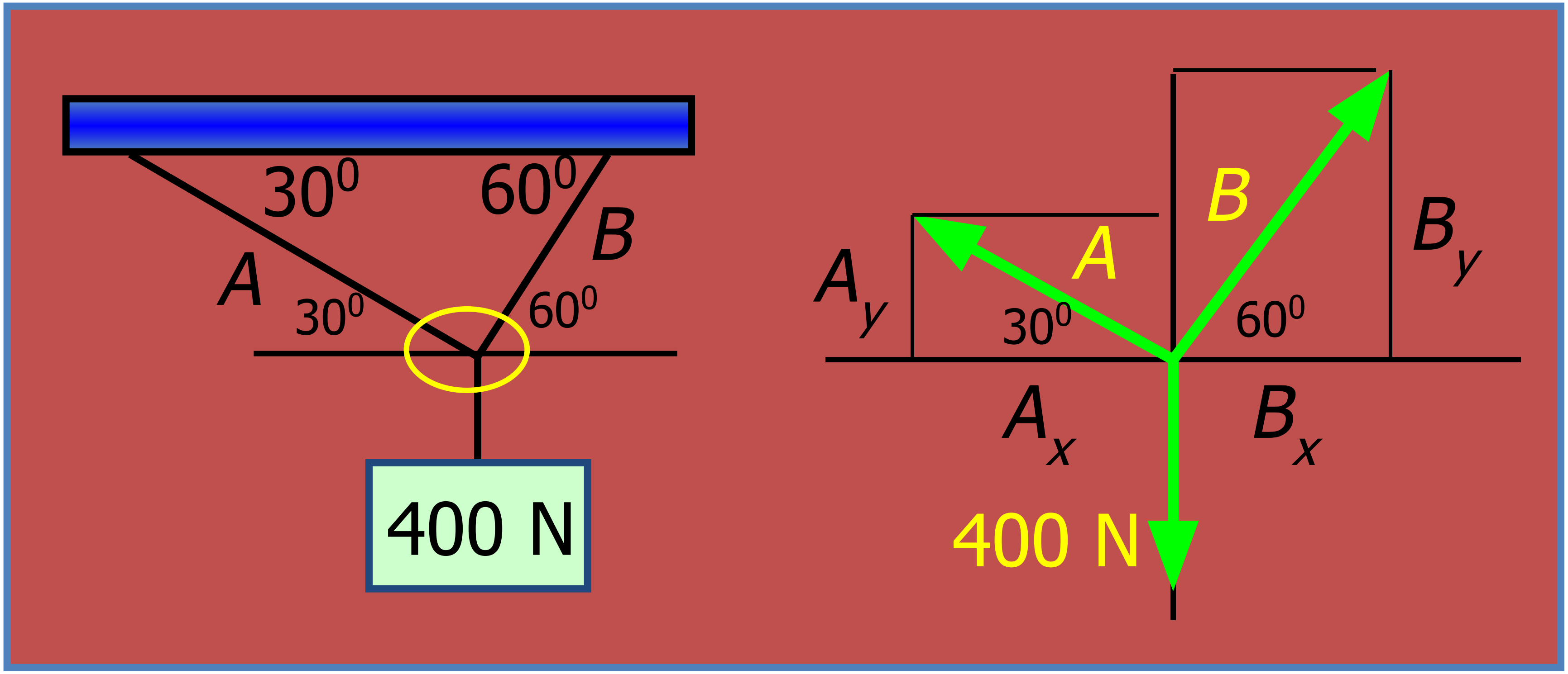
Problem Solving Strategy

1. Draw a sketch and label all information.
2. Draw a free-body diagram.
3. Find components of all forces (+ and -).
4. Apply First Condition for Equilibrium:

$$\boxed{\Sigma F_x = 0 \; ; \quad \Sigma F_y = 0}$$

5. Solve for unknown forces or angles.

Example 3. Find Tension in Ropes A and B.



1. Draw free-body diagram.

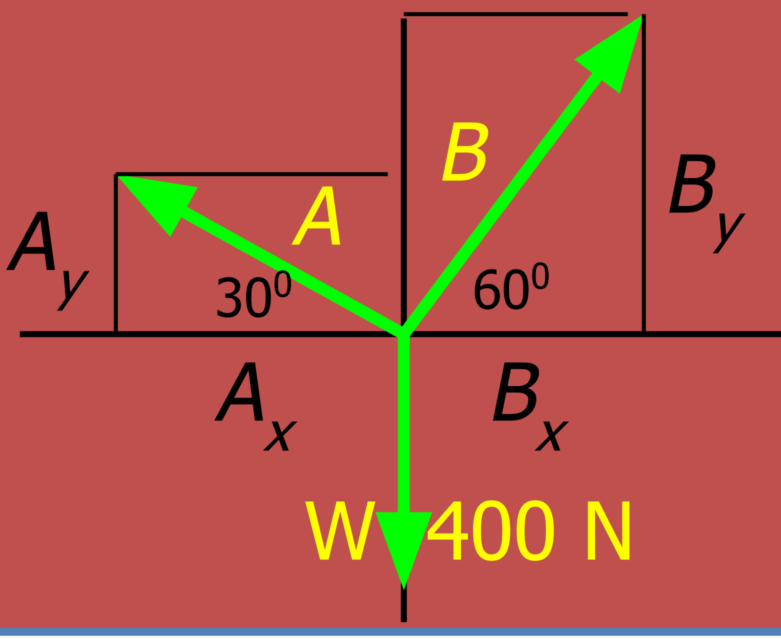
2. Determine angles.

3. Draw/label components.

Example 3. Find the tension in ropes A and B.

First Condition for Equilibrium:

$$\Sigma F_x = 0 \; ; \quad \Sigma F_y = 0$$



4. Apply 1st Condition for Equilibrium:

$$\Sigma F_x = B_x - A_x = 0 \quad \Rightarrow \quad B_x = A_x$$

$$\Sigma F_y = B_y + A_y - W = 0 \quad \Rightarrow \quad B_y + A_y = W$$

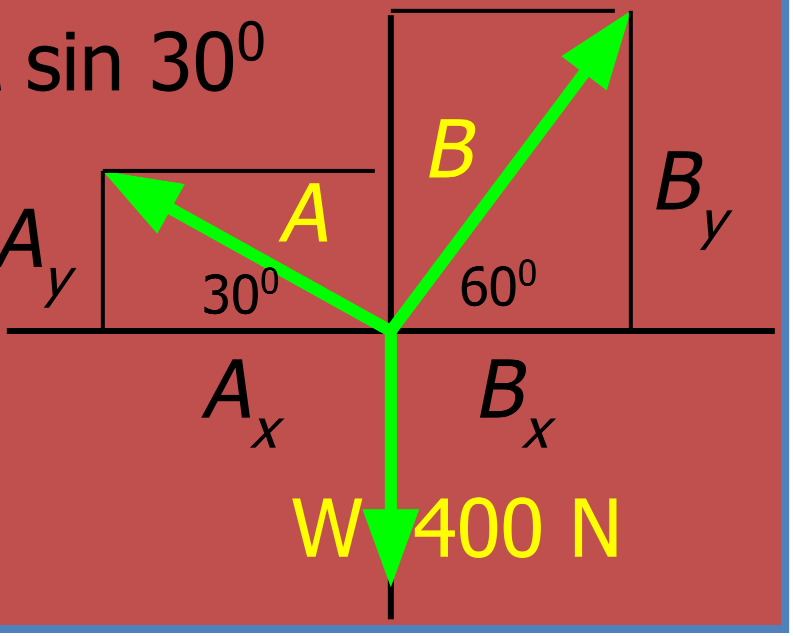
Example 3. Find the tension in ropes A and B.

$A_x = A \cos 30^0$; $A_y = A \sin 30^0$

$B_x = B \cos 60^0$

$B_y = B \sin 60^0$

$W_x = 0$; $W_y = -400$ N

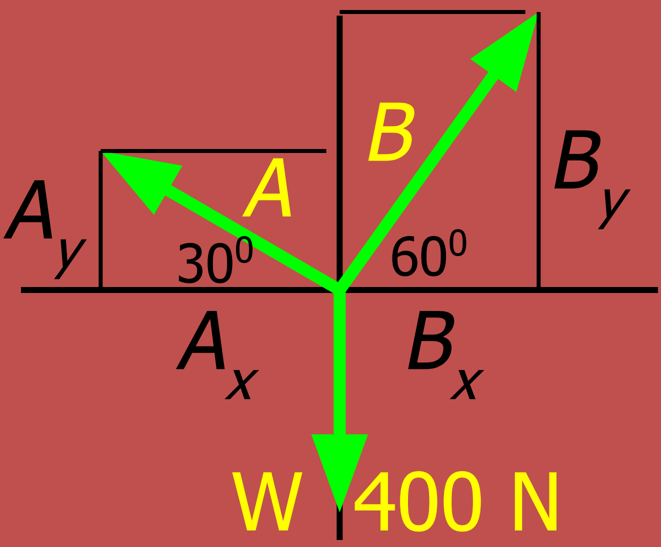


Using Trigonometry, the first condition yields:

$B_x = A_x \Longrightarrow B \cos 60^0 = A \cos 30^0$

$B_y + A_y = W \Longrightarrow A \sin 30^0 + B \sin 60^0 = 400$ N

Example 3 (Cont.) Find the tension in A and B.



$$B\cos 60^0 = B\cos 30^0$$

$$A\sin 30^0 + B\sin 60^0 = 400\text{ N}$$

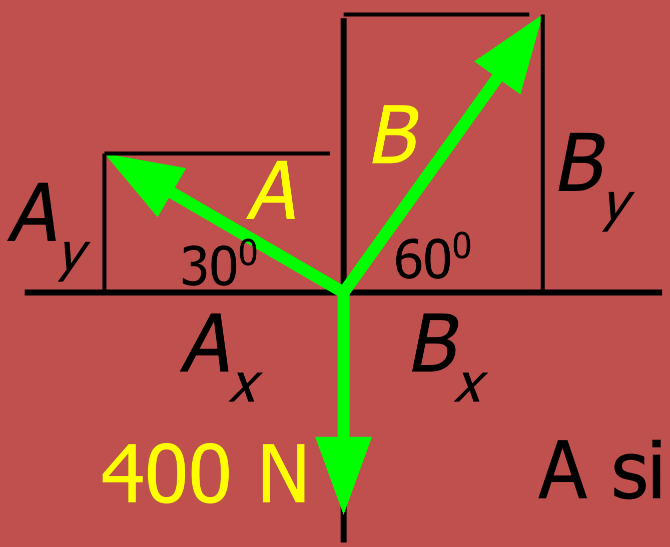
We now solve for *A* and *B*: Two Equations and Two Unknowns.

