Engineering Mechanics

Part II: Dynamics

Dr. Bahaa Saleh

Course Topics

- **Chapter 1**: **Introduction to dynamics**
- **Chapter 2**: **Kinematics of a Particle:**

Topic # 1: **Particle motion along a straight line**

Topic # 2: **Particle motion along a curved path**

Topic # 3: **Dependent motion of connected particles**

Topic # 4: **Relative motion of two particles**

- **Chapter 3**: **Kinetics of a Particle:**

Topic # 1: **Force and Acceleration**

Topic # 2: **Work and energy**

Topic # 3: **Impulse and momentum**

Course Topics – Cont.

- **Chapter 4: Planer Kinematics of a Rigid Body.**
- **Chapter 5: Planar Kinetics of a Rigid Body: Force and Acceleration.**
- **Chapter 6: Introduction to Mechanical Vibration.**

Chapter 3:

Kinetics of a Particle

Topic # 1:

Force and Acceleration

First Law

Law of inertia - a body in motion will stay in motion and a body at rest will stay at rest unless acted upon by a net external force.

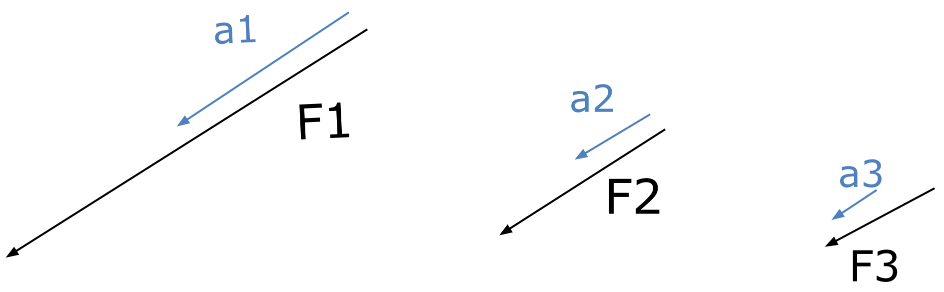
- A particle originally at rest, or moving in a straight line with a constant velocity, will remain in this state provided the particle is not subjected to an unbalanced force.
- Static law

Second Law

- A particle acted upon by an unbalanced force F experiences an acceleration that has the same direction as the force and a magnitude that is directly proportional to the force

$$F \alpha \; a$$

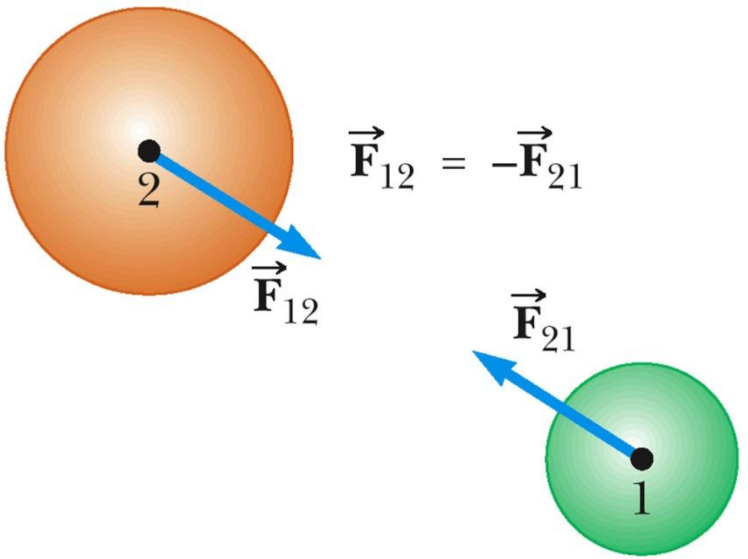
$$F = m\,a$$



Third Law

- **<u>Law of Action-Reaction</u>** - For every action, there is an equal and opposite reaction.

- The mutual forces of action and reaction between two particles are **equal**, **opposite**, and **collinear**.



$$\vec{F}_{12} = -\vec{F}_{21}$$

Summary of Newton's laws

1- Law of inertia a body in motion will stay in motion and a body at rest will stay at rest unless acted upon by a net external force.

2- Law of force-acceleration A particle acted upon by an unbalanced force F experiences an acceleration a that has the same direction as the force and a magnitude that is directly proportional to the force

$$F = m\,a$$

3- Law of action-reaction for every action, there is an equal and opposite reaction $mg = F_N$