We will first solve the horizontal equation for *B* in terms of the unknown *A*:

$$B = \frac{A\cos 30^0}{\cos 60^0} = 1.73A$$

$$B = 1.732\,A$$

<u>Example 3 (Cont.)</u> Find Tensions in A and B.



$B = 1.732\ A$

Now apply Trig to:

$A_y + B_y = 400$ N

A sin 60^0 + B sin 60^0 = 400 N

$B = 1.732\ A$

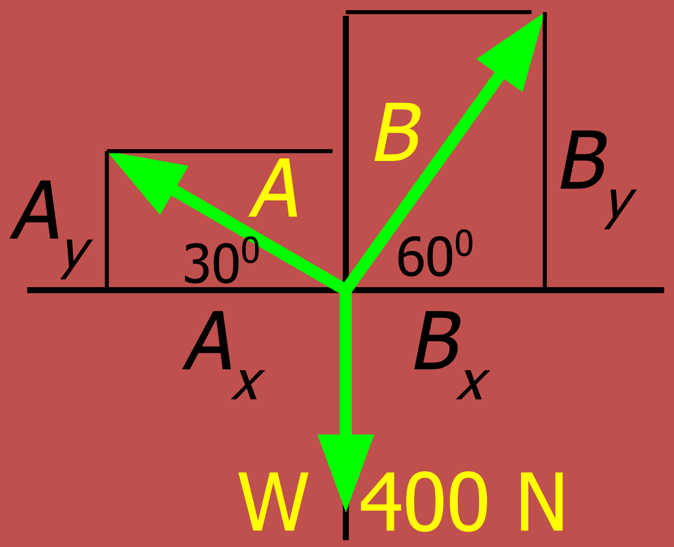
$A \sin 30^0 + B \sin 60^0 = 400$ N

$A \sin 30^0 + (1.732\ \mathrm{A}) \sin 60^0 = 400$ N

$0.500\ A + 1.50\ A = 400$ N

$\boxed{A = 200\ \mathrm{N}}$

Example 3 (Cont.) Find B with A = 200 N.



$A = 200\ N$

$B = 1.732\ A$

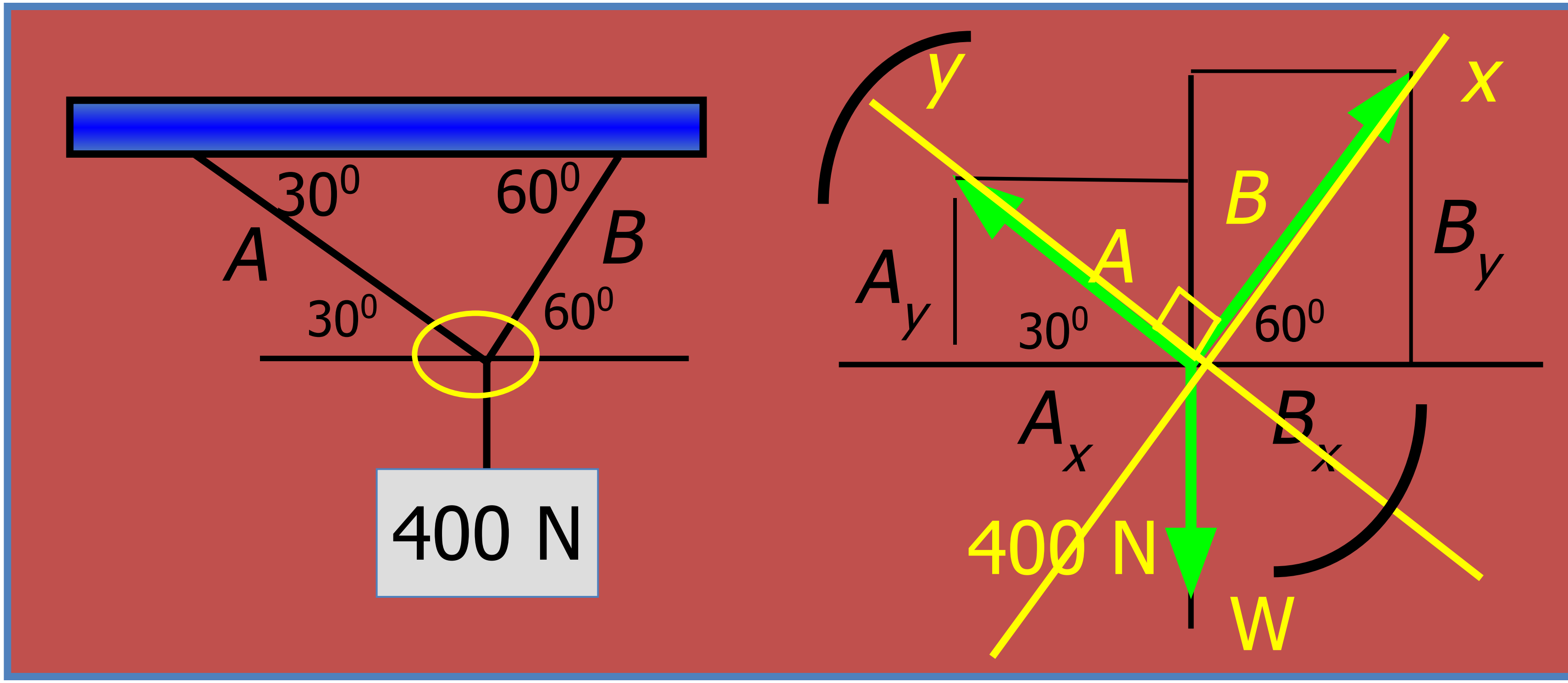
$B = 1.732(400\ N)$

$B = 346\ N$

Rope tensions are: $A = 200\ N$ *and* $B = 346\ N$

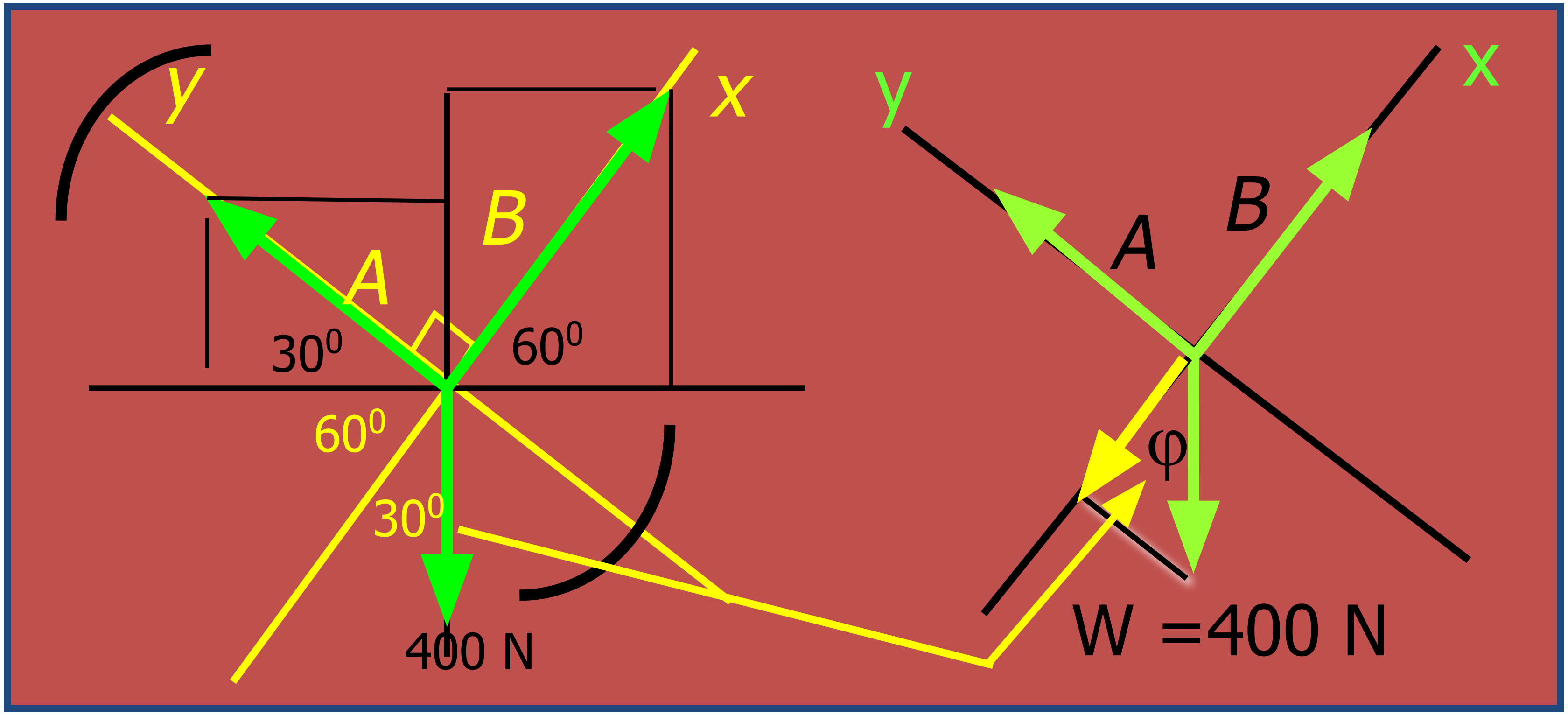
This problem is made much simpler if you notice that the angle between vectors B and A is 90^0 and rotate the x and y axes.

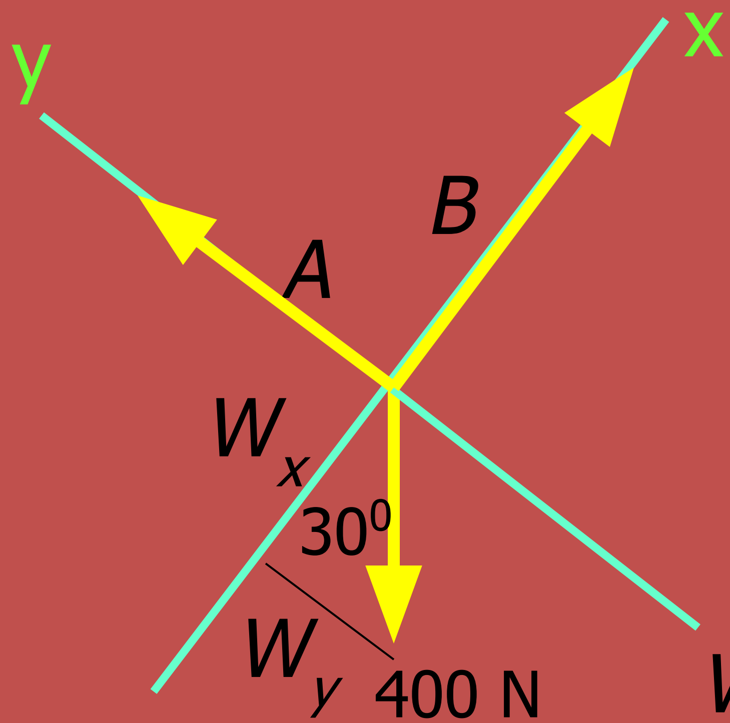
Example 4. Rotate axes for same example.



We recognize that *A* and *B* are at right angles, and choose the x-axis along *B* – not horizontally. The y-axis will then be along *A*.

Since A and B are perpendicular, we can find the new angle φ from geometry.





$W_x = (400\text{ N}) \cos 30^0$

$W_y = (400\text{ N}) \sin 30^0$

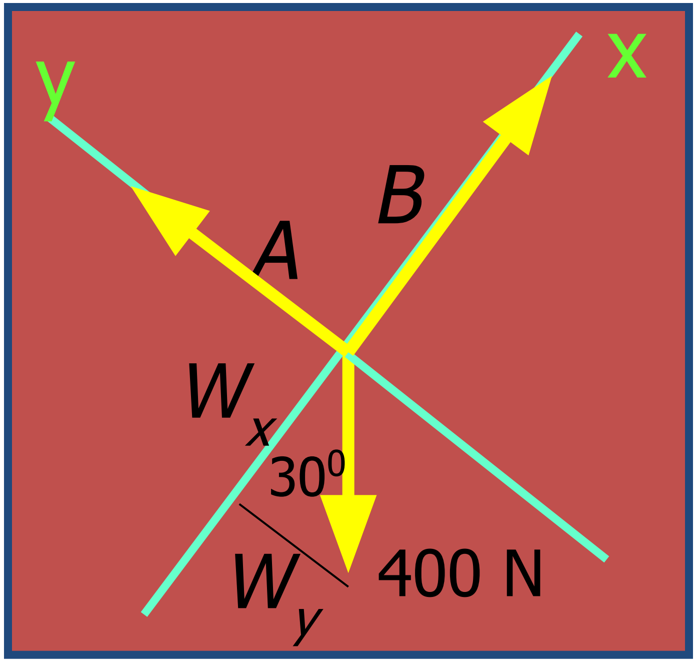
Thus, the components of the weight vector are:

$W_x = 346\text{ N};\quad W_y = 200\text{ N}$

Apply the first condition for Equilibrium, and . . .

$$B - W_x = 0 \qquad \text{and} \qquad A - W_y = 0$$

Example 4 (Cont.) We Now Solve for A and B:



$$\Sigma F_x = B - W_x = 0$$

$$B = W_x = (400\text{ N})\cos 30^0$$

$$\boxed{B = 346\text{ N}}$$

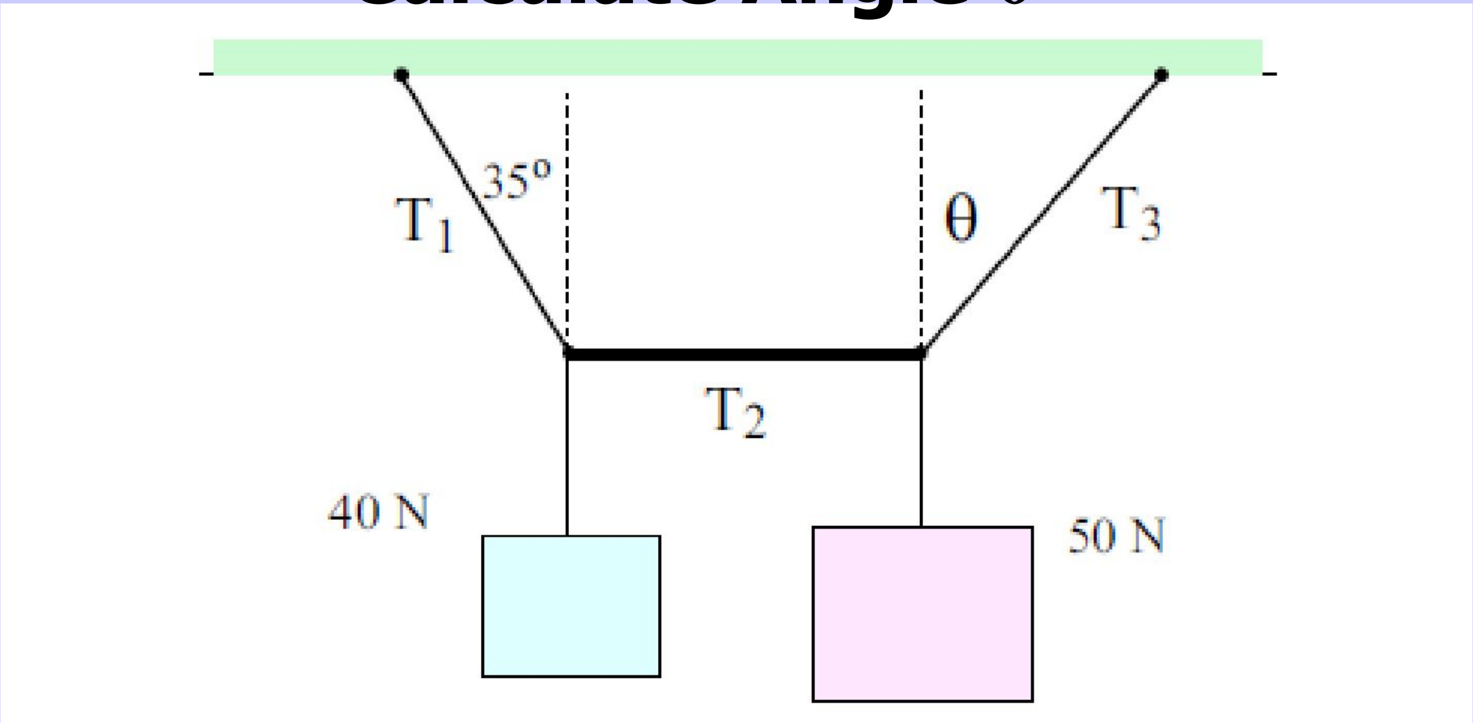
$$\Sigma F_y = A - W_y = 0$$

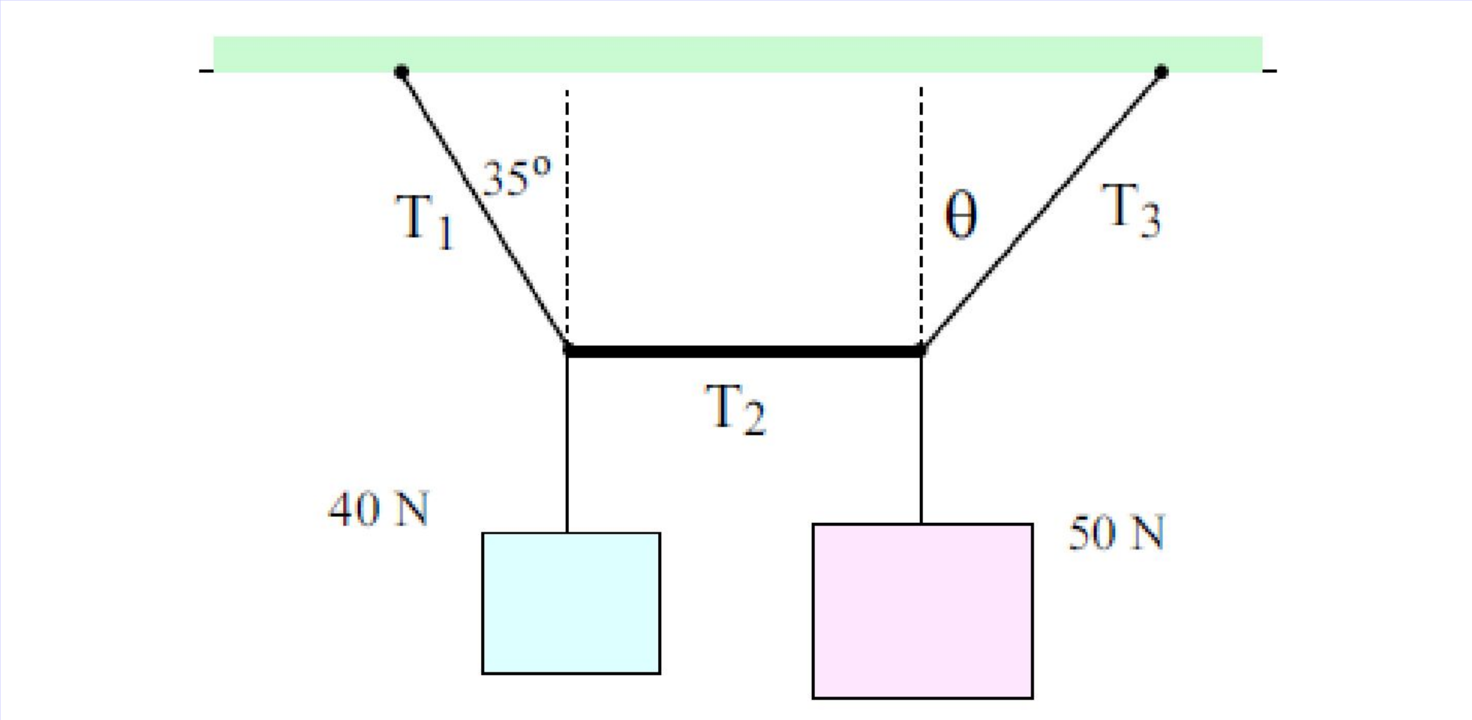
$$A = W_y = (400\text{ N})\sin 30^0$$

$$\boxed{A = 200\text{ N}}$$

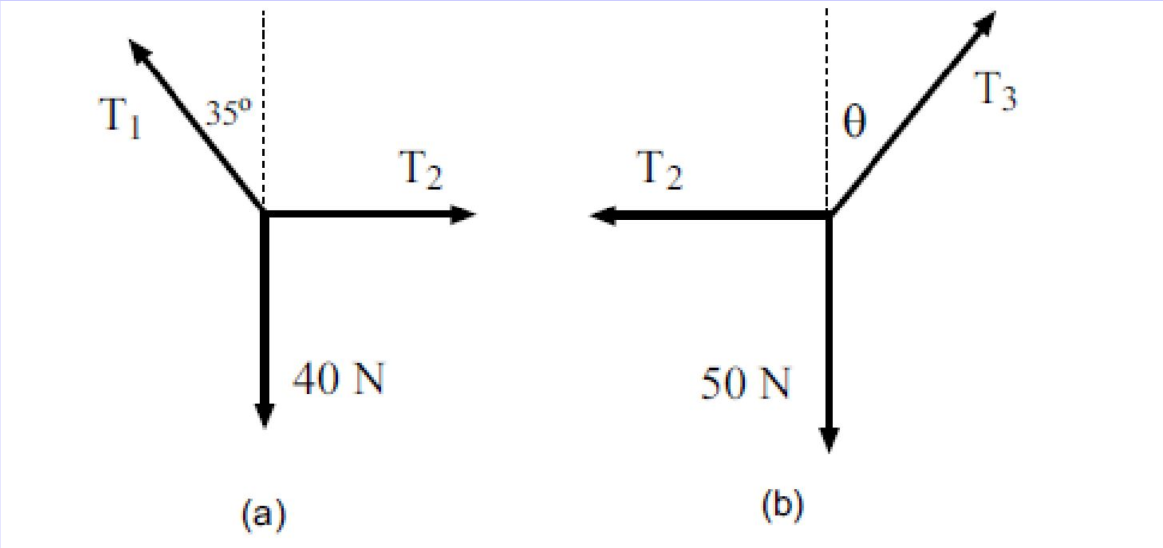
Before working a problem, you might see if rotation of the axes helps.