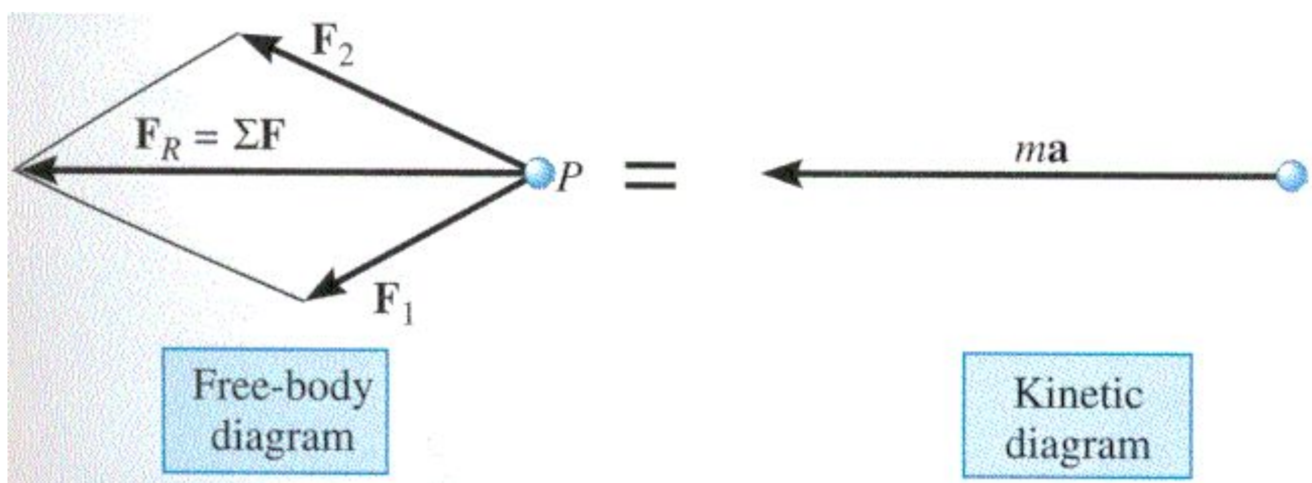
The Equation of Motion

Free Body and Kinetic Diagram

When more than one force acts on a particle, the resultant force is determined by a vector summation of all the forces. The equation of motion may be written as

$$F_R = \sum \underline{F} = m\,\underline{a}$$

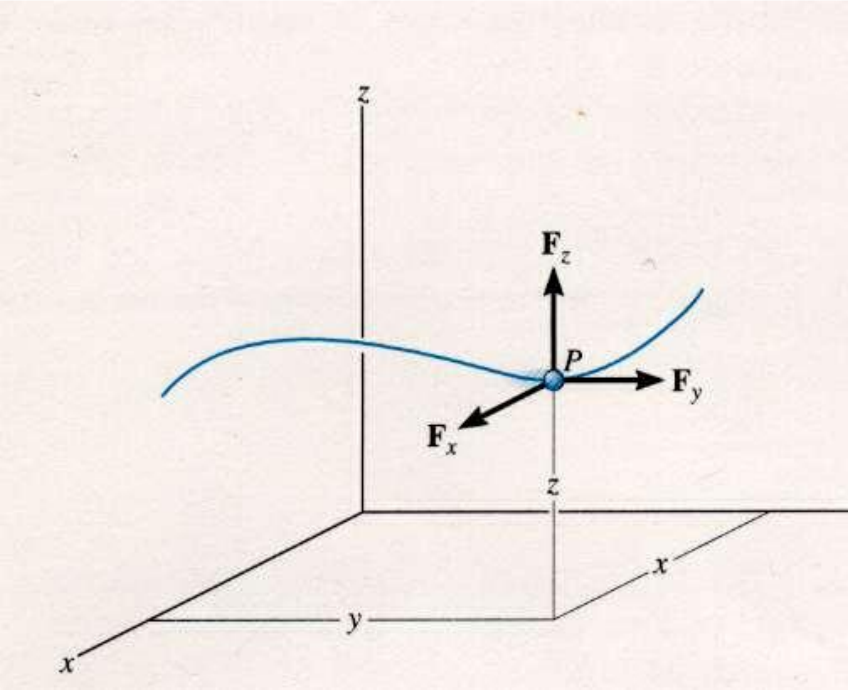
- Free-Body diagram (Force Diagram)
- Kinetic diagram (acceleration Diagram)



Equations of Motion: Rectangular Coordinates

- **When the net force is projected to separate coordinate axes the Newton's second law still holds**

$$\sum \underline{\mathrm{F}} = m\underline{\mathrm{a}}$$



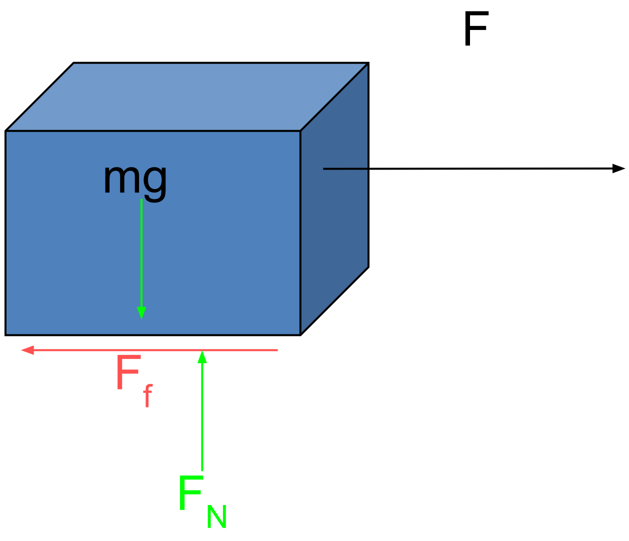
$$\sum F_x = ma_x$$
$$\sum F_y = ma_y$$
$$\sum F_z = ma_z$$

Free Body Diagram Method

- Draw each object separately
- Draw all the forces acting on that object
- Get *x* and *y* components of all the forces to calculate the net force
- Apply Newton's second law to get acceleration
- Use the acceleration in any motion analysis and establish a Kinetic Diagram

Normal & Frictional Force

$mg = F_N$

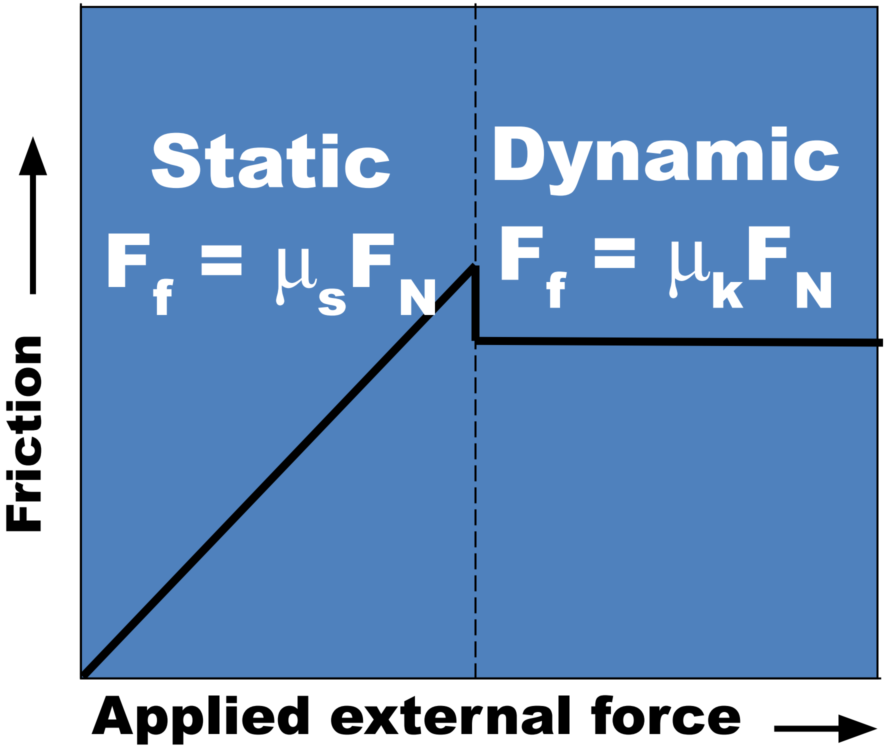


Static Friction (μ_s)

- Static friction – parallel force on the surface when there is no relative motion between the 2 objects
- Static friction force can vary from zero to Maximum

The coefficient of static friction is material dependent.

$$F_{\mathrm{f}} = \mu_s F_N$$

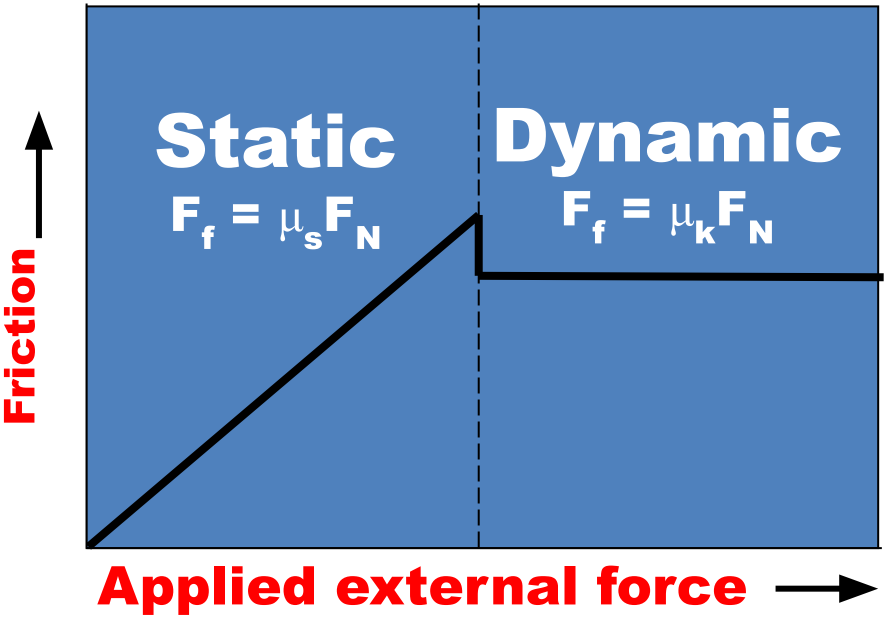


Kinetic Friction (μ_k)

- **Kinetic friction – parallel force on the surface when there is relative motion between the 2 objects**
- **Kinetic friction force is constant**