Calculate Angle θ



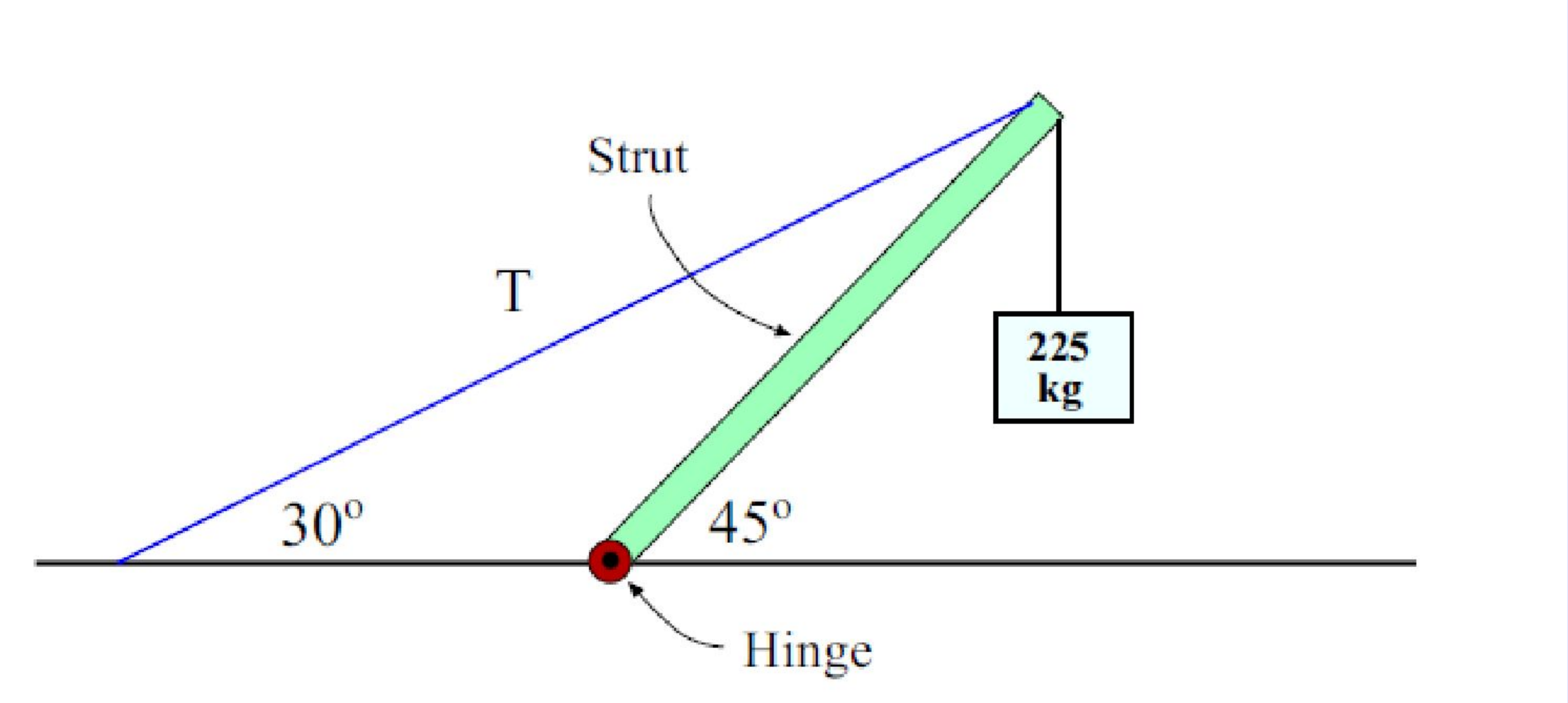


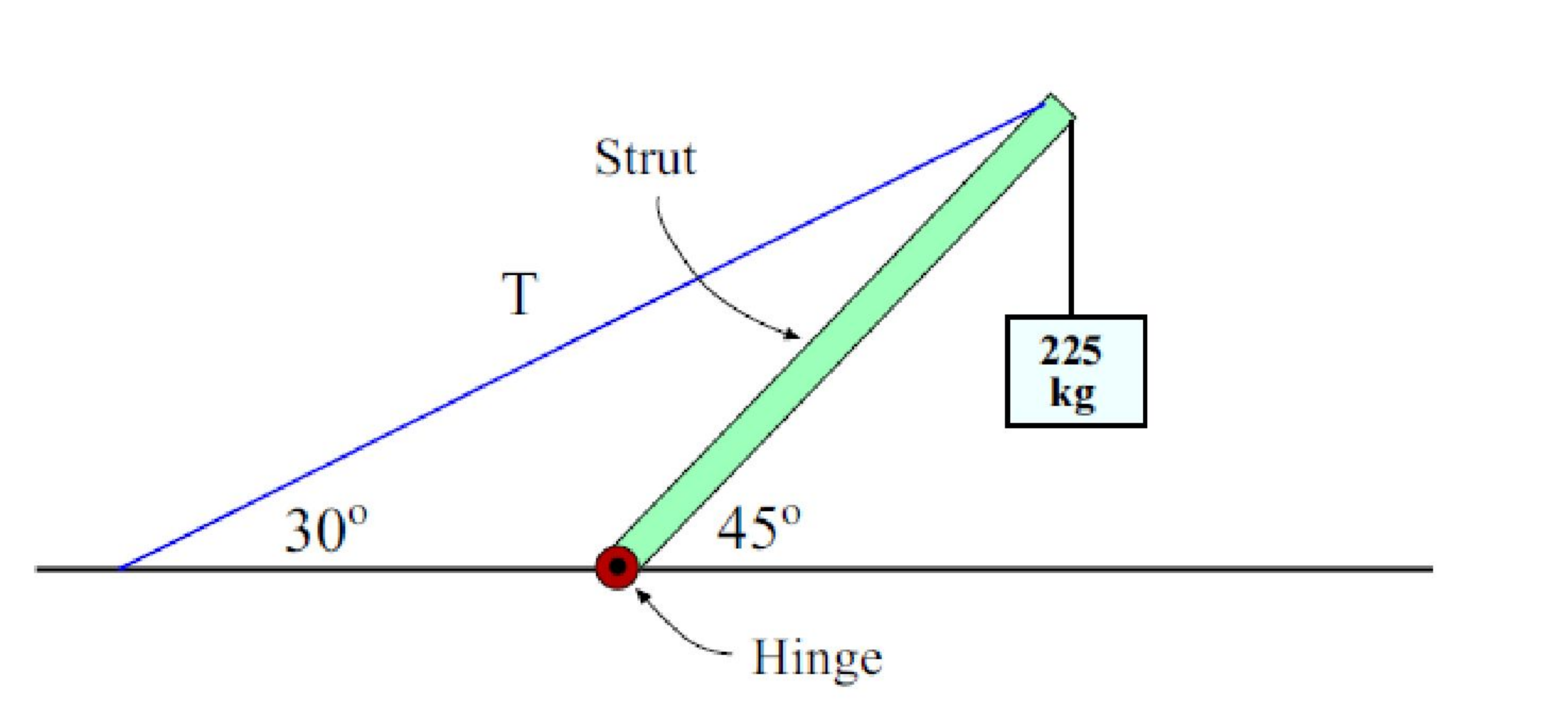
35°
T1
θ
T3
T2
40 N
50 N

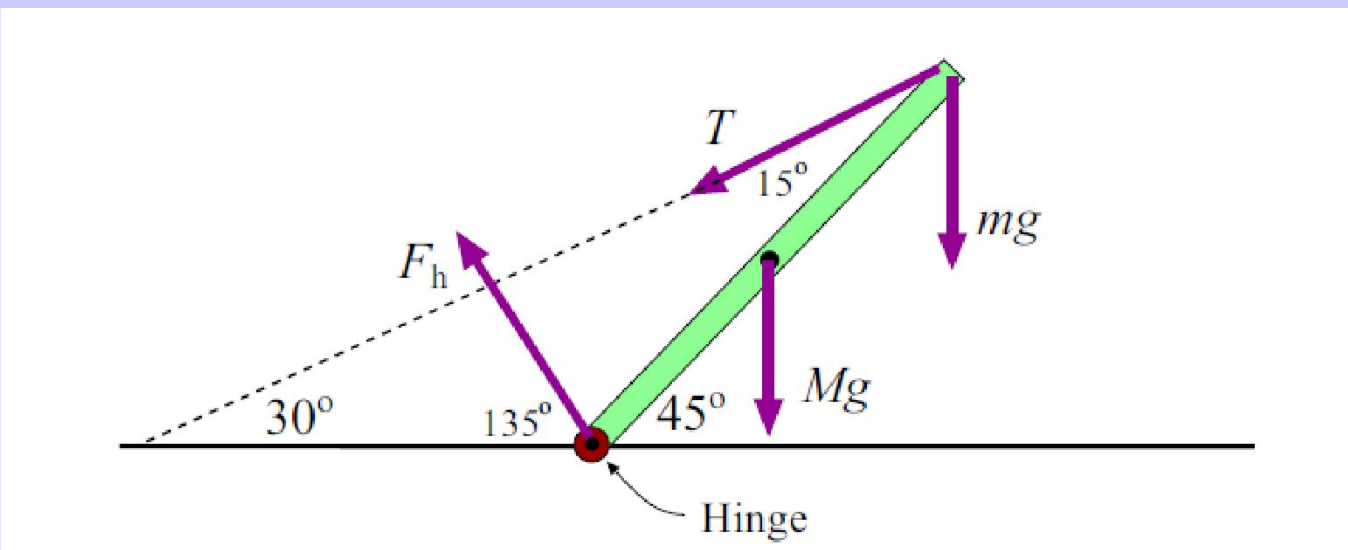


T1
35°
T2
40 N
T2
θ
T3
50 N
(a)
(b)

Calculate Reaction Force on the Hinge




Strut
T
225 kg
30°
45°
Hinge


T
15°
mg
F_h
Mg
30°
135°
45°
Hinge

Summary

- <u>Newton's First Law:</u> An object at rest or an object in motion at constant speed will remain at rest or at constant speed in the absence of a resultant force.