$$F_{\mathrm{f}} = \mu_k F_N$$

- **The coefficient of Kinetic friction**
- **is material dependent.**

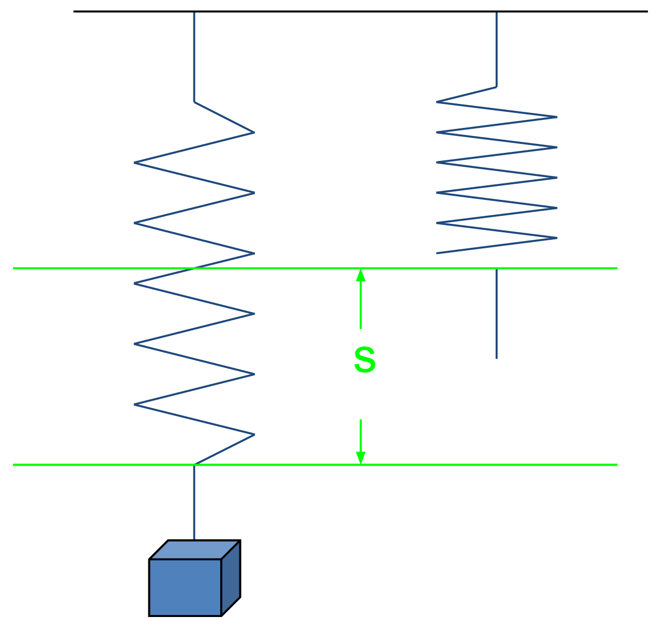


Spring Force

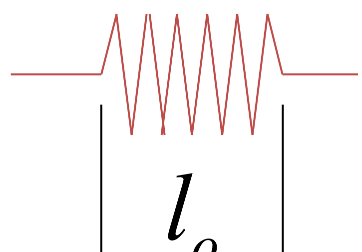
- Spring force

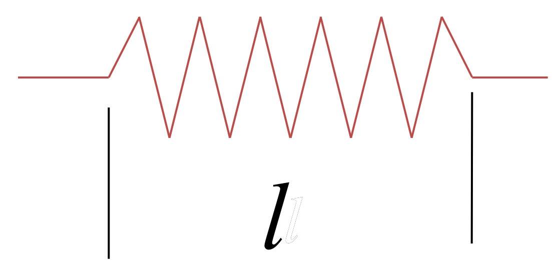
$$F_s = ks$$

- k : spring stiffness (N/m)
- s : stretched or compressed length

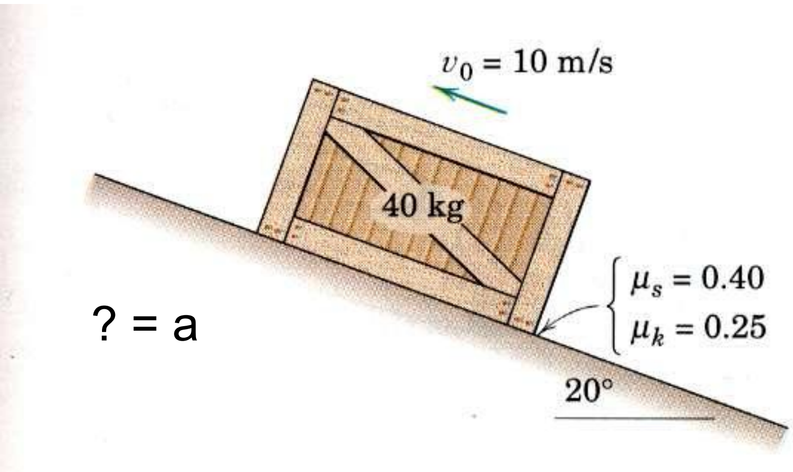


$$s = l - l_o$$

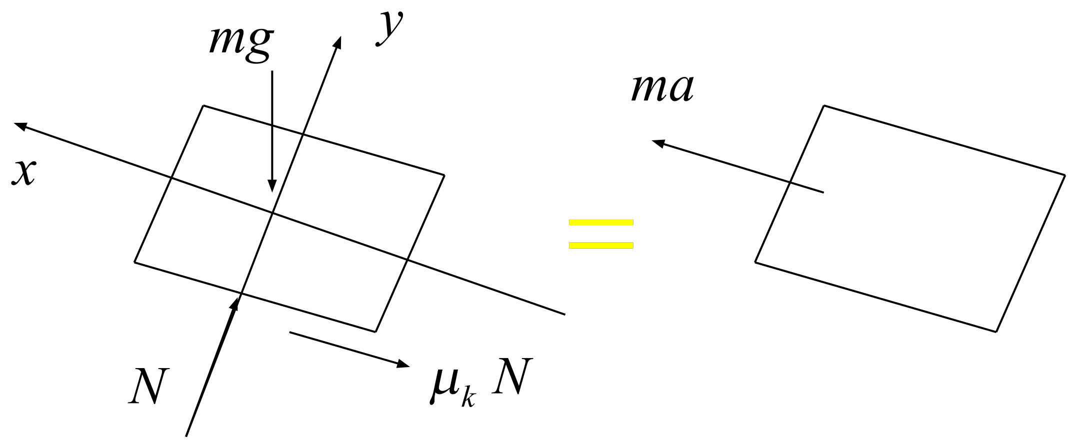




Problem



? = a



$$N - mg\cos\theta = 0$$

$$\sum F_y = 0;$$

$$N - 40 \cdot 9.81 \cdot \cos 20° = 0$$

$$N = 369 N$$

$$\sum F_x = ma_x$$

$$-\mu_k N - mg\sin\theta = ma$$

$$-0.25(369) - 40 \cdot 9.81 \cdot \sin 20° = 40 a_x$$

$$\therefore a_x = -5.66\ m/s^2$$

Chapter 3:
Kinetics of a Particle

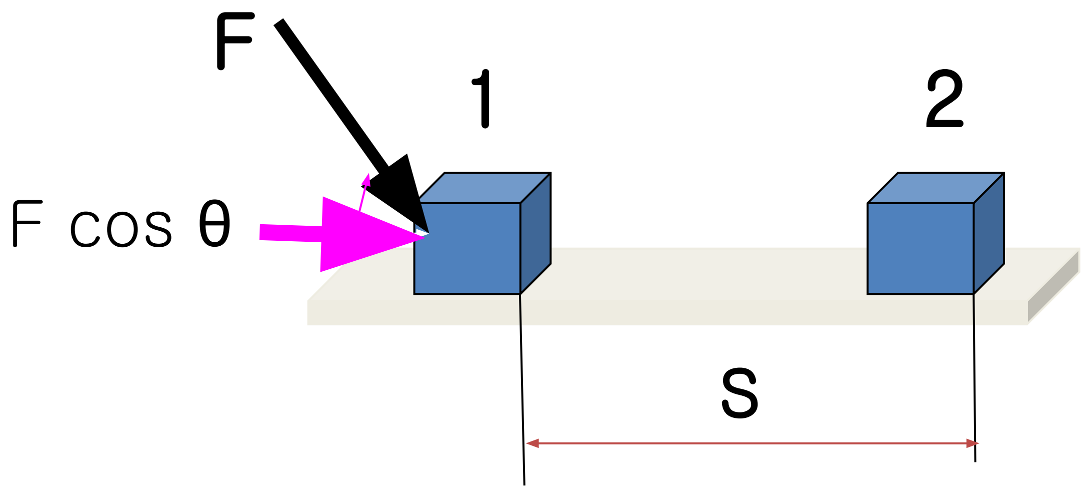
Topic # 2:
Work and Energy

Work of external Force $W_{1\rightarrow 2}$

- A force does work when it moves through a displacement in the direction of the force.
- Work is positive when the force component is in the same direction as its displacement, otherwise it is negative
- Forces that are functions of displacement must be integrated to obtain the work. Graphically, the work is equal to the area under the force-displacement curve.

Work of external Force $W_{1\to 2}$

Work = Force F, (N) * displacement S, (m) = Joule



$$W_{1\to 2} = +S * F\cos\theta$$

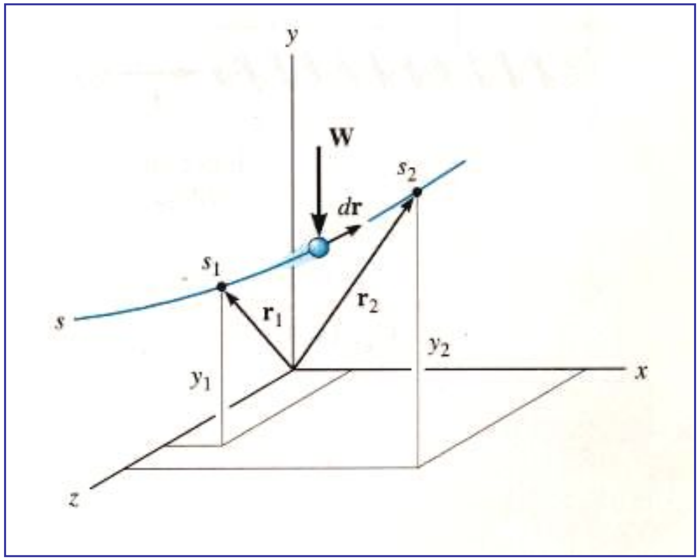
Work of a Weight

- The work of a weight is the product of the weight magnitude (W) and the vertical height from reference plane (y).

$$U_W = \pm W\ y$$

- The work is positive when the weight moves downwards (the reference plane under the body).
- The work is negative when the weight moves upwards (the reference plane over the body).
- The work is zero when the reference plane pass with the body).

Work of a Weight



$$U_{1-2} = -W(\pm\Delta y)$$

$\therefore$ magnitude of the particle's weight times its vertical displacement.

Thus, the work is independent of the path and is equal to the magnitude of the particle's weight times its vertical displacement. In the case shown in Fig. 14–4 the work is *negative*, since W is downward and Δy is upward. Note, however, that if the particle is displaced *downward* $(-\Delta y)$, the work of the weight is *positive*. Why?

Work of a Spring Force

The work of a spring is of the form

$$U_s = \pm \frac{1}{2} k s^2$$

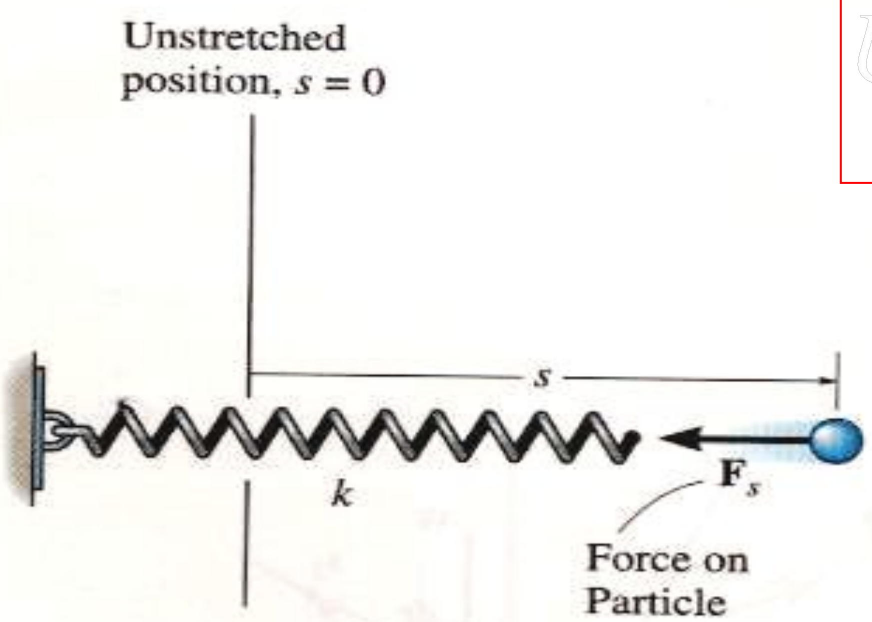
where k is the spring stiffness (N/m)

S is the stretch or compression of the spring, m

Work of a Spring Force

A mistake in sign can be avoided when applying this equation if one simply notes the direction of the spring force acting on the particle and compares it with the sense of direction of displacement of the particle if both are in the same sense, positive work results; if they are opposite to one another, the work is negative.

$$\boxed{U_s = -\left(\frac{1}{2}ks_2^{\,2} - \frac{1}{2}ks_1^{\,2}\right)}$$



$$\therefore\ U_s = \pm\frac{1}{2}k\,\Delta s^2$$

The kinetic energy

The kinetic energy (T), Joule at the initial and final points is always positive, since it involves the speed squared

$$T = \frac{1}{2} m V^2$$

Where m is the particle mass, kg

V is the particle speed in m/s

Principle of Work and Energy

$$\boxed{W_{1\to 2} = T_2 - T_1}$$

$$where\, W_{1\to 2} \;\; is\, the\, external\;\; force\, work$$

$$T_1 and\, T_2 \; are\, the\, initial\; and\;\; final\, kinetic\, energy$$

Principle of Work and Energy

$$\boxed{W_{1\to 2} + T_1 + U_1 = T_2 + U_2}$$

$$where\, W_{1\to 2} \;\; is\, the\, external \;\; force\, work$$

$$T_1 and\, T_2 \; are\, the\, initial \; and \;\; final\, kinetic\, energy$$

$$U = U_W + U_S$$

$$U_W \; is\, the\, weigth\, work \qquad U_S \; is\, the\, spring\, work$$

Procedure for Analysis

Work (Free-Body Diagram).