Summary

Second Law: Whenever a resultant force acts on an object, it produces an acceleration, an acceleration that is directly proportional to the force and inversely proportional to the mass.

Summary

- Third Law: To every action force there must be an equal and opposite reaction force.

Problem Solving Strategy

1. Draw a sketch and label all information.
2. Draw a free-body diagram.
3. Find components of all forces (+ and -).
4. Apply First Condition for Equilibrium:

$$\Sigma F_x = 0 \; ; \quad \Sigma F_y = 0$$

5. Solve for unknown forces or angles.

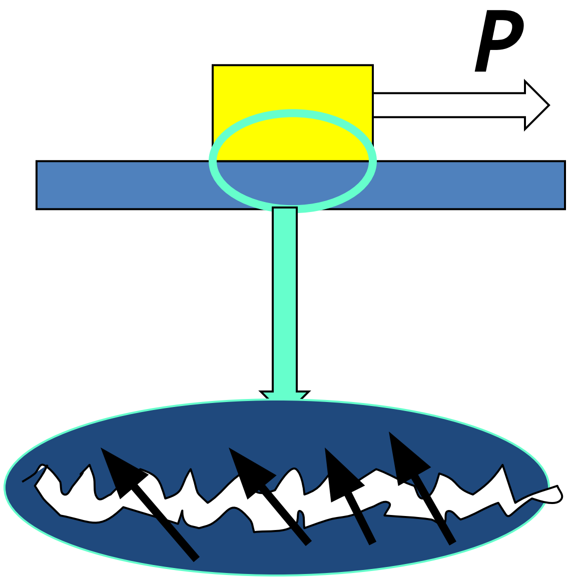
Friction and Equilibrium

Objectives

- Define and calculate the coefficients of kinetic and static friction, and give the relationship of friction to the normal force.
- Apply the concepts of static and kinetic friction to problems involving constant motion or impending motion.

Friction Forces

When two surfaces are in contact, friction forces oppose relative motion or impending motion.

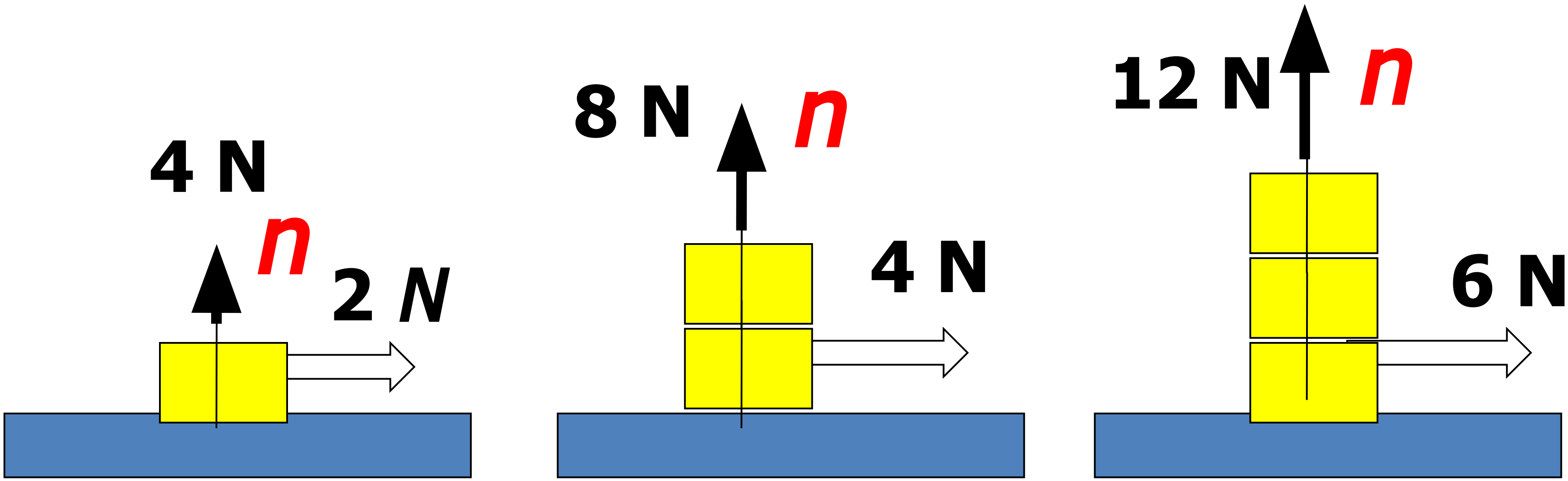


Friction forces are parallel to the surfaces in contact and oppose motion or impending motion.

***Static Friction:* No relative motion.**

***Kinetic Friction:* Relative motion.**

Friction and the Normal Force

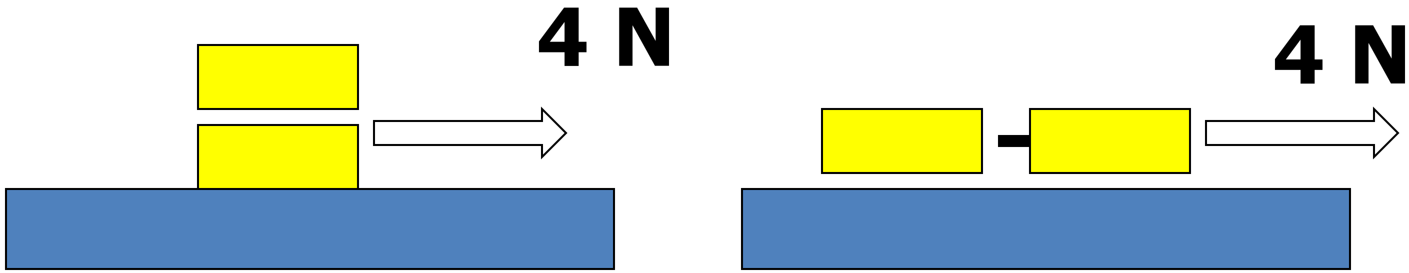


The force required to overcome static or kinetic friction is proportional to the normal force, n.

$$f_s = \mu_s n$$

$$f_k = \mu_k n$$

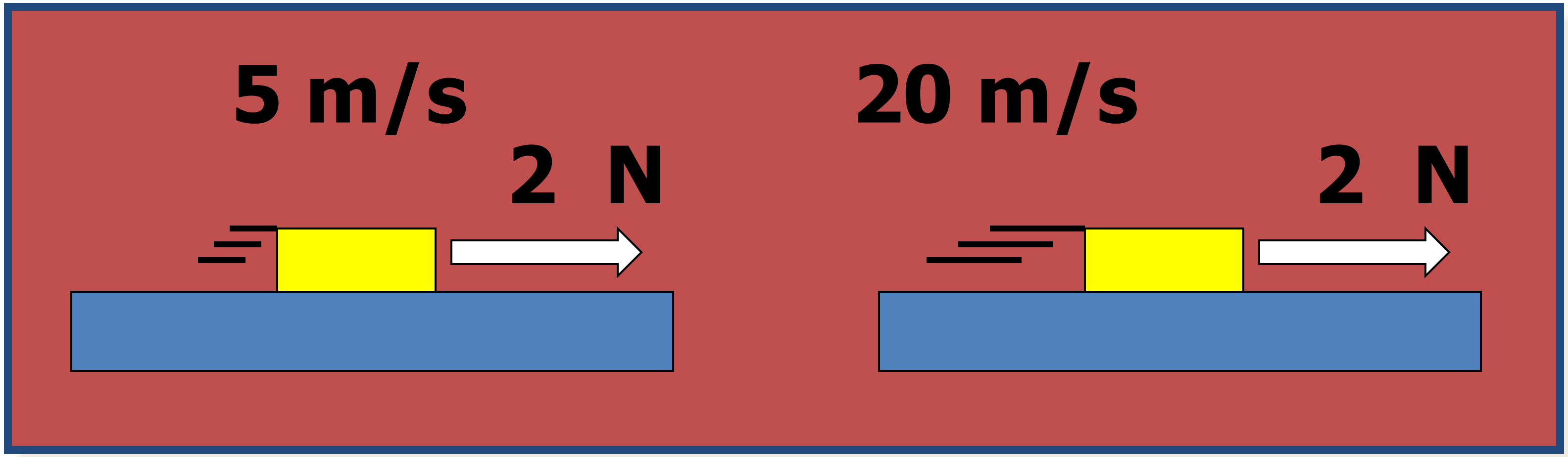
Friction forces are independent of area.



If the total mass pulled is constant, the same force (4 N) is required to overcome friction even with twice the area of contact.

For this to be true, it is essential that ALL other variables be rigidly controlled.

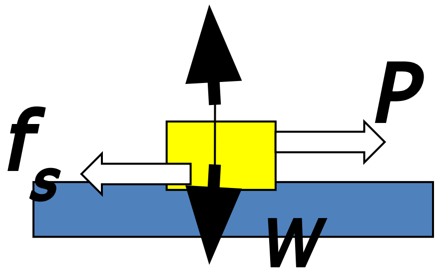
Friction forces are independent of speed.



The force of kinetic friction is the same at 5 m/s as it is for 20 m/s. Again, we must assume that there are no chemical or mechanical changes due to speed.

The Static Friction Force

When an attempt is made to move an object on a surface, static friction slowly increases to a MAXIMUM value.