- Establish the inertial coordinate system and draw a free-body diagram of the particle in order to account for all the forces that

A mistake in sign can be avoided when applying this equation if one simply notes the direction of the spring force acting on the particle and compares it with the sense of direction of displacement of the particle — if both are in the *same sense*, *positive work* results; if they are *opposite* to one another, the *work is negative*.

- The kinetic energy at the initial and final points is *always positive*, since it involves the speed squared $\left(T = \frac{1}{2}mv^2\right)$.

- A force does work when it moves through a displacement in the direction of the force.

- Work is *positive* when the force component is in the *same sense of direction* as its displacement, otherwise it is negative.

- Forces that are functions of displacement must be integrated to obtain the work. Graphically, the work is equal to the area under the force-displacement curve.

Chapter 3:

Kinetics of a Particle

Topic # 3:

Impulse and Momentum

Principle of Linear Impulse and Momentum

In this section we will integrate the equation of motion with respect to time and thereby obtain the principle of impulse and momentum.

The resulting equation will be useful for solving problems involving force, velocity, and time.

Newton's Second Law

$$\sum \underline{\mathrm{F}} = m\underline{\mathrm{a}} = m\frac{\mathrm{d}\underline{\mathrm{v}}}{\mathrm{dt}}$$

$$\sum \int_{t_1}^{t_2} \underline{\mathrm{F}}dt = m\int_{v_1}^{v_2} d\underline{\mathrm{v}}$$

$$\boxed{\sum \int_{t_1}^{t_2} \underline{\mathrm{F}}dt = m\underline{\mathrm{v}}_2 - m\underline{\mathrm{v}}_1}$$

Impulse

change in momentum

This equation is referred to as the principle of linear impulse and momentum

Principle of Linear Impulse and Momentum

$$\sum \int_{t_1}^{t_2} \underline{\mathrm{F}}\, dt = m\underline{\mathrm{v}}_2 - m\underline{\mathrm{v}}_1$$

For problem solving, pervious equation will be rewritten in the form

$$\boxed{m\underline{\mathrm{v}}_1 + \sum \int_{t_1}^{t_2} \underline{\mathrm{F}}\, dt = m\underline{\mathrm{v}}_2}$$

- which states that the initial momentum of the particle at time t1 plus the sum of all the impulses applied to the particle from t1 to t2 is equivalent to the final momentum of the particle at time t2.

Principle of Linear Impulse and Momentum

$$\boxed{m\underline{\mathrm{v}}_1 + \sum \int_{t_1}^{t_2} \underline{\mathrm{F}}\, dt = m\underline{\mathrm{v}}_2}$$

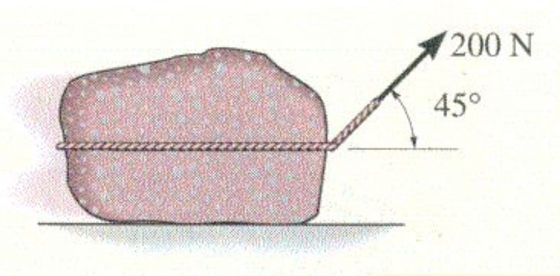
If each of the vectors in above Eq. is resolved into its x, y, z components, we can write the following three scalar equations of linear impulse and momentum.

$$m(v_x)_1 + \sum \int_{t_1}^{t_2} F_x \, dt = m(v_x)_2$$

$$m(v_y)_1 + \sum \int_{t_1}^{t_2} F_y \, dt = m(v_y)_2$$

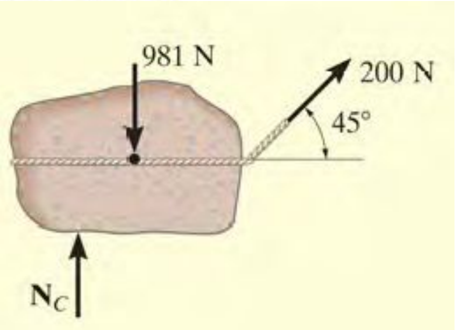
$$m(v_z)_1 + \sum \int_{t_1}^{t_2} F_z \, dt = m(v_z)_2$$

The 100-kg stone shown in Fig. is originally at rest on the smooth horizontal surface. If a towing force of 200 N, acting at an angle of 45°, is applied to the stone for 10 s, determine the final velocity and the normal force which the surface exerts on the stone during this time interval.