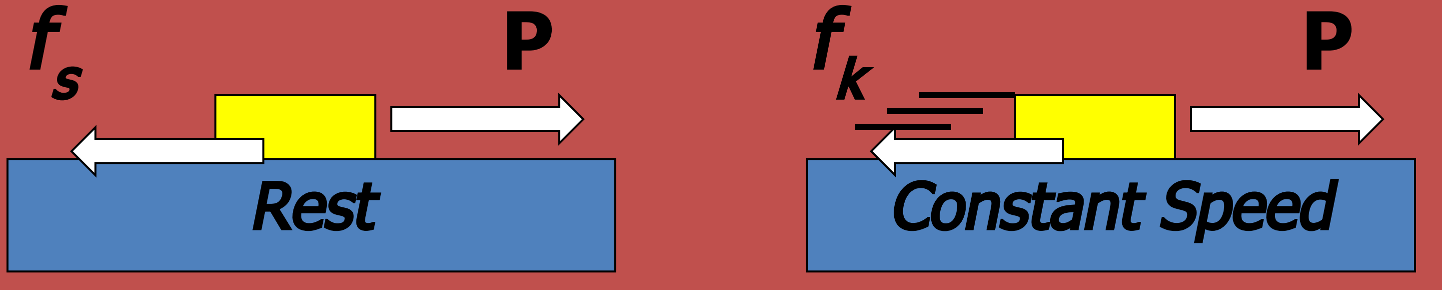
$$f_s \leq \mu_s n$$

In this module, we refer only to the maximum value of static friction and simply write:

$$f_s = \mu_s n$$

Constant or Impending Motion

For motion that is impending and for motion at constant speed, the resultant force is zero and $\Sigma F = 0$. (Equilibrium)

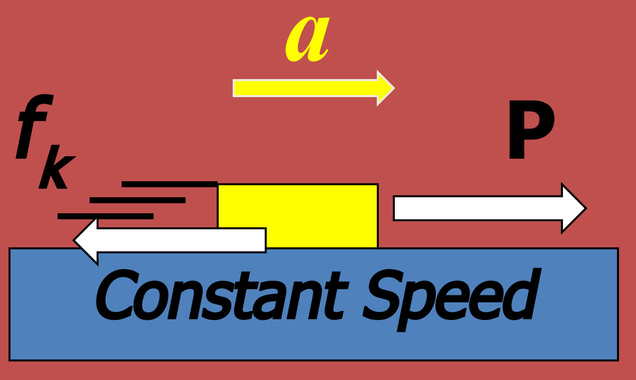


$$P - f_s = 0 \qquad P - f_k = 0$$

Here the weight and normal forces are balanced and do not affect motion.

Friction and Acceleration

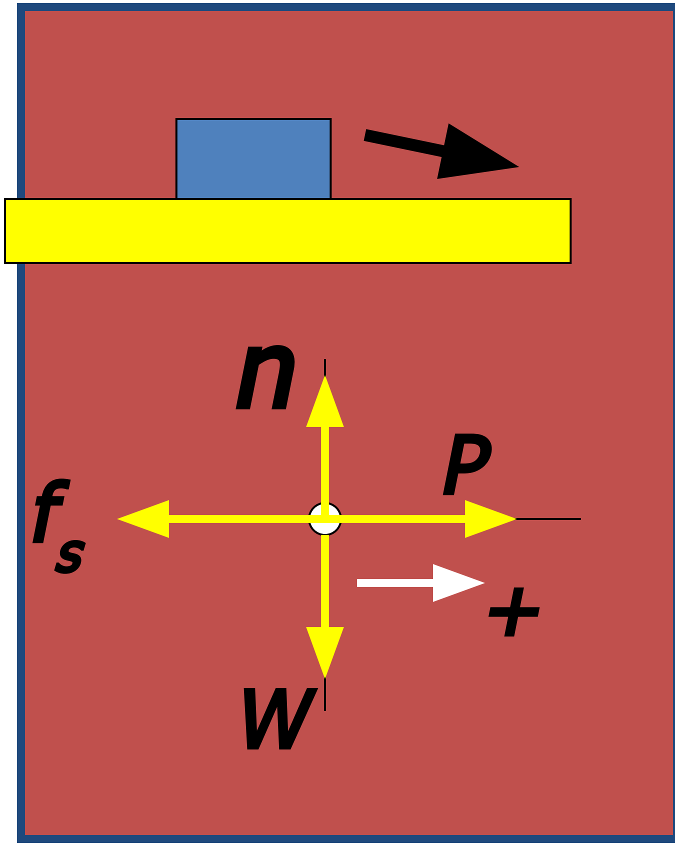
When P is greater than the maximum f_s the resultant force produces acceleration.



$$f_k = \mu_k n$$

Note that the kinetic friction force remains constant even as the velocity increases.

EXAMPLE 1: If $\mu_k = 0.3$ and $\mu_s = 0.5$, what horizontal pull P is required to just start a 250-N block moving?



1. Draw sketch and free-body diagram as shown.

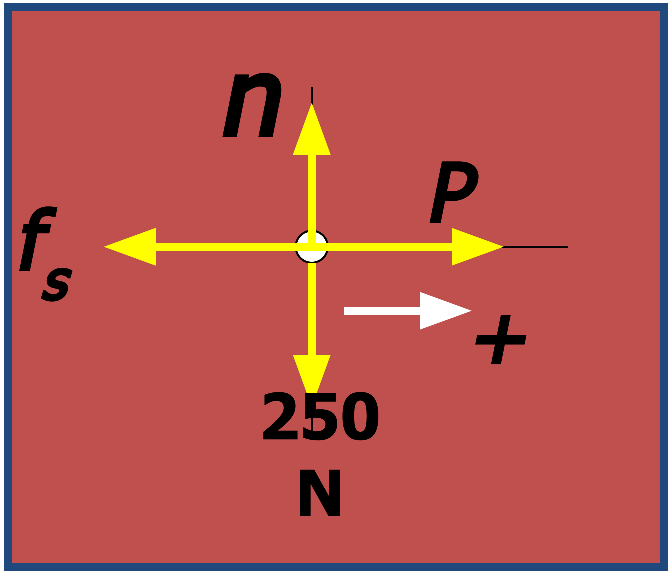
2. List givens and label what is to be found:

$\mu_k = 0.3$; $\mu_s = 0.5$; $W = 250$ N

Find: $P = ?$

3. Recognize for impending motion: $P - f_s = 0$

EXAMPLE 1(Cont.): $\mu_s = 0.5$, W = 250 N. Find *P* to overcome f_s *(max)*. Static friction applies.



For this case: $P - f_s = 0$

4. To find P we need to know f_s, which is:

$f_s = \mu_s n$

$n = ?$

5. To find n:

$\Sigma F_y = 0$

$n - W = 0$

$W = 250$ N

$n = 250$ N

EXAMPLE 1(Cont.): $\mu_s = 0.5$, $W = 250$ N. Find P to overcome f_s *(max)*. Now we know $n = 250$ N.

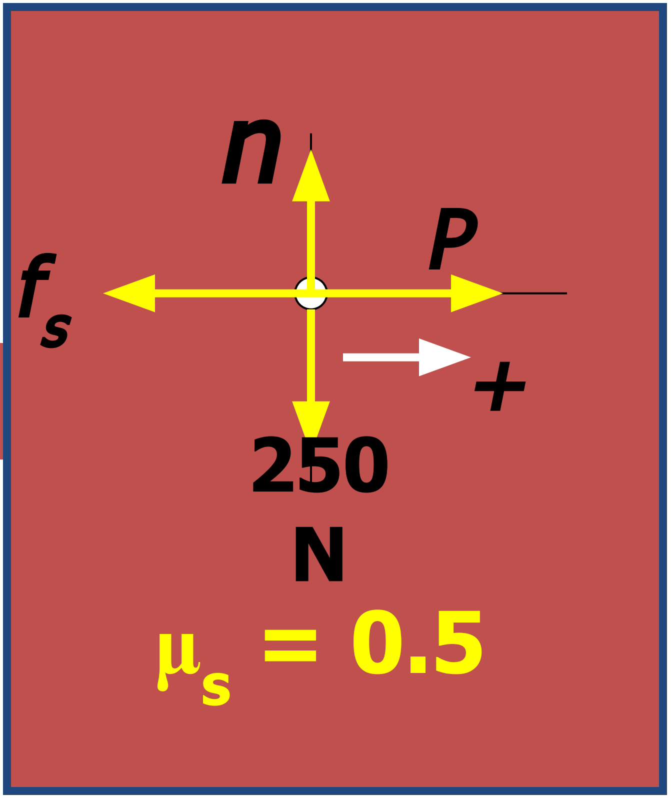
6. Next we find f_s from:

$$f_s = \mu_s n = 0.5\ (250\text{ N})$$

7. For this case: $P - f_s = 0$

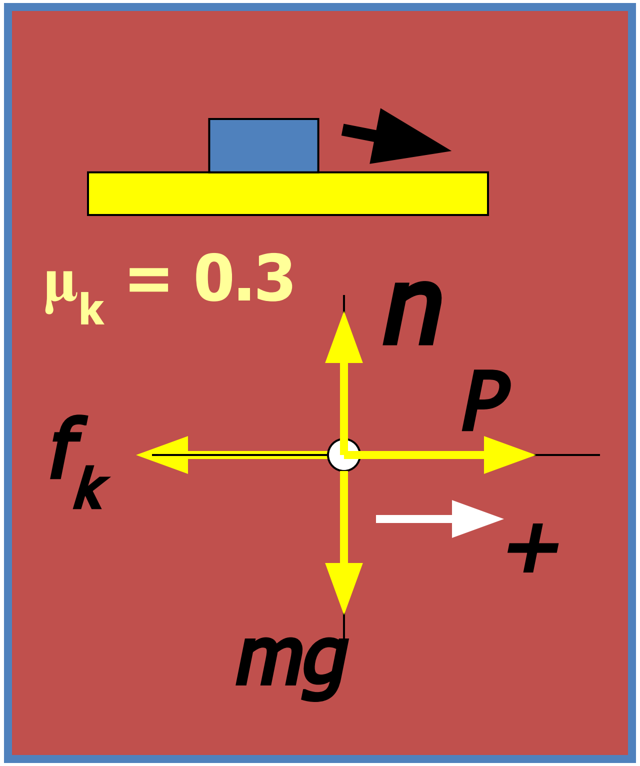
$$P = f_s = 0.5\ (250\text{ N})$$

$$\boxed{P = 125\text{ N}}$$



This force (125 N) is needed to just start motion. Next we consider P needed for constant speed.

EXAMPLE 1(Cont.): If $\mu_k = 0.3$ and $\mu_s = 0.5$, what horizontal pull P is required to move with constant speed? (Overcoming kinetic friction)



$$\Sigma F_y = ma_y = 0$$

$$n - W = 0 \qquad n = W$$

Now: $f_k = \mu_k n = \mu_k W$

$$\Sigma F_x = 0; \quad P - f_k = 0$$

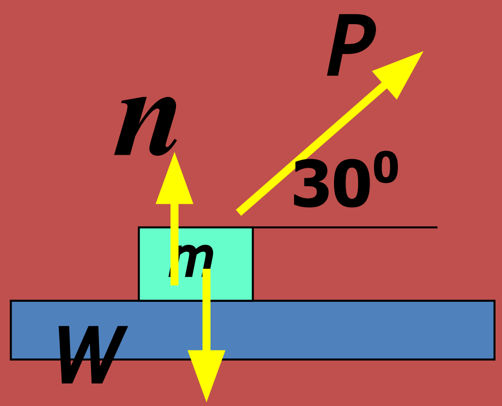
$$P = f_k = \mu_k W$$

$$P = (0.3)(250\text{ N})$$

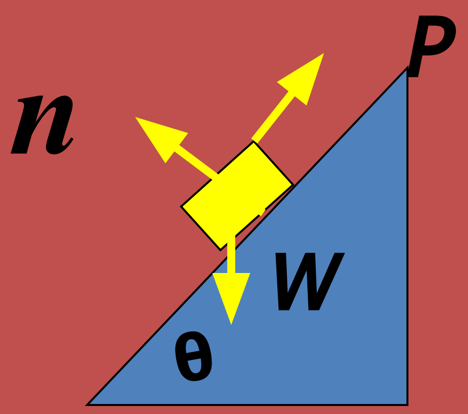
$$\boxed{P = 75.0\text{ N}}$$

The Normal Force and Weight

The normal force is NOT always equal to the weight. The following are examples:



Here the normal force is less than weight due to upward component of P.