m=100 kg
At rest smooth
t=10 s
?=V_2
?=N

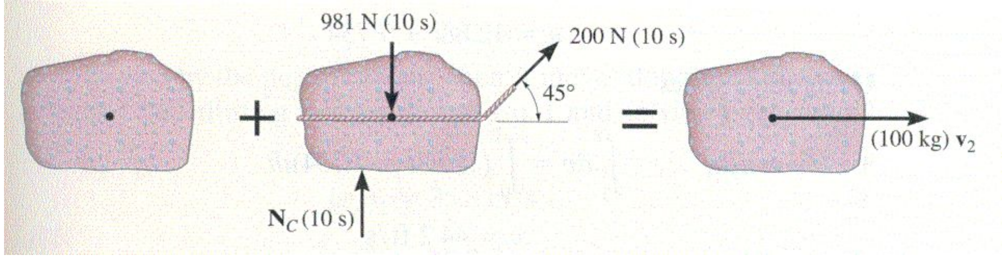
$$(\overset{+}{\rightarrow}) \quad m(\upsilon_x)_1 + \sum \int_{t_1}^{t_2} F_x \, dt = m(\upsilon_x)_2$$



$$0 + 200N \ \cos 45^o (10s) = (100\,kg)\upsilon_2$$

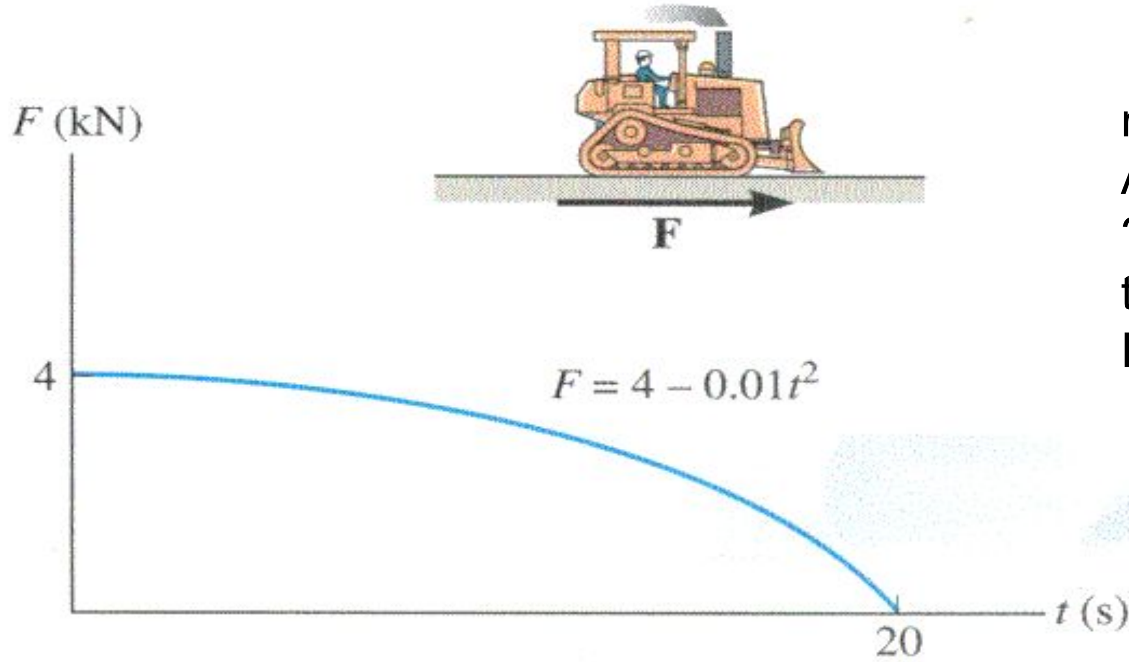
$$\upsilon_2 = 14.1\,m/s$$

$$(+\uparrow) \quad m(\upsilon_y)_1 + \sum \int_{t_1}^{t_2} F_y \, dt = m(\upsilon_y)_2$$



$$0 + N_c(10s) - 981N(10s) + 200N(10s)\sin 45^o = 0$$

$$N_c = 840\ N$$



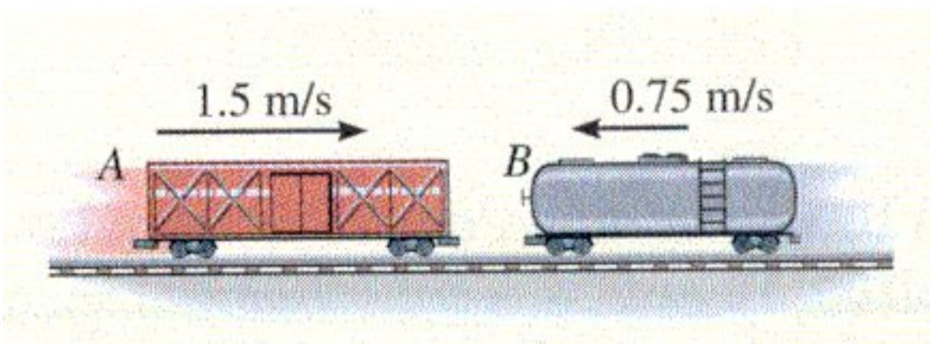
m = 28 Mg
At rest
? = V
t = 4 s
F = 4 - 0.01 t^2