Here the normal force is equal to only the component of weight perpendicular to the plane.

For Friction in Equilibrium:

- Draw free-body diagram for each body.
- Choose x or y-axis along motion or impending motion and choose direction of motion as positive.
- Identify the normal force and write one of following:

$$f_s = \mu_s n \quad or \quad f_k = \mu_k n$$

- For equilibrium, we write for each axis:

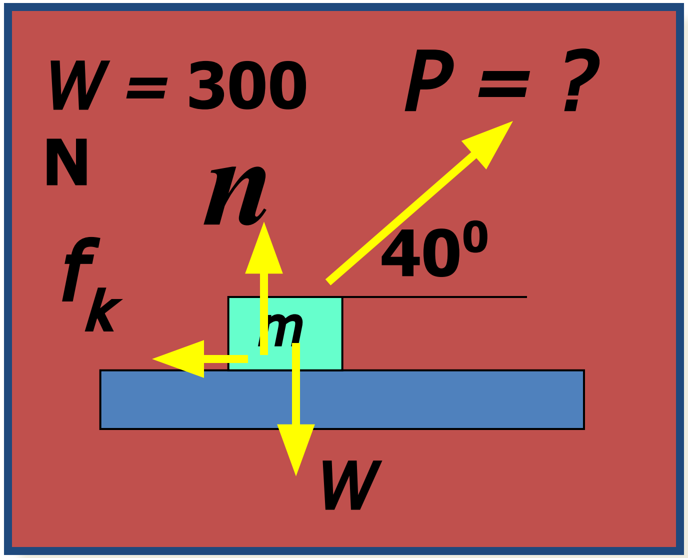
$$\Sigma F_x = 0 \qquad \Sigma F_y = 0$$

- Solve for unknown quantities.

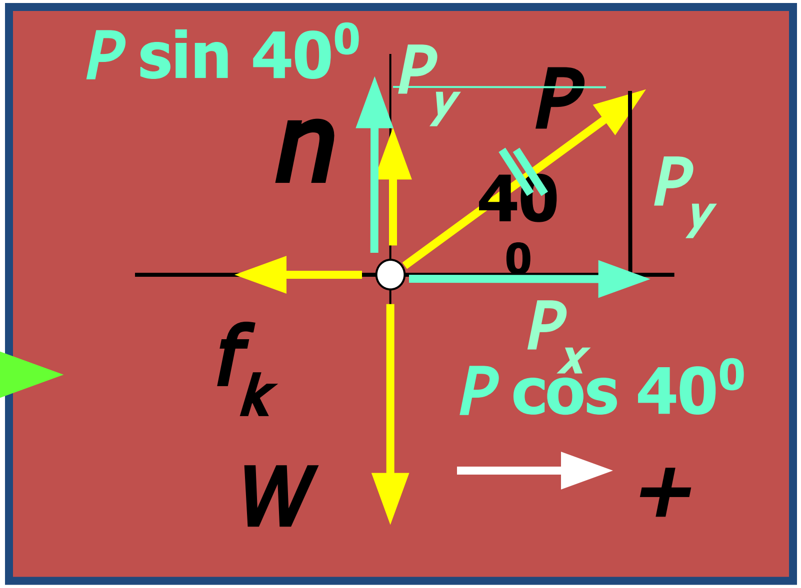
Example 2. A force of 60 N drags a 300-N block by a rope at an angle of 40^0 above the horizontal surface. If $u_k = 0.2$, what force P will produce constant speed?

1. Draw and label a sketch of the problem.

2. Draw free-body diagram.



The force P is to be replaced by its components P_x and P_y.

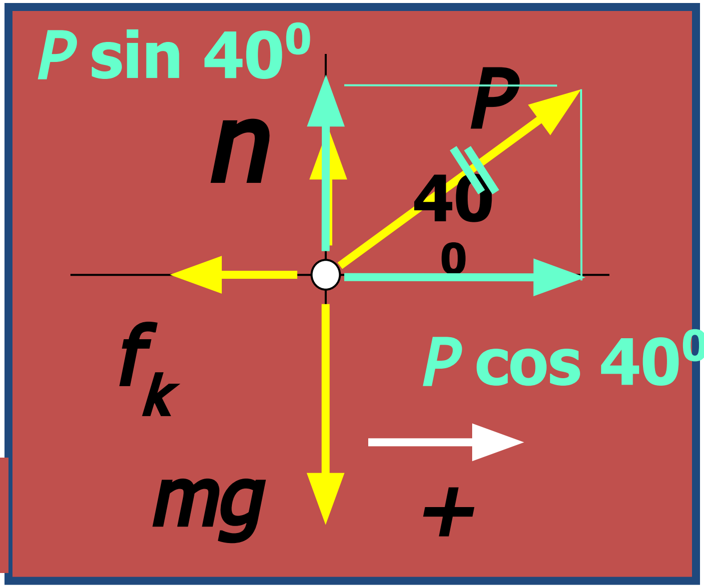


Example 2 (Cont.). P = ?; W = 300 N; u_k = 0.2.

3. Find components of P:

$P_x = P \cos 40^0 = 0.766P$

$P_y = P \sin 40^0 = 0.643P$



$P_x = 0.766P$; $P_y = 0.643P$

Note: Vertical forces are balanced, and for constant speed, horizontal forces are balanced.

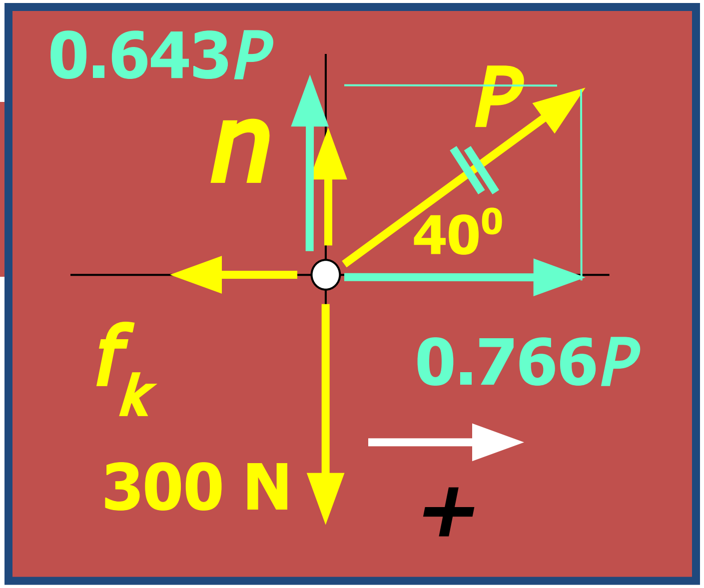
$$\boxed{\sum F_x = 0}$$

$$\boxed{\sum F_y = 0}$$

Example 2 (Cont.). P = ?; W = 300 N; u_k = 0.2.

$P_x = 0.766P \quad P_y = 0.643P$

4. Apply Equilibrium con- ditions to vertical axis.



$$\Sigma F_y = 0$$

$$n + 0.643P - 300\text{ N} = 0$$

[P_y and n are up (+)]

$$n = 300\text{ N} - 0.643P;$$

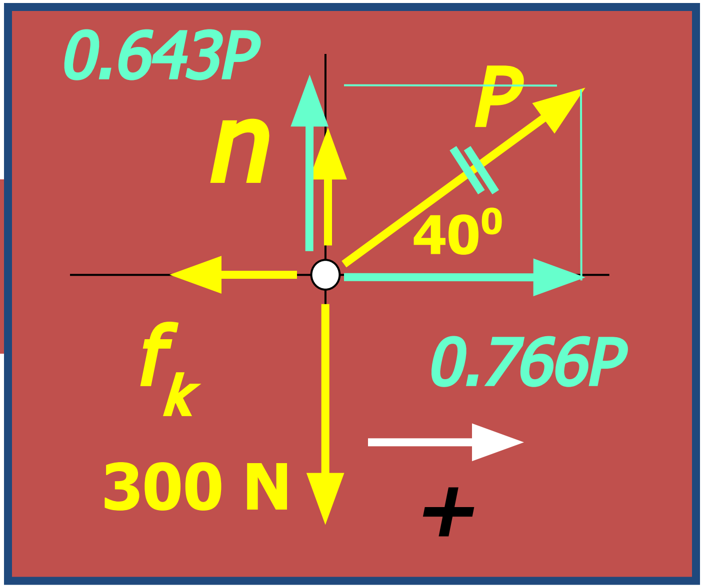
Solve for n in terms of P

$$\boxed{n = 300\text{ N} - 0.643P}$$

Example 2 (Cont.). P = ?; W = 300 N; u_k = 0.2.

$$n = 300\text{ N} - 0.643P$$

5. Apply $\Sigma F_x = 0$ **to con- stant horizontal motion.**



$$\Sigma F_x = 0.766P - f_k = 0$$

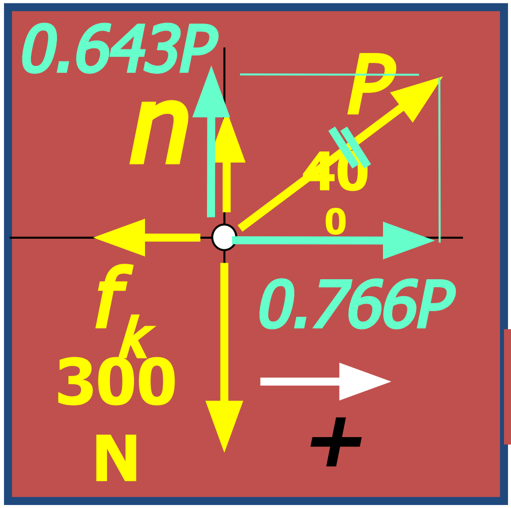
$$f_k = \mu_k n = (0.2)(300\text{ N} - 0.643P)$$

$$f_k = (0.2)(300\text{ N} - 0.643P) = 60\text{ N} - 0.129P$$

$$0.766P - f_k = 0;$$

$$0.766P - (60\text{ N} - 0.129P) = 0$$

Example 2 (Cont.). $P = ?$; $W = 300$ N; $u_k = 0.2$.



$$0.766P - (60\text{ N} - 0.129P) = 0$$

6. Solve for unknown P.

$$0.766P - 60\text{ N} + 0.129P = 0$$

$$0.766P + 0.129P = 60\text{ N}$$

$$0.766P + 0.129P = 60\text{ N}$$

$$0.895P = 60\text{ N}$$

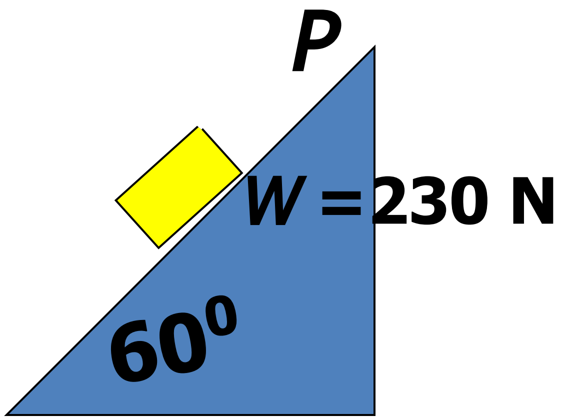
$$P = 67.0\text{ N}$$

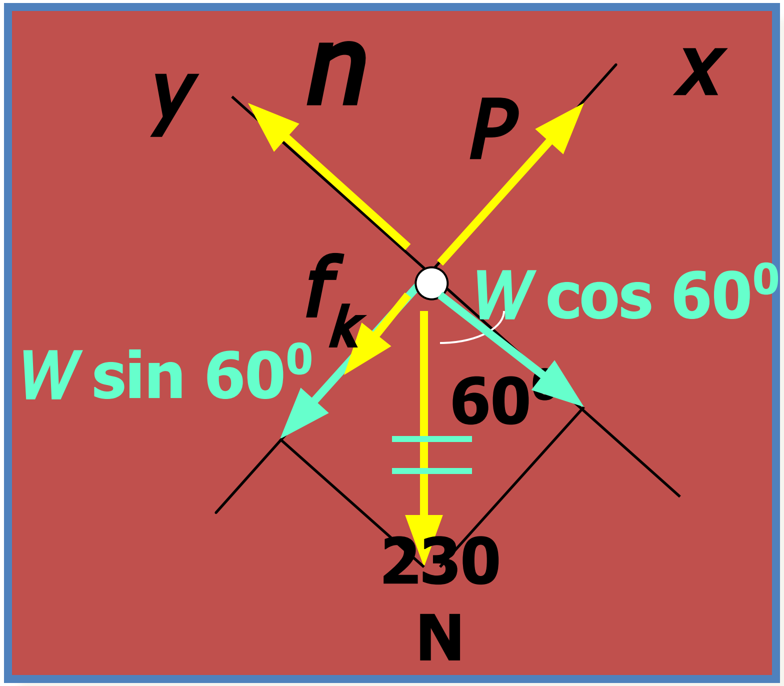
If $P = 67$ N, the block will be dragged at a constant speed.

$$P = 67.0\text{ N}$$

Example 3: What push P up the incline is needed to move a 230-N block up the incline at constant speed if $\mu_k = 0.3$?

Step 1: **Draw free-body including forces, angles and components.**





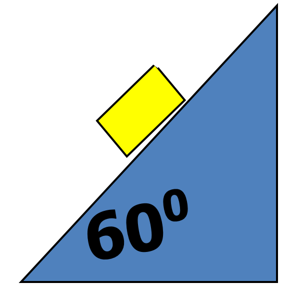
Step 2: $\Sigma F_y = 0$

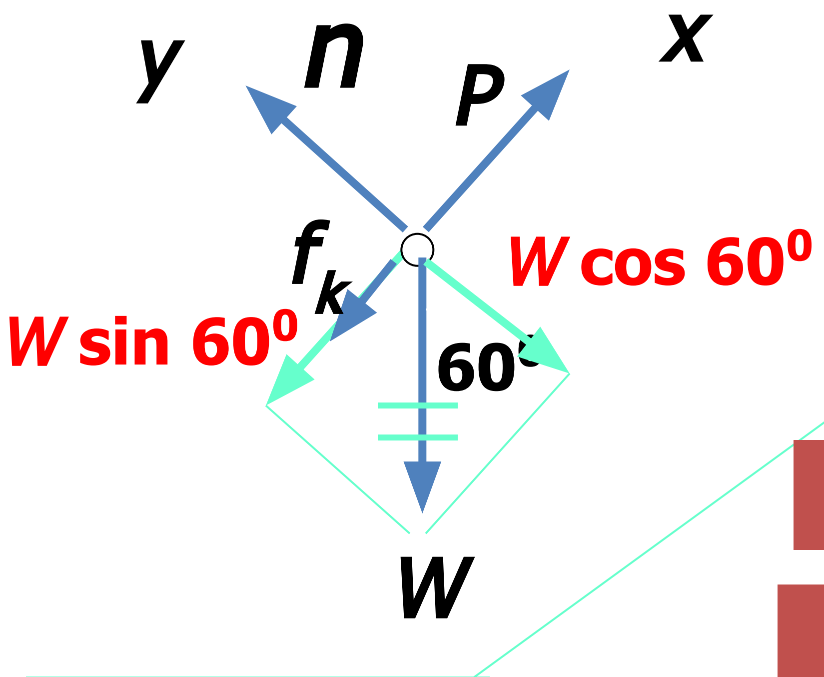
$$n - W \cos 60^0 = 0$$

$$n = (230\ \text{N}) \cos 60^0$$

$$\boxed{n = 115\ \text{N}}$$

Example 3 (Cont.): Find P to give move up the incline (W = 230 N).





$n = 115$ N

$W = 230$ N

Step 3. Apply $\Sigma F_x = 0$

$$P - f_k - W \sin 60^0 = 0$$

$$f_k = \mu_k n = 0.2(115 \text{ N})$$

$$f_k = 23 \text{ N}, \quad P = \ ?$$

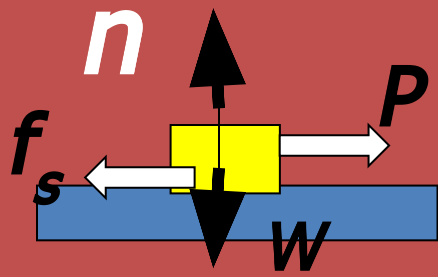
$$P - 23 \text{ N} - (230 \text{ N})\sin 60^0 = 0$$

$$P - 23 \text{ N} - 199 \text{ N} = 0$$

$$\boxed{P = 222 \text{ N}}$$

Summary

- **The maximum force of static friction is the force required to just start motion.**



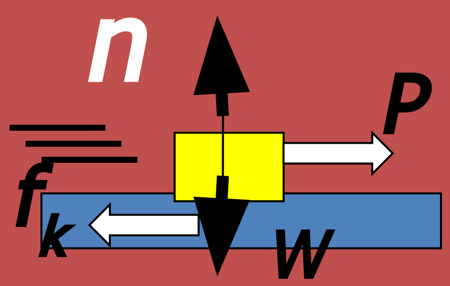
$$f_s \le \mu_s n$$

Equilibrium exists at that instant:

$$\Sigma F_x = 0; \quad \Sigma F_y = 0$$

Summary: Important Points (Cont.)

- **The force of kinetic friction is that force required to maintain constant motion.**



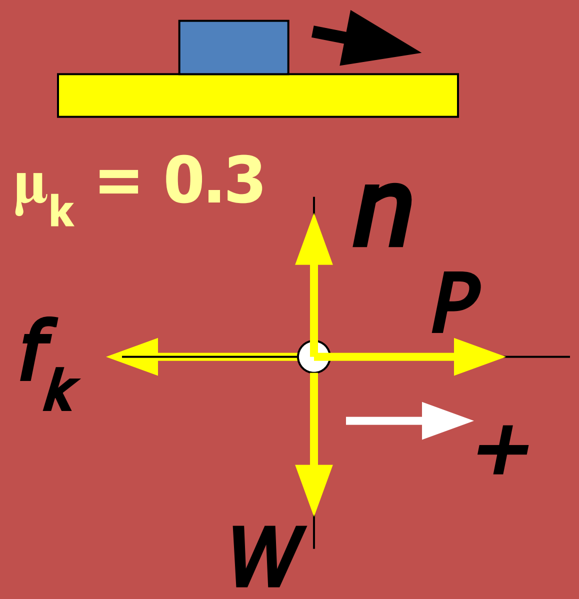
$$f_k = \mu_k n$$

- **Equilibrium exists if speed is constant, but f_k does not get larger as the speed is increased.**

$$\Sigma F_x = 0; \quad \Sigma F_y = 0$$

Summary: Important Points (Cont.)

- **Choose an *x* or *y*-axis along the direction of motion or impending motion.**



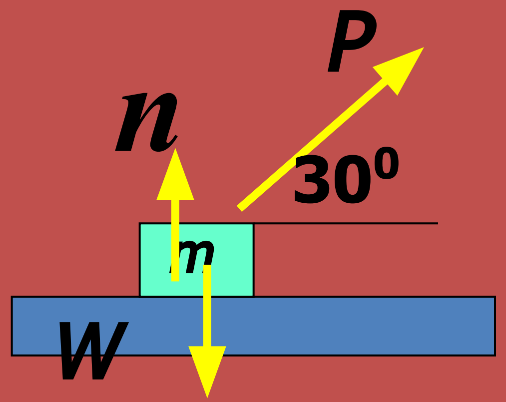
The ΣF will be zero along the x-axis and along the y-axis.

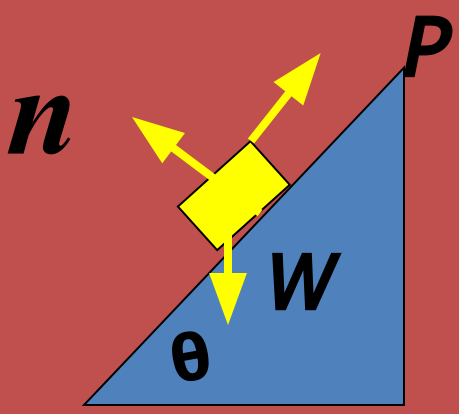
In this figure, we have:

$$\boxed{\Sigma F_x = 0; \quad \Sigma F_y = 0}$$

Summary

- the normal force *n* is **not** always equal to the weight of an object.





It is necessary to draw the free-body diagram and sum forces to solve for the correct *n* value.

$$\boxed{\Sigma F_x = 0; \quad \Sigma F_y = 0}$$

Summary

Static Friction: ***No relative motion.***

$$f_s \le \mu_s n$$

Kinetic Friction: ***Relative motion.***

$$f_k = \mu_k n$$

Procedure for solution of equilibrium problems is the same for each case:

$$\Sigma F_x = 0 \quad \Sigma F_y = 0$$