$$m(\upsilon_x)_1 + \sum \int_{t_1}^{t_2} F_x \, dt = m(\upsilon_x)_2$$

$$0 + \int_0^4 (4 - 0.01t^2)\,(10^3)\,dt = 28(10^3)\upsilon_2$$

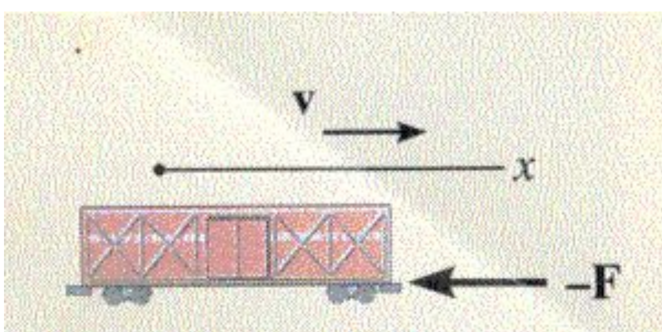
$$\upsilon_2 = 0.564\ m/s$$

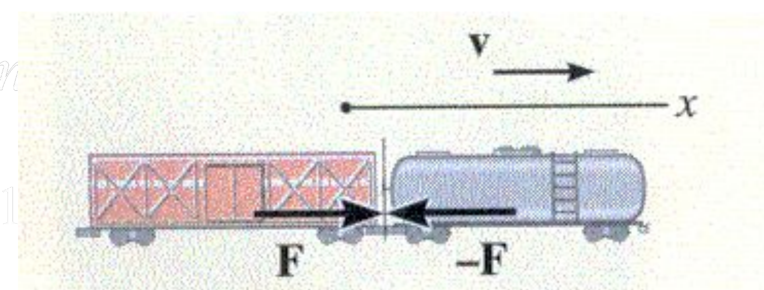
The 15-Mg boxcar A is coasting at 1 .5 mls on the horizontal track when it encounters a 12-Mg tank car B coasting at 0.75 mls toward it as shown in Fig. 15-Sa. If the cars collide and couple together, determine (a) the speed of both cars just after the coupling, and (b) the average force between them if the coupling takes place in O.S s.



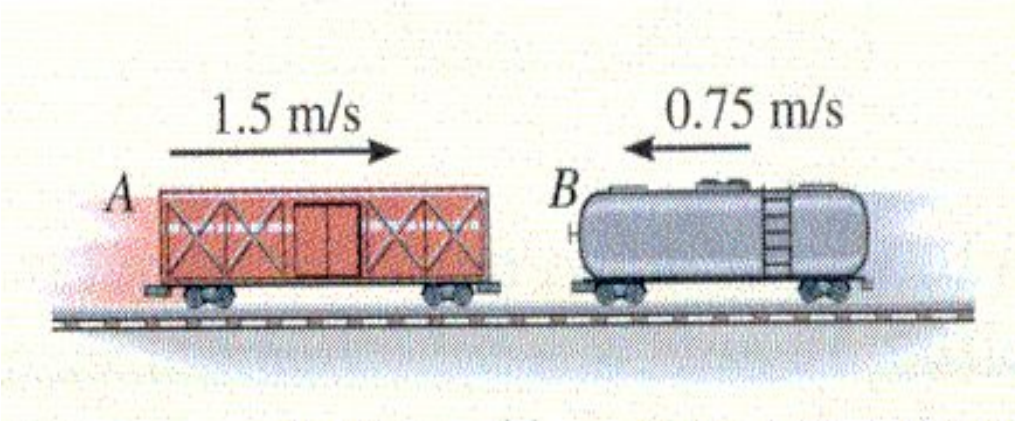
m_A=15 Mg
m_B=12 Mg
Couple together
V_2=? After coupling
F_{avg} = ? In 0.8 s

$v_2 = 0.5\ m/s$



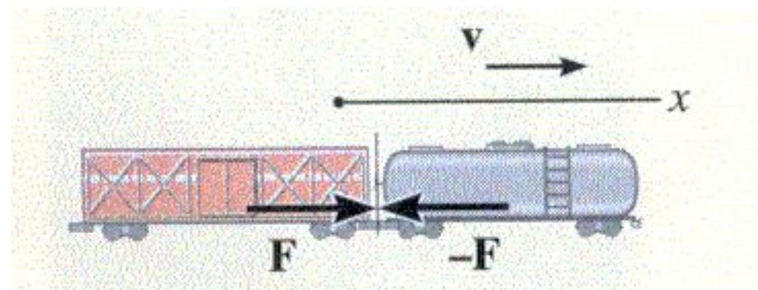


$F_{avg} = 18.8\ kN$



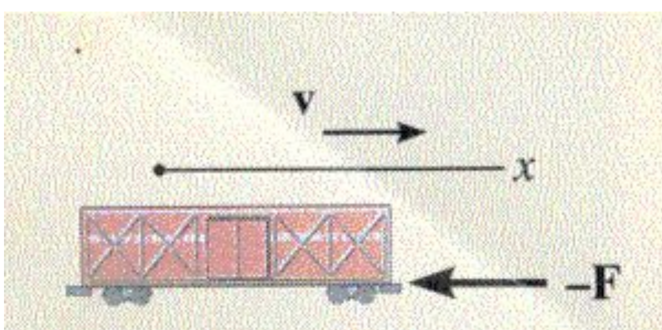
m_A=15 Mg
m_B=12 Mg
Couple together
V_2=? After coupling
F_{avg} = ? In 0.8 s

$$\left(\overset{+}{\rightarrow}\right) \quad m_A(\upsilon_A)_1 + m_B(\upsilon_B)_1 = (m_A + m_B)\upsilon_2$$



$$(15000)(1.5) - (12000)(0.75) = (27000)\,\upsilon_2$$

$$\upsilon_2 = 0.5 \;\; m/s \quad \rightarrow$$



$$(\overset{+}{\rightarrow}) \quad m(\upsilon_p)_1 + F_{avg}(t) = m(\upsilon_p)_2$$

$$(15000)(1.5) - F_{avg}(0.8) = (15000)(0.5)$$

$$F_{avg} = 18.8\,kN$$

Engineering Mechanics

Part II: Dynamics

Prof. Dr. Mahmoud S. Youssef
Dr. Bahaa Saleh